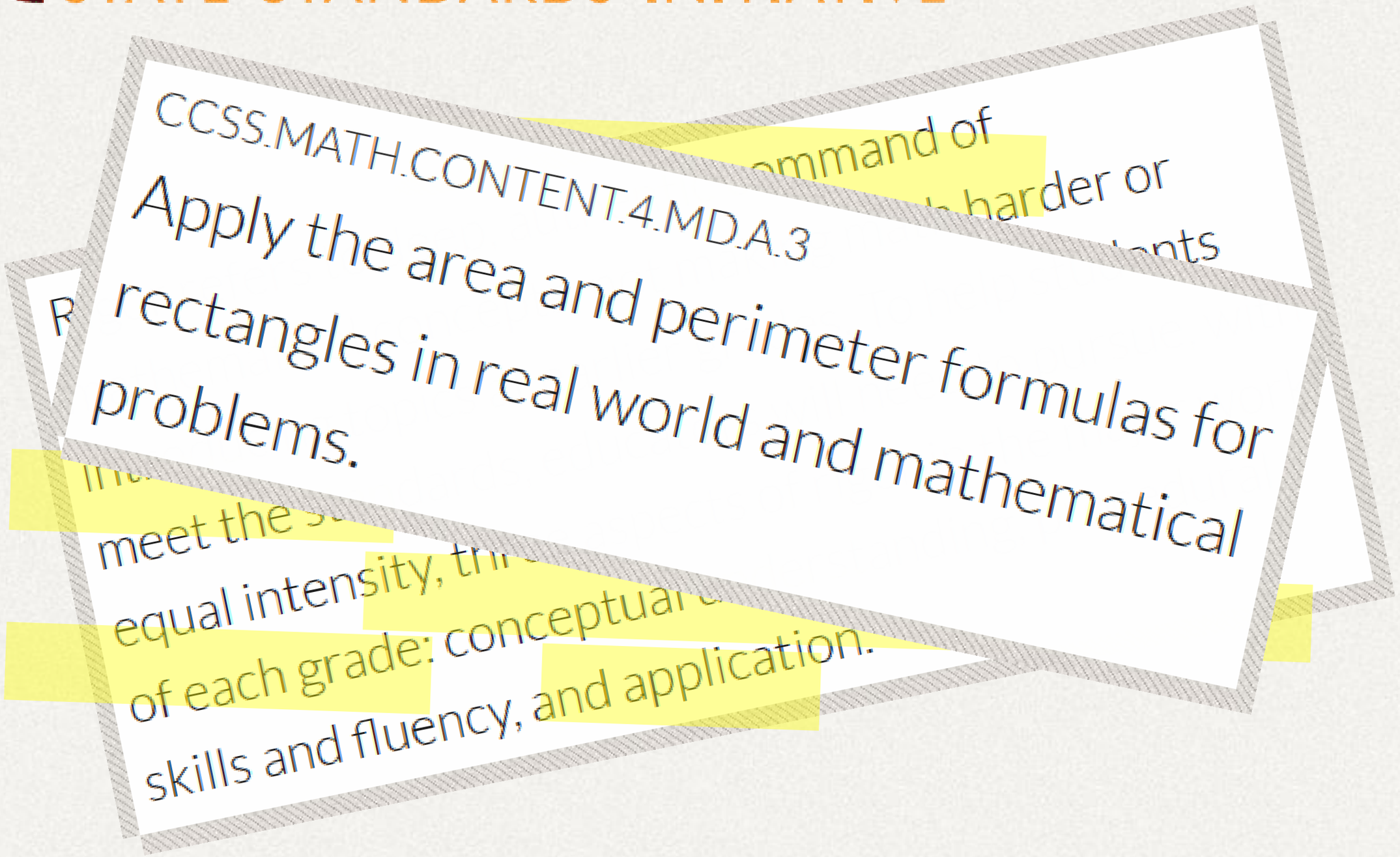
Differentiating with Depth of Knowledge

ROBERT KAPLINSKY

@robertkaplinsky

COMMON CORE
STATE STANDARDS INITIATIVE



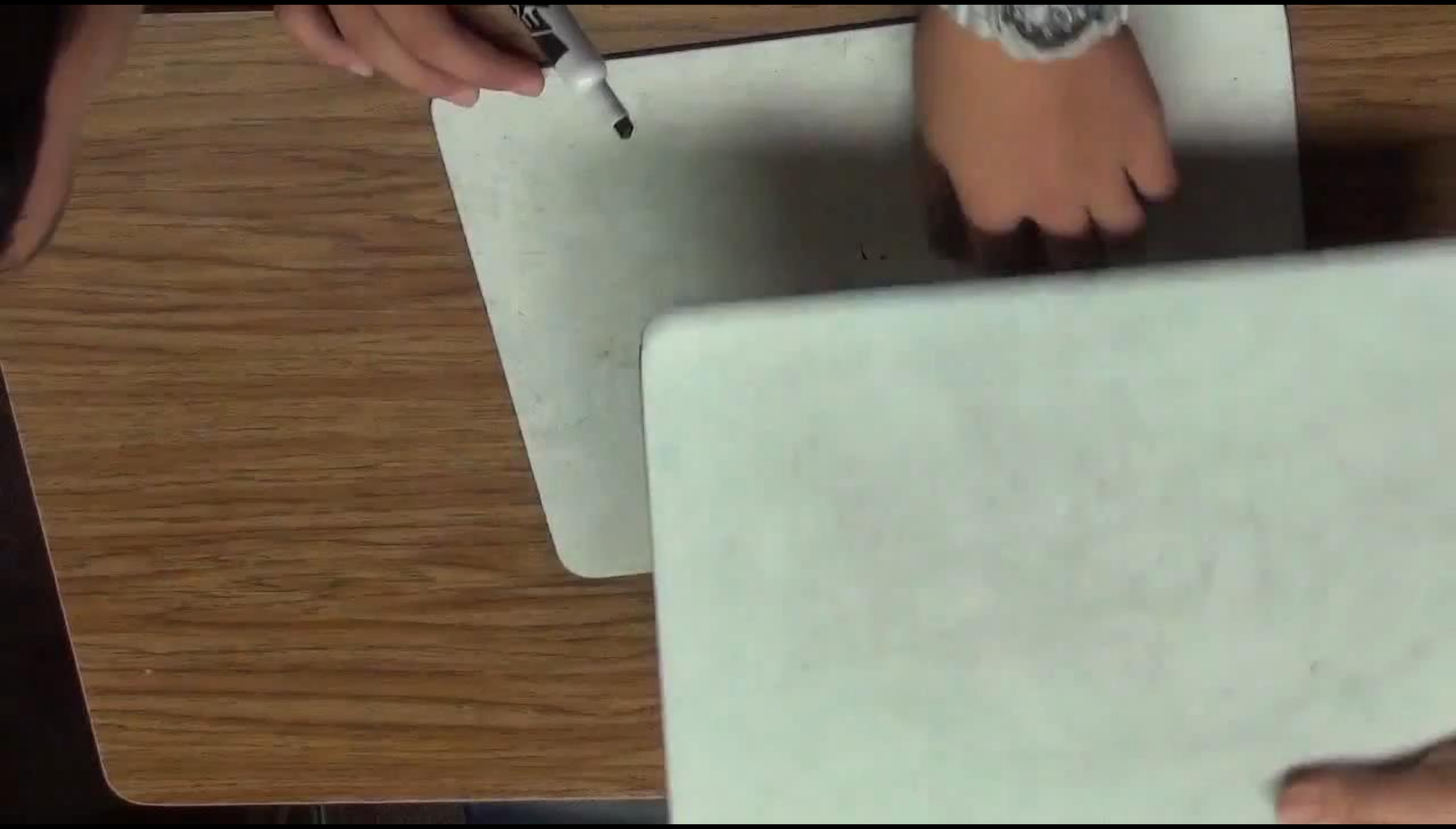
Source: http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

What is the perimeter of a rectangle that measures 8 units by 4 units?

Three Components of Rigor

- ☑ Procedural Skill and Fluency
- ☑ Conceptual Understanding
- ☒ Application

List the dimensions of a rectangle with a perimeter of 24 units.



