Evergreen Public Schools

ROBERT KAPLINSKY

@robertkaplinsky

THIS UNIT NOT LABELED FOR INDIVIDUAL RETAIL SALE

grade AA

Ralphs

butter

NET WT. 4 OZ. (113g)

- [ ] 1/3 cup butter
- [ ] 1/3 cup white sugar
- [ ] 3 tablespoons and 1-3/4 teaspoons packed brown sugar
- [ ] 1/3 cup peanut butter
- [ ] 1/4 teaspoon vanilla extract

How can we tell where to cut the butter so you have 1/3 of a cup?

THIS UNIT NOT LABELED FOR INDIVIDUAL RETAIL SALE.

Ingredients: Pasteurized Cream, Salt.

DISTRIBUTED BY: RALPHS GROCERY CO. LOS ANGELES, CALIF. 90054

1 Tbsp.	2 Tbsp.	3 Tbsp.	4 Tbsp.	5 Tbsp.	6 Tbsp.	7 Tbsp.	8 Tbsp.

1/4 cup

1 FIRST QUALITY 1

Ralphs

grade AA

butter

NET WT. 4 OZ. (113g)

- [ ] 1/2 cup butter
- [ ] 1/2 cup white sugar
- [ ] 1/3 cup packed brown sugar
- [ ] 1/2 cup peanut butter
- [ ] 1/2 teaspoon vanilla extract

How can we tell where to cut the butter so you have 1/2 of a cup?

THIS UNIT NOT LABELED FOR INDIVIDUAL RETAIL SALE.

Ingredients: Pasteurized Cream, Salt.

DISTRIBUTED BY: RALPHS GROCERY CO. LOS ANGELES, CALIF. 90054

1 Tbsp.	2 Tbsp.	3 Tbsp.	4 Tbsp.	5 Tbsp.	6 Tbsp.	7 Tbsp.	8 Tbsp.
			1/4 cup				1/2 cup

1 FIRST QUALITY 1

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1 Tbsp.	2 Tbsp.	3 Tbsp.	4 Tbsp.	5 Tbsp.	6 Tbsp.	7 Tbsp.	8 Tbsp.
			1/4 cup	1/3 cup			1/2 cup

