## Hall County Schools <br> Robert Kaplinsky July 30,2014




DOUBLE-DOUBLE

## 2004-10-31

YOUR GUEST NUMBER IS
98

$$
\begin{aligned}
& \text { IN-N=OUT BURGER LAS VEGAS EASTERN } \\
& 2004=10-31 \\
& 1651598 \\
& 8: 21 \text { PM }
\end{aligned}
$$

Cashier: SAM

## GLEST <br> \#: 98

## Counter-Eat In

 DblDbl98 Meat Pty KChz
2.65
88.20

Counter-Eat In
TAX 7.50 x
90.85

Amount Due
6.81

CASH TENDEA
Change
$\$ 97.66$
$\$ .00$
$2004-10-31$

## Cashier: SAM

## GLEST <br> H: 98

## Counter-Eat In

Dblobl

98 Meat Pty XChz

2.65
88.20

Counter-Eat In TAX 7,50x 90.85

Amount Due
6.81
97.66

CASH TENOER Change
$\$ 97.66$ $\$ .00$

2008-10-31

$$
8: 21 \text { PM }
$$

|  |  | ¢ |
| :---: | :---: | :---: |
| Hamburger w/Onion | 243 | 390 |
| Cheeseburger w/Onion | 268 | 480 |
| Double-Double w/Onion | 330 | 670 |



|  |  |
| :--- | :--- |
| Layers | Cost |
| 1 | $\$ 1.75$ |
| 2 | $\$ 2.65$ |
| 3 | $\$ 3.55$ |
| 4 | $\$ 4.45$ |
| $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ |
| 20 | $\$ 18.85$ |
| $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ |
| 100 | $\$ 90.85$ |
| $\cdot$ | $\cdot$ |
| $\cdot$ | $\$ 1.75+(\mathrm{N}-1)^{*} \$ 0.90$ |
| N |  |

bun + produce + meat + cheese + meat + cheese $=\$ 2.65$
bun + produce + meat + cheese
= \$1.75
meat + cheese $=\$ 0.90$

## The Reality

- Students needed guidance to figure out a layer's cost
- Not every class is ready to go straight to $100 \times 100$
- Common wrong answers included:
- $\$ 175.00$ ( $\$ 1.75 \times 100$ cheeseburgers)
- \$132.50 (\$2.65 x 50 Double-Doubles)
- Students had equations that had more than X patties
- Students were surprised to see three different equations:
- Starting with a Double-Double
- Starting with a cheeseburger
- Starting with produce and bun only
 NHA


## Math Question

- Solve the problem on your own. Do not work or share your answer with anyone else.
- You will have 30 seconds to complete it.
- Write your answer down on a paper.
- Pay attention to the emotions you feel while solving the problem and write those emotions down as well.

$$
\begin{gathered}
\text { There are } 125 \\
\text { sheep and } 5 \text { dogs } \\
\text { in a flock. How old } \\
\text { is the shepherd? }
\end{gathered}
$$

## Of the 32 students I interviewed...

- $75 \%$ of them gave me numerical responses
- 2 students calculated the answer to be $130(125+5)$
- 2 students calculated the answer to be 120 (125-5)
- 12 students calculated the answer to be $25(125 \div 5)$
- 0 students calculated the answer to be 625 ( $125 \times 5$ )
- 4 students stated that they guessed their answer (90, 5,42 , and 50)
- 4 students tried to divide 125 by 5 but could not correctly implement the procedure


## Takeaways

- Making sense of mathematics
- Intellectual autonomy
- Intellectual autonomy is about being able to think for yourself and not being dependent on others for the direction and control of one's thinking.


## What Does the NHTSA Say?

## Kev Statistics and Consumer Insights:

- Motor vehicle crashes are the leading cause of death for children age 1 through 12 years old. ${ }^{1}$


## According to a NHTSA study, 3 out of 4 kids are not as secure in the car as they should be because their car seats are not being used correctly.

${ }^{1}$ Source: Based on the latest mortality data currently available from the CDC's National Center for Health Statistics.