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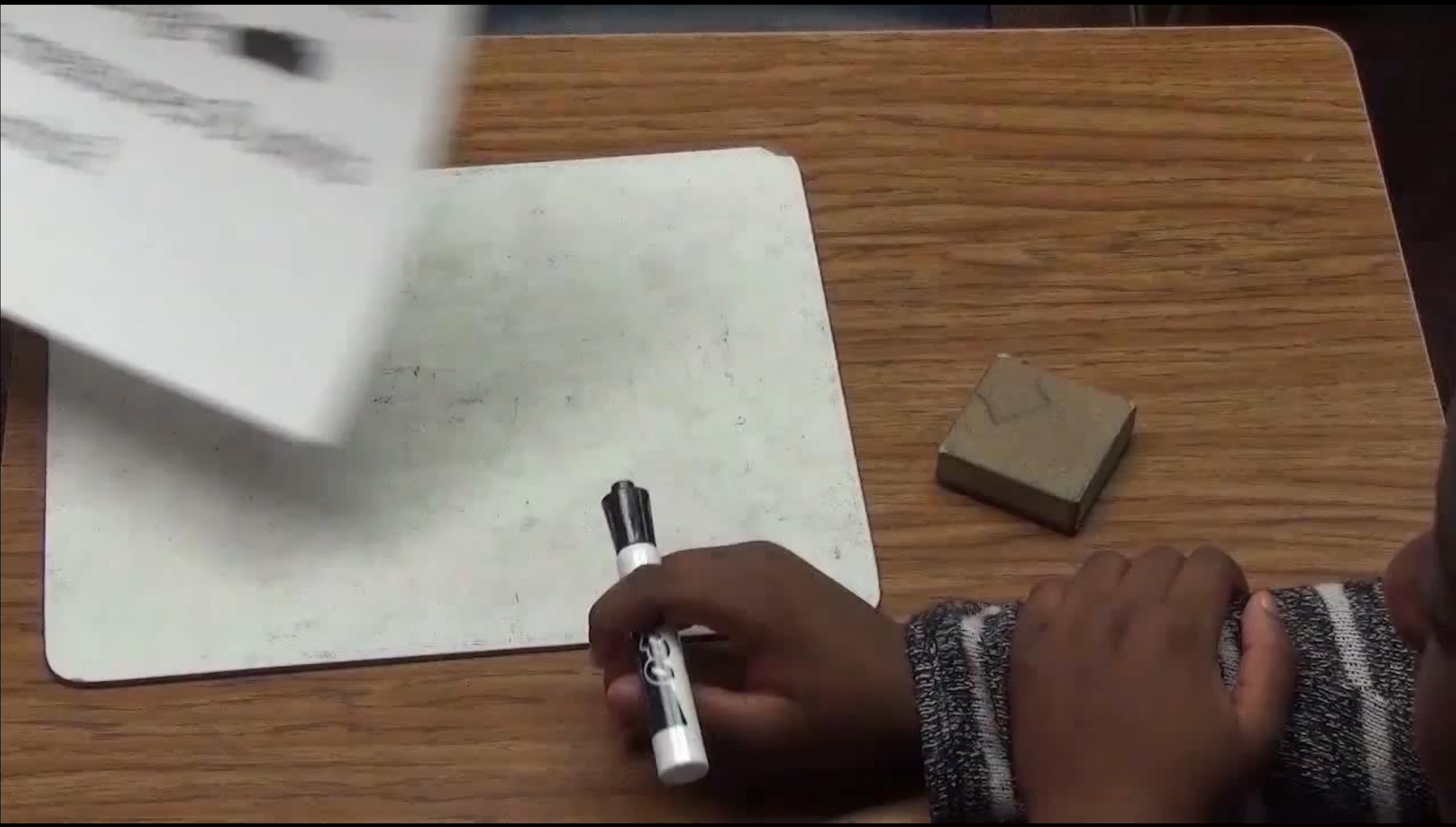
WHY?



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of a rectangle with a perimeter of 24 units.

Three Components of Rigor

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What is the perimeter of a rectangle [redacted] that measures 8 units by 4 units?