1 FIRST QUALITY 1

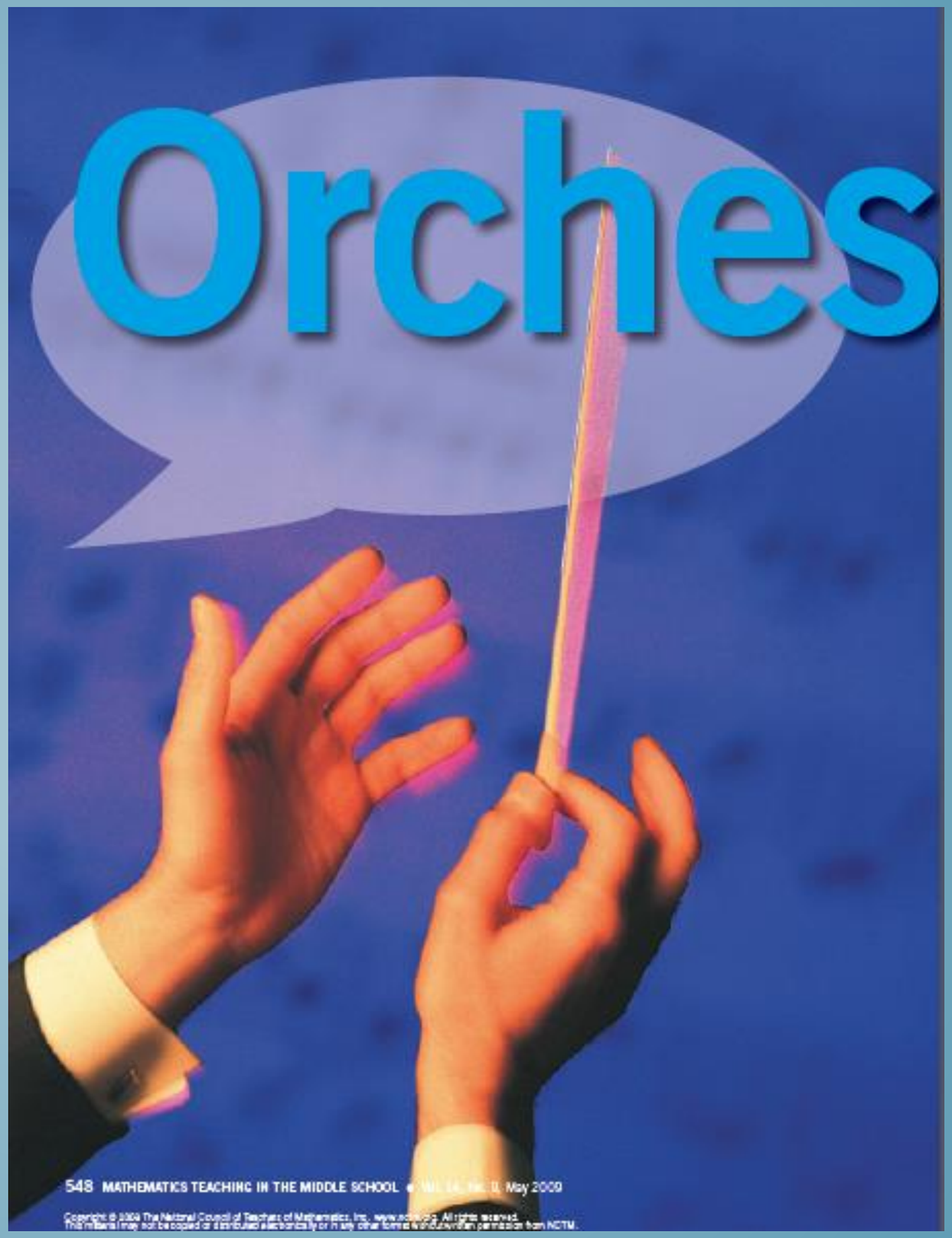
Ralphs

grade AA

butter

NET WT. 4 OZ. (113g)

FIVE PRACTICES



Discussion Questions

- "Giving students too much or too little support, or too much direction, can result in a decline in the cognitive demands of the task." (p. 550) Why?
- "By making purposeful choices about the order in which students' work is shared, teachers can maximize the chances that their mathematical goals for the discussion will be achieved." (p. 554) What ways do teachers currently select students? How would you suggest they change their selection process after reading this?
- What challenges might teachers have when trying to "connect" student solutions? (p. 554)

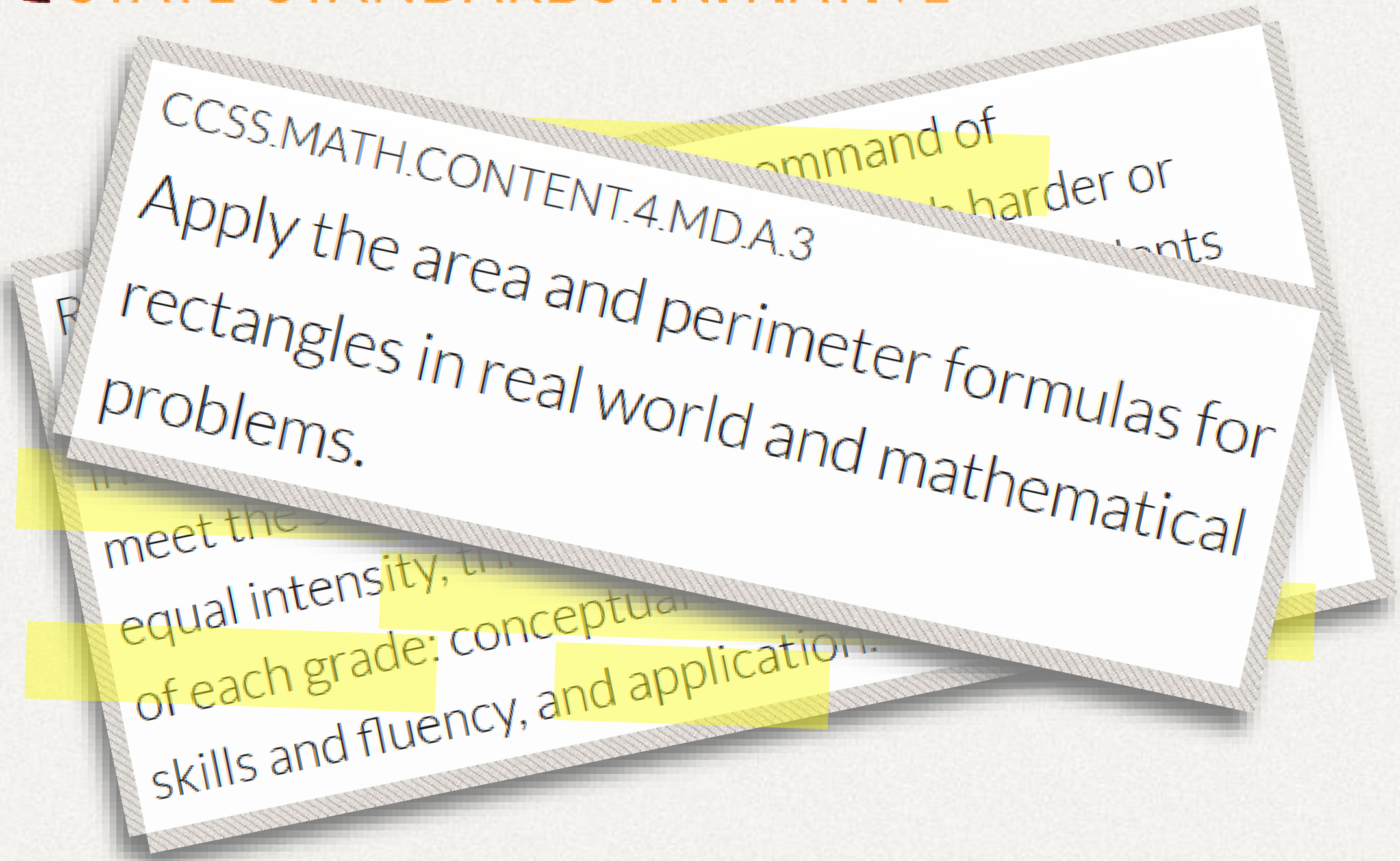
Implementing the Five Practices

1. Anticipate potential student responses to the butter stick fraction problem.
2. Review the ten student work samples that represent students in your classroom.
3. Figure out which students you would have share their mathematical work.
4. Determine the order you would have those students present their work.
5. Decide on which connections you would emphasize between the students' work and mathematical ideas.