- "because they have their child in the right seat"
- "because their car seats are not being used correctly"

IF YOUR CHILD IS IN THE RIGHT CAR SEAT.
Ad
council
VISIT SAFERCAR.GOV/THERIGHTSEAT


## KNOW FOR SURE

IF YOUR CHILD IS IN THE RIGHT CAR SEAT.

## VISIT SAFERCAR.GOV/THERIGHTSEAT

 NHTSA





## Solving Real-World Geometry Problems

## High School

- G-MG. 1 - Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- G-GMD. 3 - Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.


## Middle School

- 8.G. 9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
-7.G.6 - Solve real-world and mathematical problems involving area, volume and surface area of two- and threedimensional objects.
-6.G.2 -Apply the formulas $V=I$ wh and $V$ $=b h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.


## Elementary School

- 5.MD. 5 - Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
- 4.MD. 3 - Apply the area and perimeter formulas for rectangles in real world and mathematical problems
- 3.MD.7d - Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
- 2.MD. 1 - Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
- 1.MD. 2 - Express the length of an object as a whole number of length units.
- K.MD. 1 - Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.


# WHAT ISN'T MATHEMATICAL MODEHNG? 

- It is not modeling in the sense of, "I do; now you do."
- It is not modeling in the sense of using manipulatives to represent mathematical concepts.
- It is not modeling in the sense of a "model" being just a graph, equation, or function.
- It is not just starting with a real world situation and solving a math problem.
- It is not beginning with the mathematics and then moving to the real world.


# PROBLEM- • How often do teachers do 

 problem-based learning? lessons take?- Do teachers use problem-based lessons to introduce a topic or after you've already taught it?
- How is problem-based learning assessed?
- How much time does it take to create a problem-based lesson?


# The Four C's 

- Communication - Curiosity
- 6.G. 4 - Represent threedimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures.
7.G.6 - Solve real-world and mathematical problems involving area, volume and surface area.
- 8.G.9 - Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
- G-GMD.3-Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- G-c0. 6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.
- A-CED. 1 - Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- F-IF.7a - Graph linear and quadratic functions and show intercepts, maxima, and minima.
6.RP.2 - Understand the concept of a unit rate


## The Four C's

- Communication
- Curiosity
- Critical Thinking


## Problem Solving Framework

- Inspired by Geoff Krall's resources at emergentmath.com


## The Four C's

- Communication
- Curiosity
- Critical Thinking
- Content Knowledge


# WHAT DOES IT LOOK LIKE... 

- when students have procedural skill but not conceptual understanding or the ability to apply mathematics?
- when students struggle to process mathematics at a higher depth of knowledge?


# How far apart are the exits on this freeway: Jct 90 and Jefferson Blvd? 






## Questioning Scenarios

- The activity begins with teachers in groups of three taking the roles of teacher, student, or observer.
- The individuals playing the role of teacher and student each receive a slip of paper describing their scenario.
- The individual playing the role of observer waits to record all of the teacher's questions to the student.
- Once the activity begins, the teacher will talk to the student in the context of the scenario they read about on the slips of paper.

What did you get for the area of the circle with a radius of 2 units?

Great. Do you have any questions?

What did you get for the area of the circle with a radius of 2 units?

Great. How did you get your answer?

The radius is 2 so I plugged it into 2 pi $r$ and got 4 pi.

## Depth of Knowledge

 -What?
## Depth of Knowledge Examples

## Perimeter

- DOK 1 - What is the perimeter of a rectangle with that measures 8 units by 4 units?
- DOK 2 - List the dimensions of a rectangle with a perimeter of 24 units.
- DOK 3 - Of all the rectangles with a perimeter of 24 units, which one has the most area?

Surface Area

- DOK 1 - What is the surface area of a rectangular prism that measures 8 units by 4 units by 3 units?
- DOK 2 - List the dimensions of a rectangular prism with a surface area of 20 square units.
- DOK 3-Of all the rectangular prisms with a surface area of 20 square units, which one has the most volume?


## Depth of Knowledge

-What?

- How?
-Why?


## Depth of Knowledge - Level One

What is the circle's circumference? $\pi \approx 3.14$


$$
C=\pi d \text { or } C=2 \pi r
$$

$$
A=\pi r^{2}
$$

70 This circular stage has a radius of 25 meters.


Which equation could be used to find the area of the stage in square meters?