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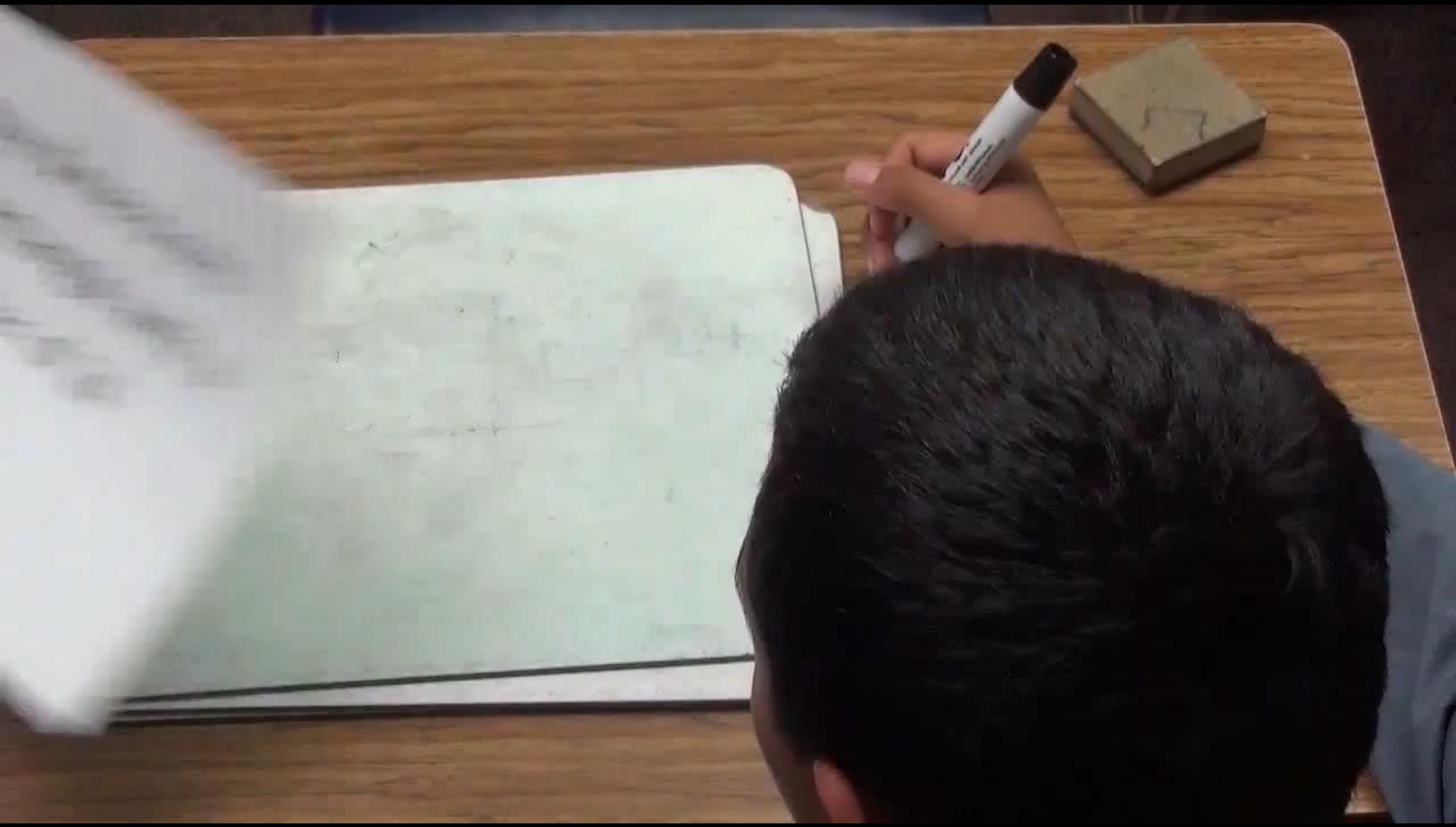