Posters

- At the top of the poster, list the selection strategy used by your group. For example:
  - Starting with the most commonly used strategy and moving to one that few students used.
  - Starting with a strategy that is more concrete and moving to strategies that are more abstract.
  - Incorporating wrong answers to address common misconceptions.
- Attach those students' work to the poster in the order that you would present it.
- Next to the student work list the questions you would ask the student(s) or ideas that you would want to come out as a result of showing that student's work.

COMMON CORE
STATE STANDARDS INITIATIVE



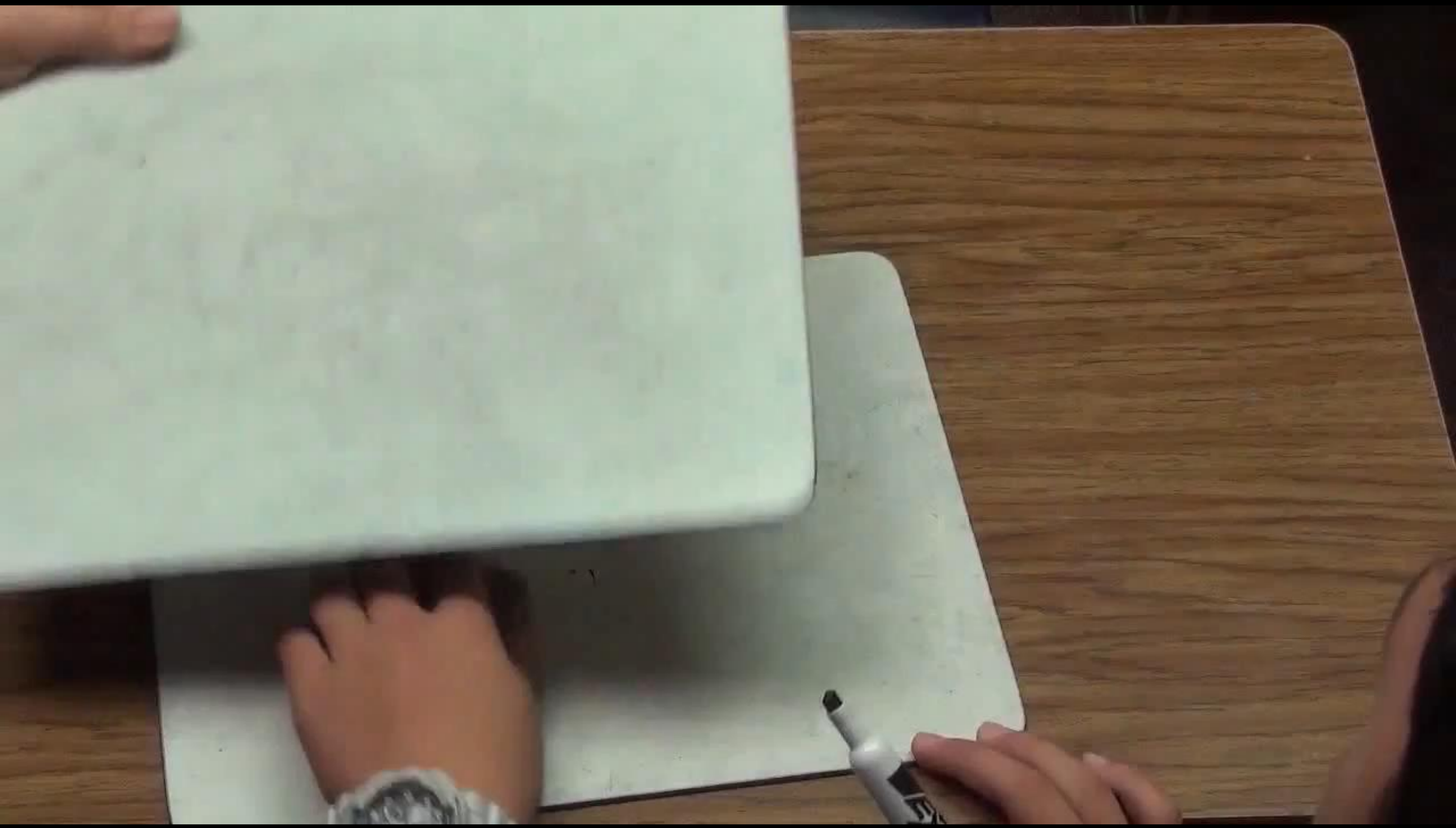
Source: http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

What is the perimeter of a rectangle that measures 8 units by 4 units?