A $\quad A=25 \pi$
B $\quad A=50 \pi$
C $\quad A=\pi \cdot 25^{2}$
D $\quad A=\pi \cdot 50^{2}$

72 The top part of this hat is shaped like a cylinder with a diameter of 7 inches.


Which measure is closest to the length of the band that goes around the outside of the hat?

A 10.1 inches
B 11.0 inches
C 22.0 inches
D 38.5 inches

Source: $6^{\text {th }}$ Grade CST Released Test Questions - http://www.cde.ca.gov/ta/tg/sr/documents/cstrtqmath6.pdf


## Student Data Facts

- 396 seventh grade students were assessed
- 68.26\% correctly answered the circumference question
- $78.59 \%$ correctly answered the area question


# Mathematics Preliminary Summative Assessment Blueprint 

 Target Sampling Mathematics Grade 7-Table 6bAssessment Consortium

| Claim | Content Category | Assessment Targets | DOK | Minimum \# Scored Tasks |  | Minimum \# Items per Item Type |  | Min/Max Number of Items |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | CAT | PT/ECR | SR | CR |  |
|  |  | A. Analyze proportional relationships and use them to solve real-world and mathematical problems. | 1,2 | $p(9)=1.0$ |  |  |  |  |
| Cinnertiner |  | E. Draw, construct, and describe geometrical figures and describe the relationship between them. |  |  |  |  |  |  |
|  |  | F. Solve real-life and problems involvin area, surface area | $\begin{aligned} & \text { lath } \\ & \text { angl } \\ & \text { and } \end{aligned}$ | matic mea olum | al <br> ure, |  |  |  |
|  |  | problems involving angle measure, area, surface area, and volume. | 1,2 |  | 0 | 2 | 1 | 5/8 |
|  |  | G. Use random sampling to draw inferences about a population. | 1,2 | $p(2)=1.0$ |  |  |  |  |
|  |  | H. Draw informal comparative inferences about two populations. | 1,2 |  |  |  |  |  |
|  |  | I. Investigate chance processes and develop, use, and evaluate probability models. | 1,2 |  |  |  |  |  |



## Depth of Knowledge - Level Two

Which circle is bigger? How do you know?
Circle A
Circle B

Area $=36$ units $^{2}$

$$
\begin{array}{cc}
C=\pi \cdot 2 \cdot r & A=\pi \cdot r^{2} \\
36 \approx 6.28 \cdot r & A \approx 3.14 \cdot 5.73^{2} \\
\frac{36}{6.28} \approx r & A \approx 3.14 \cdot 32.83 \\
5.73 \text { units } \approx r & A \approx 103.15 \text { units }^{2}
\end{array}
$$

Circumference $=36$ units

## SBAC Constructed Response Rubric

- For full credit (2 points):
- Student reaches the correct conclusion. AND
- Student provides sufficient reasoning to support this conclusion.
- For partial credit (1 point):
- Student reaches the correct conclusion but does not provide sufficient reasoning to support this conclusion. OR
- Student does not reach the correct conclusion but provides reasoning to support this conclusion that contains a minor conceptual or computation error.



## Video Facts

- Of the ten students interviewed:
-Ten correctly answered both of the DOK 1 questions.
- One earned two points on the DOK 2 question.
- Six earned one point on the DOK 2 question.
- Three earned zero points on the DOK 2 question.


## Student Data Facts

- Of the 396 seventh grade students who were assessed, $12.12 \%$ earned two points on the DOK 2 question.
- $97.92 \%$ of the students who correctly answered the DOK 2 question also correctly answered both of the two DOK 1 questions.
- $10.61 \%$ of the students who correctly answered both of the two DOK 1 questions also correctly answered the DOK 2 question.


## More Student Data Facts

- $28.28 \%$ of the students earned only one point.
- All of them earned one point by choosing Circle $B$ and providing insufficient reasoning.
- 59.59\% of the students earned no points.


## Assessing Deeper Understanding

Make the smallest difference using the numbers 1 through 9 no more than one time each.