Three Components of Rigor

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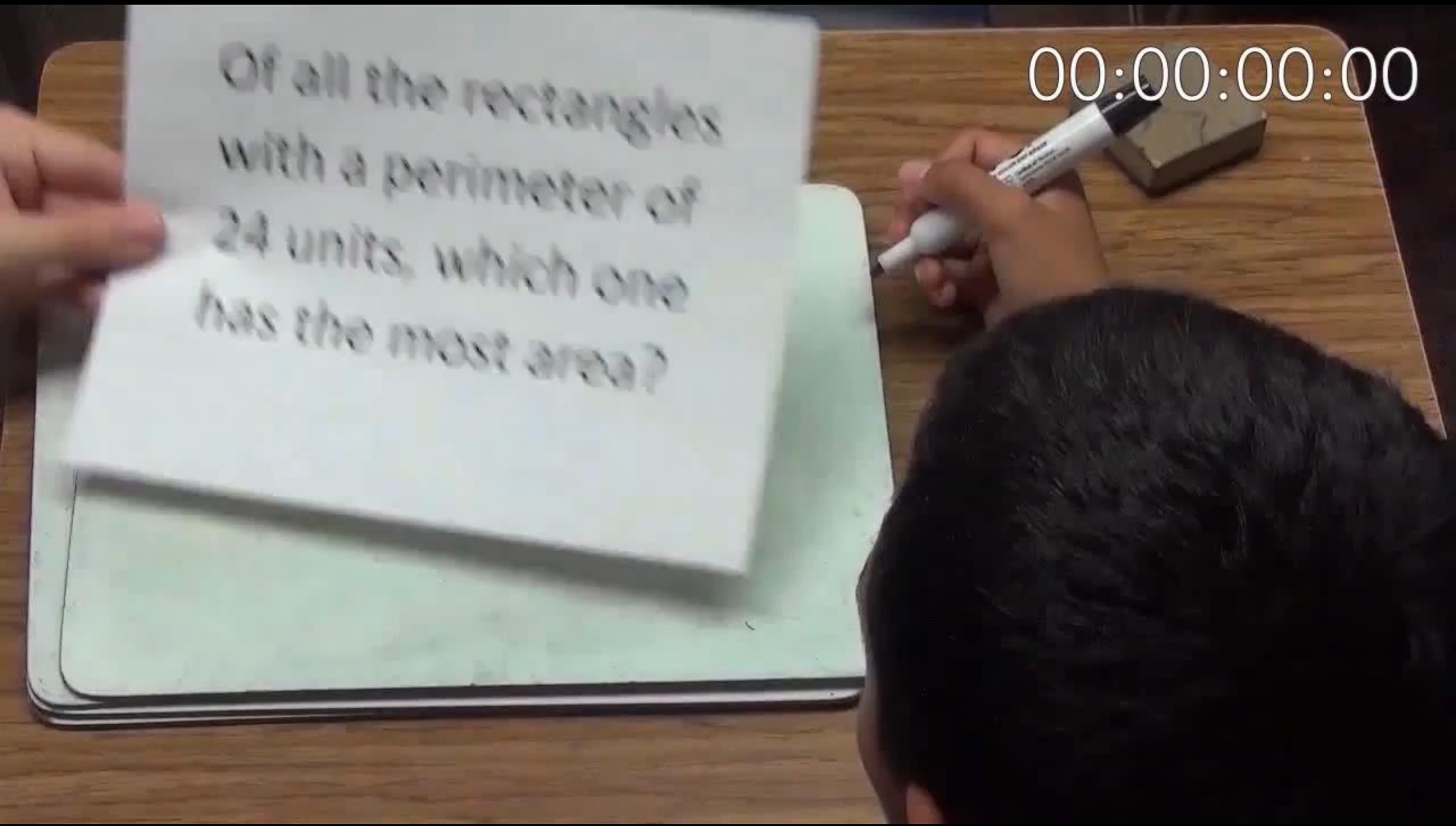
- ☒ Application

Of all the rectangles with a perimeter of 24 units, which one has the most area?


Of all the rectangles with a perimeter of 24 units, which one has the most area?
00:00:00:00

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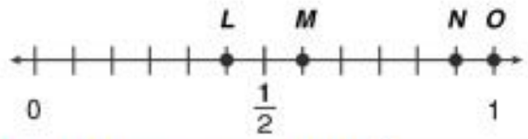
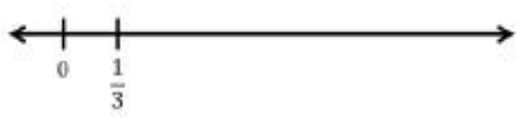
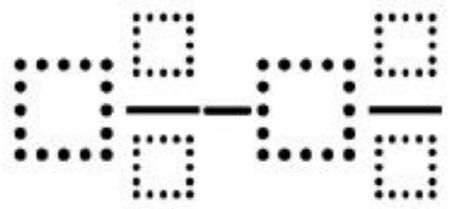
Defining the Problem

- Students appear to demonstrate "deep, authentic command of mathematical concepts" when given commonly used problems.
- However with more challenging problems, the same students seem to no longer demonstrate that command.

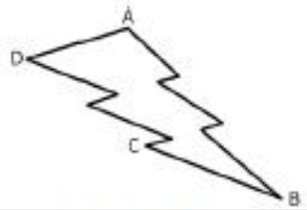
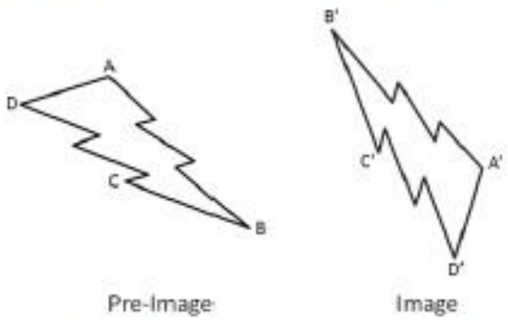
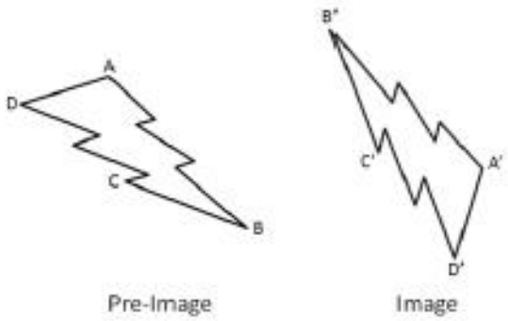
Addressing the Problem