Components of Rigor

- ☑ Procedural Skill and Fluency
- ☑ Conceptual Understanding

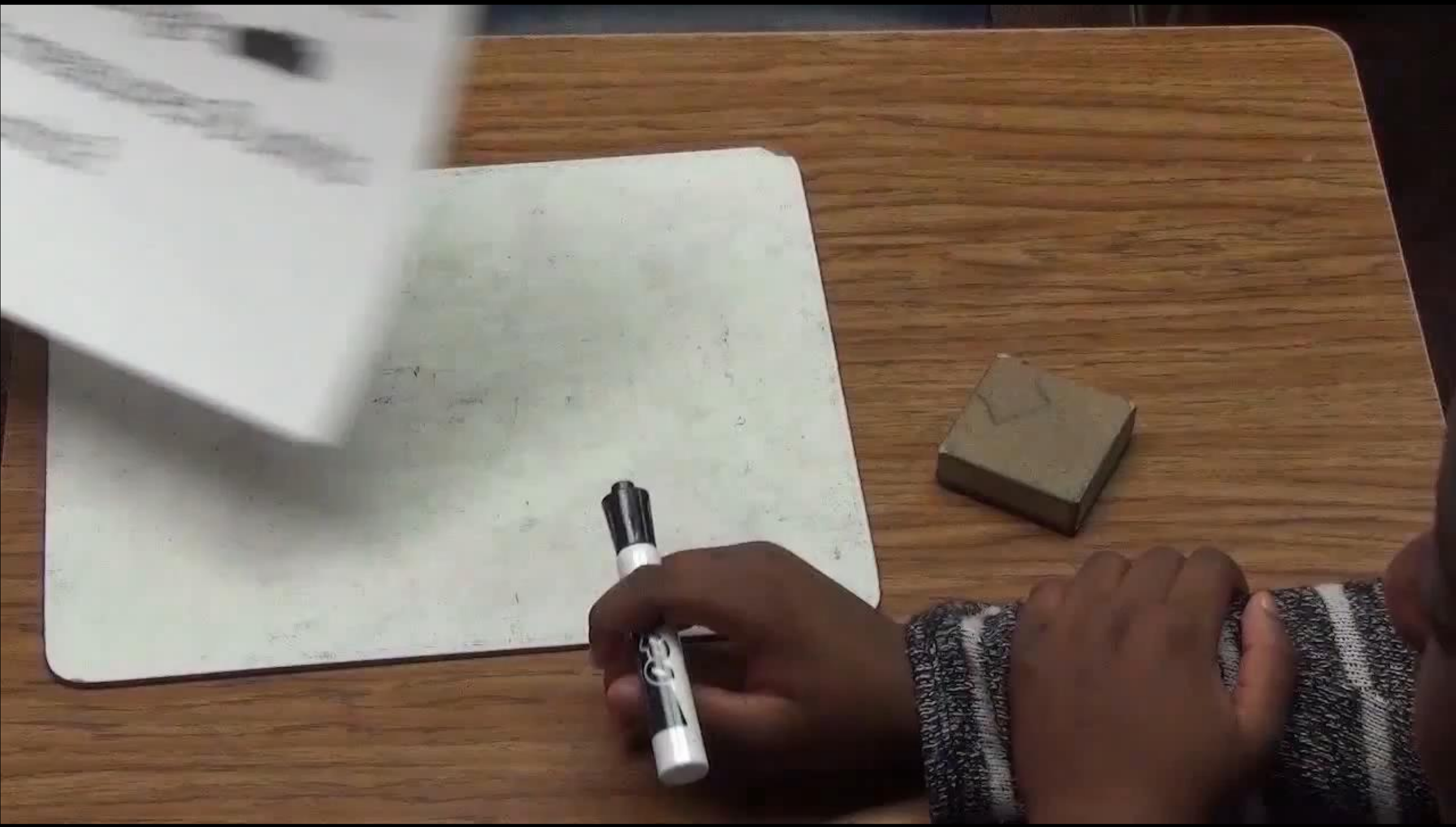
List the dimensions of a rectangle with a perimeter of 24 units.



Components of Rigor

- [x] Procedural Skill and Fluency
- [x] Conceptual Understanding

WHY?



Components of Rigor

- ☑ Procedural Skill and Fluency
- ☑ Conceptual Understanding

List the

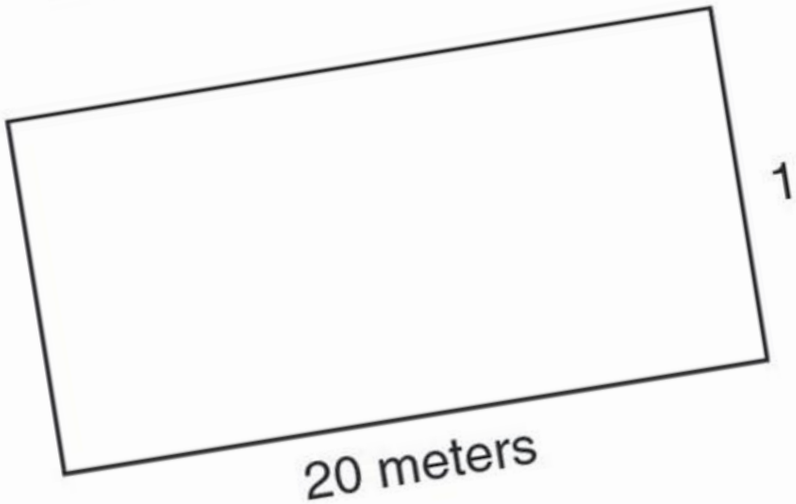
of a rectangle with a perimeter of 24 units.