## STUDENT WORK




## Open Middle Problems

- Open middle problems require a higher depth of knowledge than most problems that assess procedural and conceptual understanding.
- They often have a "closed beginning" meaning that they all start with the same initial problem
- They often have a "closed end" meaning that they all end with the same answer
- They have an "open middle" meaning that there are multiple ways to approach and ultimately solve the problem


## www.openmiddle.com

## Problem-Based Lesson Resources

- Problem-based lesson search engine:


## http://robertkaplinsky.com/prbl-search-engine/

- My lessons: http://www.robertkaplinsky.com/lessons
- Dan Meyer: http://threeacts.mrmeyer.com
- Andrew Stadel: http://tinyurl.com/mrstadel
- Geoff Krall: http://tinyurl.com/PrBLmaps
- Dan Meyer's TED talk: http://tinyurl.com/meyer-TED


How Many Sheets Do You Need To Break Out Of Prison?
Goperations with rationalinumiberso ENTE


Robert graduated from University of


Math content expert

California, Los Angeles (UCLA) with a Bachelors of Science in Mathematics. He has taught mathematics to students at the

Lessons elementary, middle, and high school levels. As



How Much Is One Third Of A Cup Of Butter?



How Do Skytypers Write Messages?


Robert Kaplinsky's Problem-Based Lessons
File Edit View Insert Format Data Tools Help All changes saved in Drive
두
\$ $\% \quad 123$
Arial
10
$\mathrm{B} \quad I \quad \mathrm{~A}$

- 田
 $\Sigma$


## Task Name

How Can We Water All Of The Grass?
How Much Money IS That?!
How Much Money Should Dr. Evil Demand?
How Tall Is Mini-Me?
How Did They Make Ms. Pac-Man?
Which Ticket Option Is The Best Deal?
How Far Apart Are The Freeway Exits?
Do We Have Enough Paint?
How Many Stars Are There In The Universe?
What Rides Can You Go On?
Do You Have Enough Money?
Which Bed Bath \& Beyond Coupon Should You Use?
Is Gas Cheaper With Cash Or Credit Card?
Where's The Nearest Toys R Us?
How Sharp Is The iPhone 5's Retina Display?
When Should She Take Her Medicine?
How Biq Are Sunspots?
What Michael's Coupon Should I Use?
Is It Cheaper To Pay Monthly or Annually?
How Biq Is The 2010 Guatemalan Sinkhole?
How Can You Win Every Prize At Chuck E. Cheese's?
How Many Royal Flushes Will You Get?
How Much Does The Paint On A Space Shuttle Weigh?
How Did Motel 6 Go From $\$ 6$ to $\$ 66$ ?
How Much Does The Aluminum Foil Prank Cost?
How Many Laps Is A 5k Race?
Which Toilet Uses Less Water?
How Did Someone Get A \$103,000 Speeding Ticket In Finland? Which Pizza Is A Better Deal?
How Biq Is The World's Largest Deliverable Pizza?
How Many Sheets Do You Need To Break Out Of Prison?
Do Hybrid Cars Pay For Themselves?
How Many Hot Dogs Did They Eat?!
How Much Purple Ribbon Will You Need? Are We There Yet?
Which Chinese Food Coupon Should I Use?
How Biq Is The Vehicle That Uses Those Tires?
Where Would The Angry Birds Have Landed?
How Many Movies Can You See In One Day?
Which Carrots Should You Buy?
How Fast Can You Throw A Baseball?