- First, we must have a clear understanding about why these problems are different from one another.
- Next, we need to practice implementing these problems such that all students are engaged in a problem that is at the right challenge level for them.
- Last, we need a source that can provide us with a variety of free problems.

DOK | Distinguishing Between Depth of Knowledge Levels in Mathematics

Topic	Adding Whole Numbers	Money	Fractions on a Number Line	Area and Perimeter	Subtracting Mixed Numbers
CCSS Standard(s)	• 1.NBT.4 • 2.NBT.5	• 2.MD.8	• 3.NF.2	• 3.MD.8 • 4.MD.3	• 5.NF.1
DOK 1 Example	Find the sum. $44 + 27 =$	If you have 2 dimes and 3 pennies, how many cents do you have	Which point is located at $\frac{7}{12}$ below? 	Find the perimeter of a rectangle that measures 4 units by 8 units.	Find the difference. $5\frac{1}{2} - 4\frac{2}{3} =$
DOK 2 Example	Fill in the boxes below using the whole numbers 1 through 9, no more than one time each, so that you make a true equation. $\square\square + 53 = \square\square$	Make 47¢ in three different ways with either quarters, dimes, nickels, or pennies.	Label the point where $\frac{3}{4}$ belongs on the number line below. Be as precise as possible. 	List the measurements of three different rectangles that each has a perimeter of 20 units.	Create three different mixed numbers that will make the equation true by using the whole numbers 1 through 9, no more than one time each,. You may reuse the same whole numbers for each of the three mixed numbers. $5\frac{4}{5} - \square\frac{\square}{\square} = 3\frac{1}{20}$
DOK 3 Example	Make the largest sum by filling in the boxes below using the whole numbers 1 through 9, no more than one time each. $\square\square + \square\square =$	Make 47¢ using exactly 5 coins with either quarters, dimes, nickels, or pennies.	Create 5 fractions using the whole numbers 0 through 9, no more than one time each, as numerators and denominators and correctly place them all on a number line.	What is the greatest area you can make with a rectangle that has a perimeter of 24 units?	Make the smallest difference by filling in the boxes below using the whole numbers 1 through 9, no more than one time each. 