Components of Rigor

- [x] Procedural Skill and Fluency
- [x] Conceptual Understanding

STAR

71 **A basketball court is shaped like a rectangle 20 meters long and 10 meters wide.**



What is the perimeter in meters of the court?

A 30 meters

B 50 meters

C 60 meters

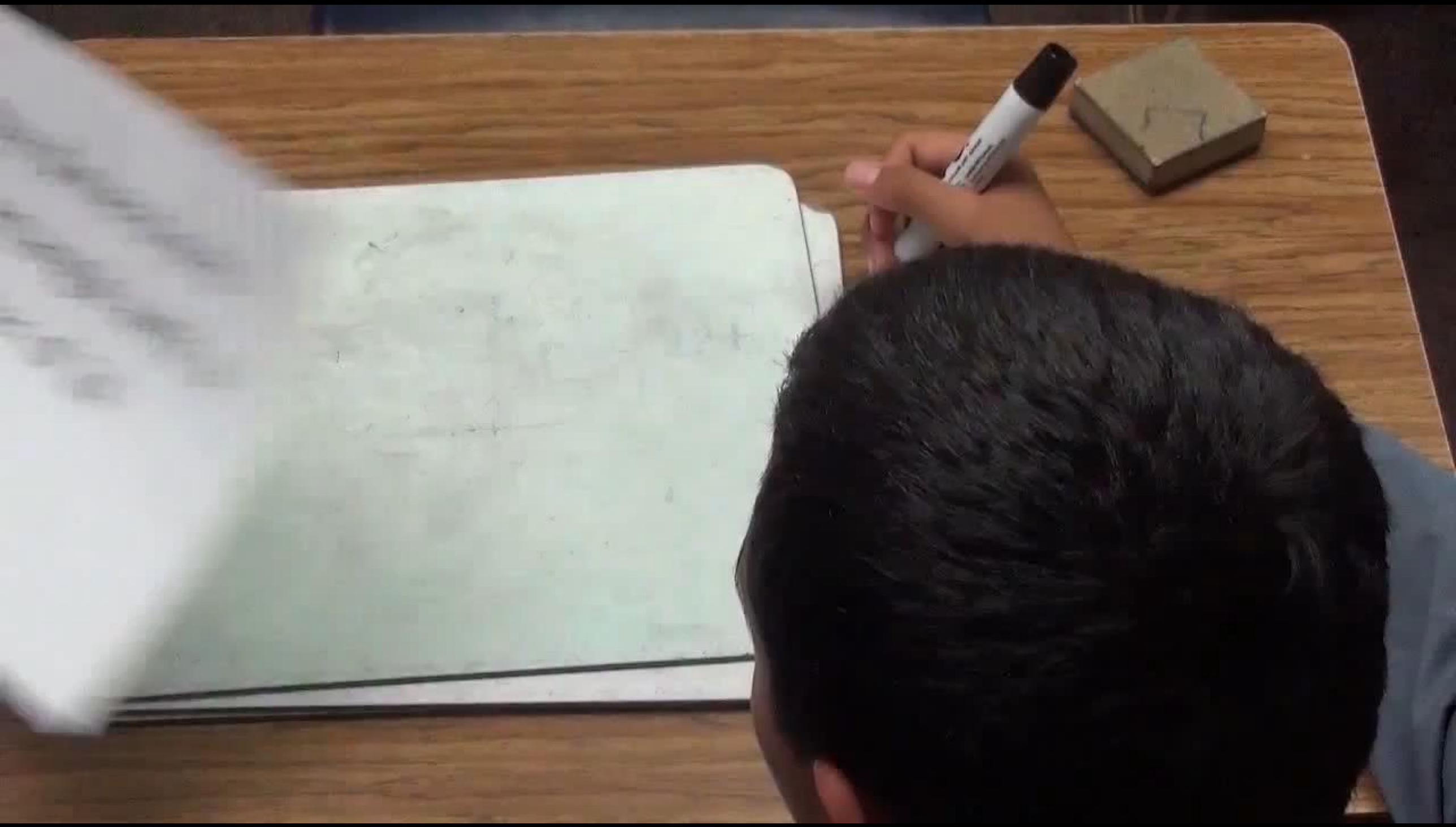
D 200 meters

Source: http://www.cde.ca.gov/ta/tg/sr/documents/cstrtqmath3.pdf

What is the perimeter of a rectangle [redacted] that measures 8 units by 4 units?