| B | c | D | E | F |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Concept / Skill | Standard 1 | Standard 2 | Standard 3 | Standard 4 | St |
| Circles, Pythagorean Theorem, trigonometric ratios | 7.G. 4 | 8.G. 7 | G-SRT. 8 | G-MG. 1 | G |
| Volume of rectangular prism | 5.MD. 3 | 5.MD. 4 | 5.MD. 5 | 5.MD.5b | 5. |
| Exponential Growth | N-RN. 2 | A-SSE. 1 | A-SSE.3c | A-SSE. 4 | A- |
| Scale and Dividing Decimals | 5.NF. 5 | 5.NF.5a | 5.NF.5b | 6.NS. 3 |  |
| Transformations (Rotations, Reflections, and Translations) | 8.G. 1 | 8.G. 2 | 8.G. 3 | 8.G. 4 | G |
| Unit Rates and Ratios | 6.RP. 2 | $6 . \mathrm{RP} .3$ | 6.RP.3a | 6.RP.3b |  |
| Fractions on a Number Line and Subtracting Fractions | 3.NF. 2 | 3.NF.2b | 4.NF. 2 | 4.NF.3a | 4. |
| Area | 3.MD. 5 | 3.MD. 6 | 3.MD. 7 |  |  |
| Scientific Notation | 8.EE. 3 | 8.EE. 4 |  |  |  |
| Inequalities and Measurement | 2.MD. 1 | 6.NS.7a | 6.NS.7b |  |  |
| Money | 2.MD. 8 |  |  |  |  |
| Percent Discount | 7.RP. 3 |  |  |  |  |
| Percent Discount | 7.RP. 3 |  |  |  |  |
| Pythagorean Theorem (Distance in coordinate system) | 8.G. 8 | G-SRT. 8 | G-GPE. 7 |  |  |
| Pythagorean Theorem (Length of a side) | 8.G. 7 | G-SRT. 8 | G-GPE. 7 |  |  |
| Operations with Time Intervals | 4.MD. 2 |  |  |  |  |
| Converting Units, Proportions, and Scientific Notation | 5.MD. 1 | 7.RP. 2 | 7.G. 4 | 8.EE. 4 | G |
| Percent Discount | 7.RP. 3 | A-CED. 3 |  |  |  |
| Decimal Operations and/or Systems of Equations | 5.NBT. 7 | 8.EE.8c | A-CED. 3 | A-REI. 11 | F- |
| Volume of Cylinder | 5.MD. 3 | 5.MD. 4 | 5.MD. 5 | 8.G.9 | G |
| Decomposing Numbers and/or Systems of Equations | 2.NBT. 7 | 3.NBT. 2 | 3.NBT. 3 | 8.EE.8c | A- |
| Probability | 7.SP. 5 | 7.SP. 6 | 7.SP. 7 | S-MD. 5 | S- |
| Surface Area | 6.G.4 | 7.G. 6 | 8.G. 7 | G-MG. 1 | G |
| Percent Increase and Compound Interest | 7.RP. 3 | A-SSE. 1b | F-BF. 1 | F-IF.8b | F- |
| Surface Area and Unit Rates | 6.G.4 | 6.RP. 2 | 6.RP. 3 | 7.G.6 |  |
| Perimeter | 4.MD. 3 |  |  |  |  |
| Systems of Equations/Inequalities | 8.EE.8c | A-CED. 3 | A-REI. 11 | F-BF. 1 |  |
| Linear Equations | A-CED. 2 | F-BF. 1 | F-IF. 4 | F-IF. 6 |  |
| Area or Circle, Square, and Unit Rates | 3.MD. 5 | 3.MD. 6 | 3.MD. 7 | 4.MD. 3 | 6. |
| Area of Square | 3.MD. 5 | 3.MD. 6 | 3.MD. 7 | 4.NBT. 3 | 4.1 |
| Integer Operations | 5.NBT. 6 |  |  |  |  |
| Systems of Equations or Rates | 6.RP. 2 | 6.RP. 3 | 8.EE.8c | A-CED. 3 | F- |
| Linear and Quadratic Functions | 8.F. 3 | 8.F. 4 | F-BF. 1 | F-BF. 2 | F- |
| Perimeter \& Circumference | 3.MD. 8 | 4.MD. 3 | 7.G. 4 |  |  |
| Adding Times | 3.MD. 1 | 4.MD. 2 |  |  |  |
| Percent Discount | 7.RP. 3 |  |  |  |  |
| Ratio and Proportions | 7.RP. 2 |  |  |  |  |
| Create Equation From Quadratic Graph | A-CED. 1 | F-BF. 1 | F-IF. 4 | F-IF.7a | F-L |
| Adding Times | 3.MD. 1 | 4.MD. 2 |  |  |  |
| Unit Rates | 6.RP. 1 | 6.RP. 2 | 6.RP. 3 |  |  |
| Converting Units and Unit Rates | 5.MD. 1 | 6.RP. 2 |  |  |  |