DOK | Distinguishing Between Depth of Knowledge Levels in Mathematics

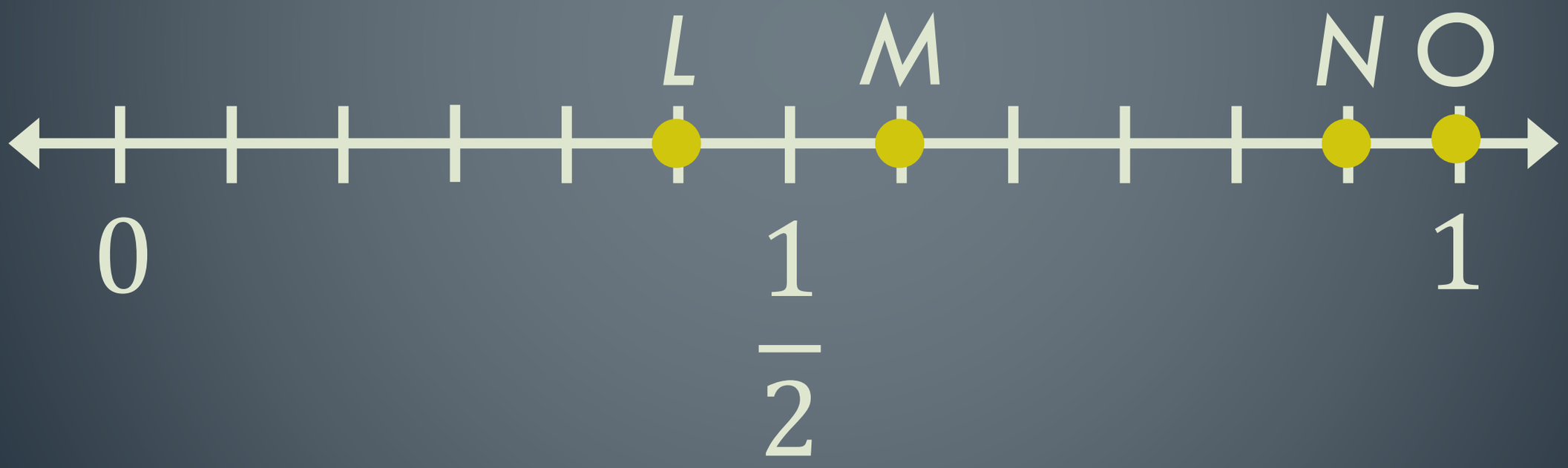
Topic	Surface Area and Volume	Probability	Transformations	Quadratics in Vertex Form
CCSS Standard(s)	• 6.G.4 • 7.G.6	• 7.SP.5 • 7.SP.7	• 8.G.1 • G-CO.5	• F-IF.7a
DOK 1 Example	Find the surface area of a rectangular prism that measures 3 units by 4 units by 5 units.	What is the probability of rolling a sum of 5 using two 6-sided dice?	Rotate the image below 90° counterclockwise and reflect it across a horizontal line. 	Find the roots and maximum of the quadratic equation below. $y = 3(x-4)^2 - 3$
DOK 2 Example	List the measurements of three different rectangular prisms that each has a surface area of 20 square units.	What value(s) have a 1/12 probability of being rolled as the sum of two 6-sided dice?	List three sequences of transformations that take pre-image ABCD to image A'B'C'D'. 	Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values.
DOK 3 Example	What is the greatest volume you can make with a rectangular prism that has a surface area of 20 square units?	Fill in the blanks to complete this sentence using the whole numbers 1 through 9, no more than one time each. Rolling a sum of ___ on two ___-sided dice is the same probability as rolling a sum of ___ on two ___-sided dice.	What is the fewest number of transformations needed to take pre-image ABCD to image A'B'C'D'? 	Create a quadratic equation with the largest maximum value using the whole numbers 1 through 9, no more than one time each. $y = -\square(x-\square)^2 + \square$

DOK Level Differences

- **Level 1: Recall & Reproduction**
  - Often a trivial application of facts.
  - Requires little to no cognitive effort beyond remembering the right formula.
  - Usually only one answer.
- **Level 2: Skills & Concepts**
  - Usually requires more than one step to solve.
  - Often multiple answers.
- **Level 3: Strategic Thinking**
  - Usually requires critical thinking about the best way to approach a problem.
  - May be multiple answers or a single optimal answer.
  - Often challenging enough to make your head hurt.
- **Level 4: Extended Thinking**
  - In mathematics these are generally represented by performance tasks or problem-based lessons.

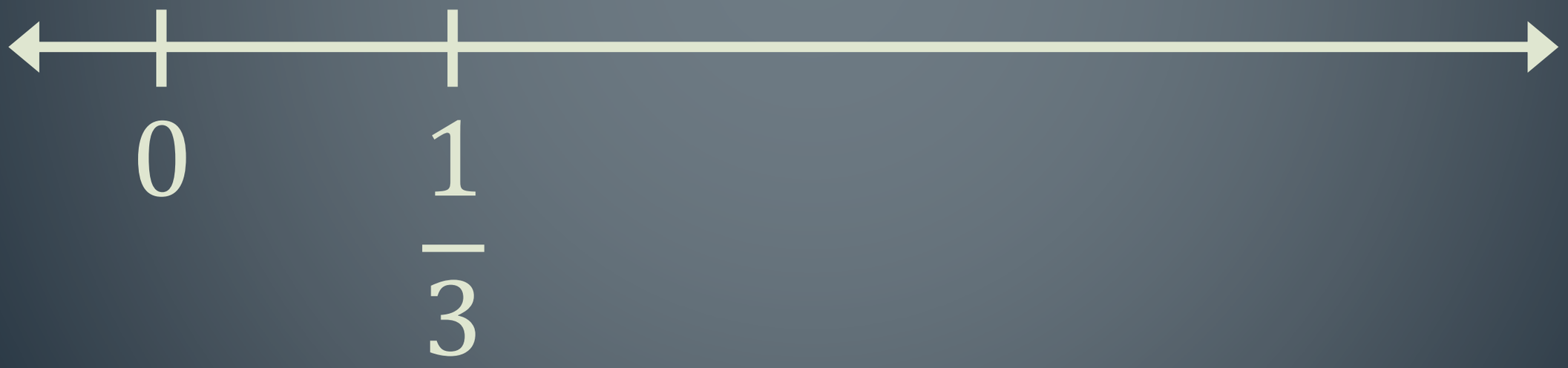
Fractions on a Number Line

Which point is located at $\frac{7}{12}$ below?