Components of Rigor

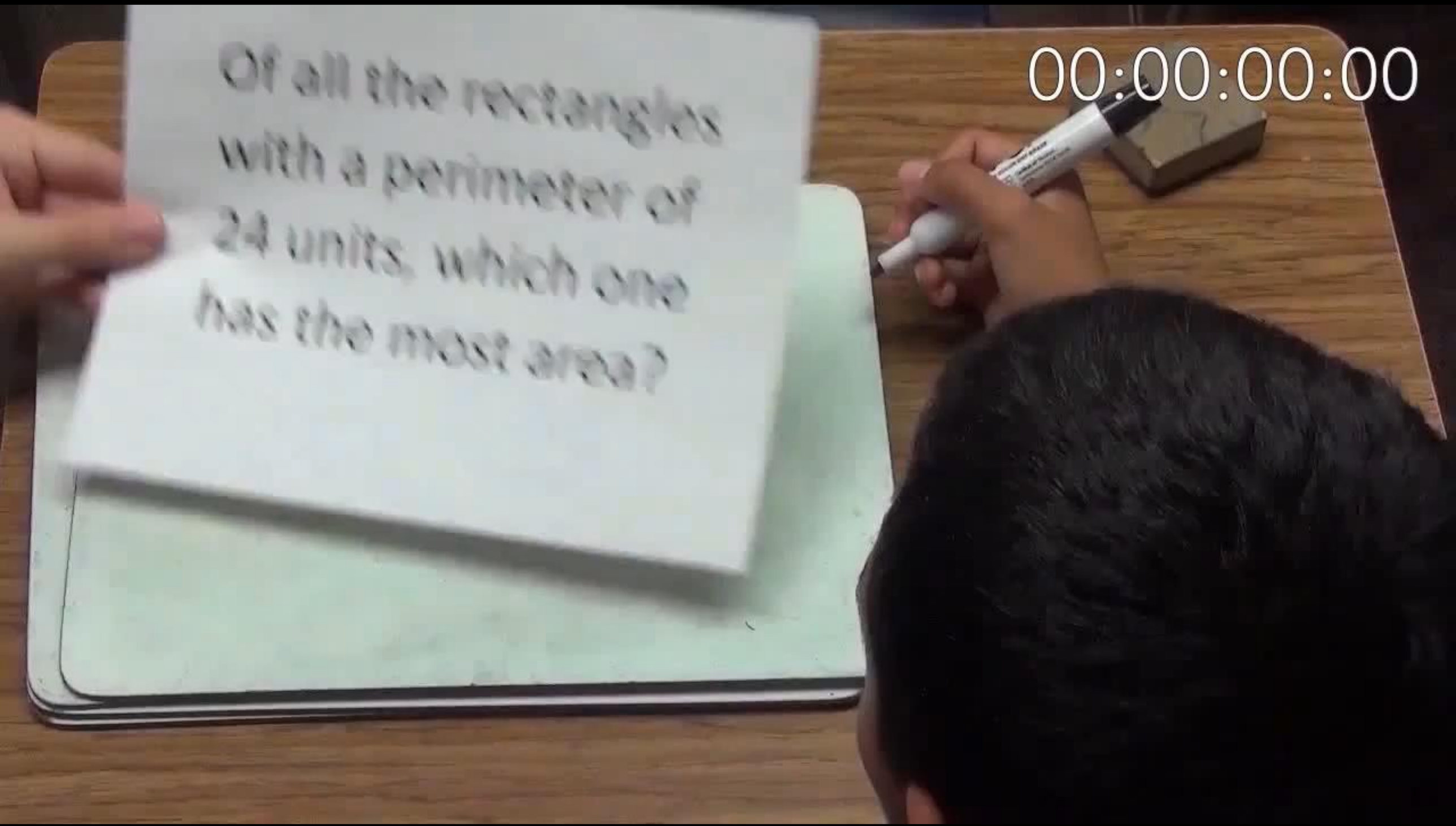
- ☑ Procedural Skill and Fluency
- ☑ Conceptual Understanding



Components of Rigor

- ☑ Procedural Skill and Fluency
- ☑ Conceptual Understanding

Of all the rectangles with a perimeter of 24 units, which one has the most area?


Of all the rectangles with a perimeter of 24 units, which one has the most area?
00:00:00:00

Components of Rigor

- [x] Procedural Skill and Fluency
- [x] Conceptual Understanding

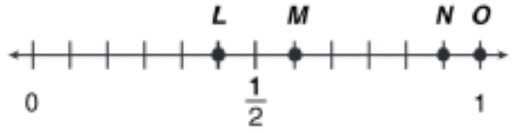
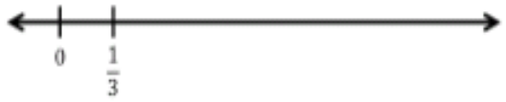
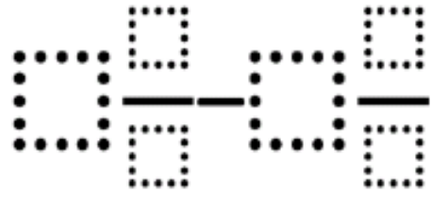
Defining the Problem

- Students appear to demonstrate "deep, authentic command of mathematical concepts" when given commonly used problems.
- However with more challenging problems, the same students seem to no longer demonstrate that command.

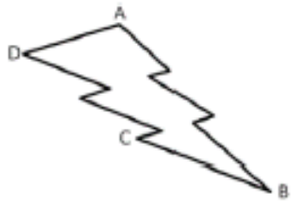
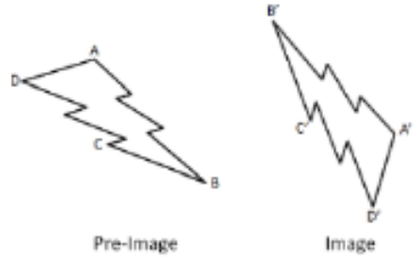
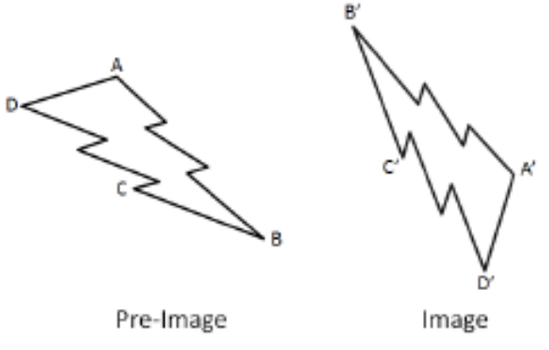
Addressing the Problem

- First, we must have a clear understanding about why these problems are different from one another.
- Next, we need to practice implementing these problems such that all students are engaged in a problem that is at the right challenge level for them.
- Last, we need a source that can provide us with a variety of free problems.

DOK | Distinguishing Between Depth of Knowledge Levels in Mathematics

Topic	Adding Whole Numbers	Money	Fractions on a Number Line	Area and Perimeter	Subtracting Mixed Numbers
CCSS Standard(s)	• 1.NBT.4 • 2.NBT.5	• 2.MD.8	• 3.NF.2	• 3.MD.8 • 4.MD.3	• 5.NF.1
DOK 1 Example	Find the sum. $44 + 27 =$	If you have 2 dimes and 3 pennies, how many cents do you have	Which point is located at $\frac{7}{12}$ below? 	Find the perimeter of a rectangle that measures 4 units by 8 units.	Find the difference. $5\frac{1}{2} - 4\frac{2}{3} =$
DOK 2 Example	Fill in the boxes below using the whole numbers 1 through 9, no more than one time each, so that you make a true equation. $\square\square + 53 = \square\square$	Make 47¢ in three different ways with either quarters, dimes, nickels, or pennies.	Label the point where $\frac{3}{4}$ belongs on the number line below. Be as precise as possible. 	List the measurements of three different rectangles that each has a perimeter of 20 units.	Create three different mixed numbers that will make the equation true by using the whole numbers 1 through 9, no more than one time each,. You may reuse the same whole numbers for each of the three mixed numbers. $5\frac{4}{5} - \square\frac{\square}{\square} = 3\frac{1}{20}$
DOK 3 Example	Make the largest sum by filling in the boxes below using the whole numbers 1 through 9, no more than one time each. $\square\square + \square\square =$	Make 47¢ using exactly 5 coins with either quarters, dimes, nickels, or pennies.	Create 5 fractions using the whole numbers 0 through 9, no more than one time each, as numerators and denominators and correctly place them all on a number line.	What is the greatest area you can make with a rectangle that has a perimeter of 24 units?	Make the smallest difference by filling in the boxes below using the whole numbers 1 through 9, no more than one time each. 

DOK | Distinguishing Between Depth of Knowledge Levels in Mathematics

Topic	Surface Area and Volume	Probability	Transformations	Factoring Quadratics	Quadratics in Vertex Form
CCSS Standard(s)	• 6.G.4 • 7.G.6	• 7.SP.5 • 7.SP.7	• 8.G.1 • G-CO.5	• A-SSE.3a	• F-IF.7a
DOK 1 Example	Find the surface area of a rectangular prism that measures 3 units by 4 units by 5 units.	What is the probability of rolling a sum of 5 using two 6-sided dice?	Rotate the image below 90° counterclockwise and reflect it across a horizontal line. 	Find the factors: $2x^2 + 7x + 3$	Find the roots and maximum of the quadratic equation below. $y = 3(x-4)^2 - 3$
DOK 2 Example	List the measurements of three different rectangular prisms that each has a surface area of 20 square units.	What value(s) have a 1/12 probability of being rolled as the sum of two 6-sided dice?	List three sequences of transformations that take pre-image ABCD to image A'B'C'D'.  Pre-Image Image	Fill in the blank with integers so that the quadratic expression is factorable. $x^2 + \_\_x + 4$	Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values.
DOK 3 Example	What is the greatest volume you can make with a rectangular prism that has a surface area of 20 square units?	Fill in the blanks to complete this sentence using the whole numbers 1 through 9, no more than one time each. Rolling a sum of ___ on two ___-sided dice is the same probability as rolling a sum of ___ on two ___-sided dice.	What is the fewest number of transformations needed to take pre-image ABCD to image A'B'C'D'?  Pre-Image Image	Fill the blank by finding the largest and smallest integers that will make the quadratic expression factorable. $2x^2 + 3x + \_\_$	Create a quadratic equation with the largest maximum value using the whole numbers 1 through 9, no more than one time each. $y = -\square(x-\square)^2 + \square$

DOK Level Differences

- **Level 1: Recall & Reproduction**
  - Often a trivial application of facts.
  - Requires little to no cognitive effort beyond remembering the right formula.
  - Usually only one answer.
- **Level 2: Skills & Concepts**
  - Usually requires more than one step to solve.
  - Often multiple answers.
- **Level 3: Strategic Thinking**
  - Usually requires critical thinking about the best way to approach a problem.
  - May be multiple answers or a single optimal answer.
  - Often challenging enough to make your head hurt.
- **Level 4: Extended Thinking**
  - In mathematics these are generally represented by performance tasks or problem-based lessons.

Probability

What is the probability of rolling a sum of 5 using two 6-sided dice?

Probability

What value(s) have a $\frac{1}{12}$ probability of being rolled as the sum of two 6-sided dice?

Author: Daniel Luevanos

Probability

Fill in the blanks to complete this sentence using the whole numbers 1 through 9, no more than one time each.

Rolling a sum of ___ on two ___-sided dice is the same probability as rolling a sum of ___ on two ___-sided dice.

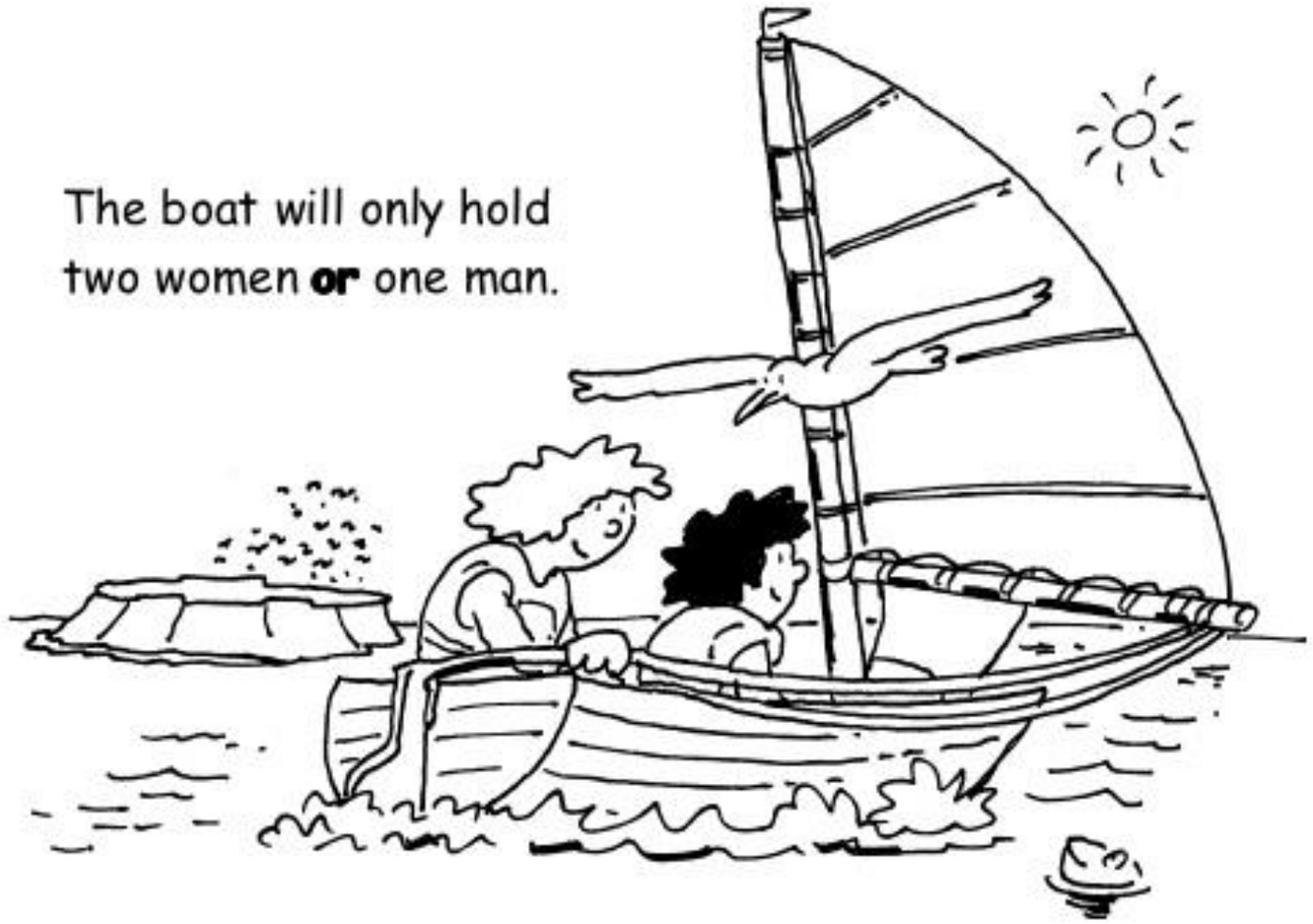
Authors: Audrey Mendivil, