# Teaching Slope Conceptually For CST And Common Core Success

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## Presentation Goals

- Attendees will better understand what students are expected to know about slope under the outgoing California Standards and the incoming Common Core State Standards.
- Attendees will review a lesson that you can do with your own students to begin to prepare them to conceptually understand slope as defined under both the California Standards and Common Core Standards.
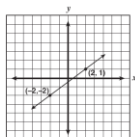
## What Would Students Say?

- If at the end of the school year you asked 7th grade students "What is slope?", what answers do you think you would get?

## Slope in California Content Standards

- 7.AF 3.3 - Graph linear functions, noting that the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.
- 7.AF 3.4 - Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.
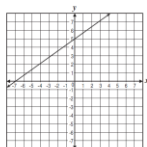
## CST Released Test Questions



Source: http://www.cde.ca.gov/ta/tg/sr/css05rtq.asp

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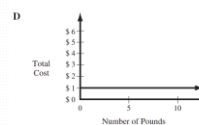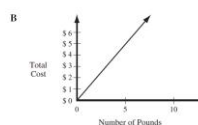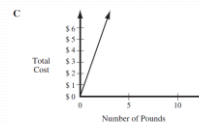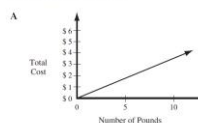
## Slope in Common Core Standards

- 8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- 8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.
- 8.SP.3 - Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

## Common Core Slope Examples

- The graphs below show the cost *y* of buying *x* pounds of fruit. One graph shows the cost of buying *x* pounds of peaches, and the other shows the cost of buying *x* pounds of plums.
- Which kind of fruit costs more per pound? Explain.
- Bananas cost less per pound than peaches or plums. Draw a line alongside the other graphs that might represent the cost *y* of buying *x* pounds of bananas.
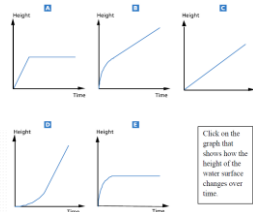
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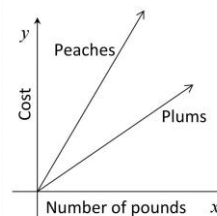
## Common Core Slope Examples

5. **Water Tank**

A water tank has shape and dimensions as shown in the diagram.

At the beginning the tank is empty. Then it is filled with water at the rate of one litre per second.

Click on the graph that shows how the height of the water surface changes over time.

Source: "Mathematics Content Specifications" from
http://www.smarterbalanced.org/smarter-balanced-assessments/

## Materials Required

- Required
  - Rulers
  - Pencils
  - Class sets of printouts
- Highly Recommended
  - Geoboards (11 x 11 pins)
  - Projection method
- Optional
  - Tension rod

## Expectations on Student Discourse

- Students should have strong oral and written explanations throughout the lesson.
- They may be hesitant to respond so a Think-Pair-Share strategy works well.
- Ask students which grind rail is the steepest and have them answer by incorporating the terms "vertical change" and "horizontal change" into a sentence like, "I think Rail #____ is the steepest because _____."
- The next student can respond with, "I agree/disagree that Rail #____ is the steepest because _____."
- This supports Math Practice 3 – Construct viable arguments and critique the reasoning of others.

## Examples of Student Responses

- These student responses give you an idea of what to expect.

Rank **3, 2, 1. 3 is the steepest because it's closer to becoming an angle.**

Rank the grind rails above from most steep to the least steep and explain why you ranked them that way:

**The first steepest is Rail 3, then #2, and lastly #1. Rail #3 is the steepest because it's the closest one to being verticle and its more deep.**

| Grind Rail #1 | Grind Rail #2 | Grind Rail #3 |
|---|---|---|
| Vertical change: 9 | Vertical change: 9 | Vertical change: 9 |
| Horizontal change: 10 | Horizontal change: 9 | Horizontal change: 8 |

Rank the grind rails above from most steep to the least steep and explain why you ranked them that way:

**The steepest grind rail is #3 the absolute value is the smallest the smaller the number at a bottom and a larger number on the top is the steepest line out of the grind rail.**

## Addressing Slope Misconceptions

- Student misconceptions to be on the lookout for include:
  - The length of a line segment affect's its slope.
    - Addressed by the treadmill pictures and Sheets 2 and 3
  - A line segment's vertical change does not affect its slope.
    - Addressed by Sheet 4 (you may need to use Geoboards too)
  - A line segment's horizontal change does not affect its slope.
    - Addressed by Sheet 5 (you may need to use Geoboards too)
  - Steepness can be defined solely by looking at the horizontal or vertical change.
    - Addressed by Sheets 2 and 3
- You may need to go back and forth between these sheets to address and readdress the misconceptions as they come up.

## Introducing Slope

- At some point in the lesson, usually during Sheets 4 or 5, students begin to realize that visually looking at a line segment is not always enough to determine steepness.
- They will then be looking for a way to measure steepness that always works, and this is the time to introduce slope.
- Generally classes fall into one of three categories:
  - The students <u>have</u> learned about slope before.
  - The students <u>have not</u> learned about slope but they <u>realize</u> they can compare the horizontal and vertical changes using a ratio or fraction.
  - The students <u>have not</u> learned about slope and they <u>do not realize</u> they can compare the horizontal and vertical changes using a ratio or fraction.
- Once slope has been introduced, go back to each of the pages and find the slope

## Grind Rail Questionnaire Goals

- Question 1 - Provide a working definition for slope which includes:
  - A measure of a line's steepness
  - Vertical change over horizontal change
  - Rise over run
- Question 2 – Apply the concept of slope to each grind rail including:
  - Build a connection between the vertical and horizontal changes and the slope
  - How to calculate slope from the vertical and horizontal changes
  - How to explain, using words, what each part of the slope represents
- Question 3 – Determine the slope of a line segment from two points including:
  - Finding the vertical and horizontal changes without counting by ones
  - More generally developing a foundation for the slope formula
  - The slope formula should be introduced here if it hasn't already come up
- Question 4 – Apply the concept of slope to each grind rail including:
  - Draw a line segment if given a slope
  - How to explain, using words, what they drew

## Tips for Using Geoboards

- Make sure that students are making a full slope triangle rather than just the grind rail.
- Help students understand how the vertical and horizontal changes are related to the triangle's sides.



## Lessons I Learned the Hard Way

- Many students lack knowledge we will need including:
  - What steepness means
  - What to count (dots or spaces) when counting vert. / hor. change.
  - How to compare fractions
- Geoboards greatly help struggling students visualize differences in steepness between similar slopes, but they also make the lesson take longer.
- Students may not realize that the vertical and horizontal changes need to be compared in a ratio, fraction, or other mathematical comparison.
- Students may think lines always have to go through the dots as opposed to just stopping and starting at a dot.
- Give students a paper printout of Sheets 2-5 as opposed to only having them view it off a screen.
- For struggling students, you may want to do one sheet at a time rather than letting them work on them all before discussing them.

## Information

- Download:
  http://robertkaplinsky.com/cmcs-slope

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