Fractions on a Number Line

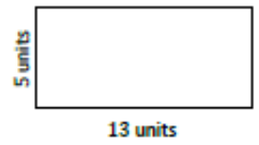
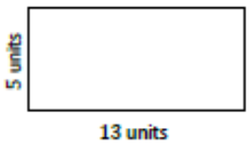
Label the point where $\frac{3}{4}$ belongs on the number line below. Be as precise as possible.



Fractions on a Number Line

Create 5 fractions using the whole numbers 0 through 9, no more than one time each, as numerators and denominators and correctly place them all on a number line.

DEPTH OF KNOWLEDGE EXTENSIONS MENU

Question #1 3.MD.8 : DOK 2 Draw three different rectangles with a perimeter of 20 units. **2 points**	**Question #2** 3.MD.7 : DOK 1 Find the rectangle's area.  **1 point**	**Question #3** 3.MD.5 : DOK 2 The length of one side of a rectangle is 6 cm and its perimeter is 16 cm. What is the area of the rectangle in square centimeters? **2 points**
Question #4 4.MD.3 : DOK 2 Which square is bigger: a square with a perimeter of 36 units or a square with an area of 36 square units? **2 points**	**Instructions** You must earn <u>at least 8 points</u> by doing the problems of your choice. You may work by yourself or in pairs but each person needs to turn in separate work. Circle the questions you have answered.	**Question #5** 4.MD.3 : DOK 3 What is the greatest area you can make with a rectangle that has a perimeter of 24 units? **3 points**
Question #6 4.MD.3 : DOK 3 What is the greatest perimeter you can make on a rectangle with an area of 24 square units? **3 points**	**Question #7** 3.MD.8 : DOK 2 What is the area of a square that has a perimeter of 20 units? **2 point**	**Question #8** 3.MD.8 : DOK 1 Find the rectangle's perimeter.  **1 point**

Lessons Learned

- Strangely little collaboration
  - Students could pick their own problems.
  - Few neighbors were working on the same problem.
  - Next time had kids pair up and pick the same problem to work on.
- The fraction sheet was chaos
  - Just because a problem is below grade level, doesn't mean they can do it.
  - Make sure students can do a DOK 1 before giving them DOK 2 and 3 problems.

Open Middle

Challenging math problems worth solving

NEW OPEN M

Exponents a[illegible] Operations

February 10, 2015 Leave

Directions: Find 3 positive i[illegible]t add up to 10. Place each number into one of the blanks to find the largest possible result. Source: Zack[illegible]ller (@zmill415) **Read More »**

Create Squares

February 10, 2015 2 Comments

Directions: Create a square with one of the vertices at (2,3). Fill in the blanks with whole numbers 0 through 9, using each number at most once, to show the rest of the vertices of the square. Bonus: Find more than one set of vertices. Source: John Mahlstedt (@jdmahlstedt) **Read More »**

Solution of Two Linear Equations

February 10, 2015 Leave a comment

Directions: Using the Integers 0-9 (without duplication), provide four sets of points that represent two distinct lines. These lines can be written as two linear equations. Then provide a fifth point that represents the intersection (or solution) of those equations. Line 1: (__, __) and (__, __) Line 2: (__, __) and (__, __) Solution (__, __) Source: Bryan Anderson **Read More »**

Bingo card

February 5, 2015 1 Comment

Directions: In a standard game of BINGO, the cards are labeled with numbers 1 through 75. If it was possible, which card would you choose: a card with all of the same number or a standard bingo card? Source: Nanette

OPEN MIDDLE WORKSHEET

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Version 1.1

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- Grade 2 (6)
  - Measurement & Data (2)
  - Number & Operations in Base Ten (4)
- Grade 3 (11)
  - Measurement & Data (6)
  - Number & Operations in Base Ten (3)
  - Number & Operations—Fractions (2)

Contact

Robert Kaplinsky

robert@robertkaplinsky.com

robertkaplinsky.com

@robertkaplinsky

OCMC15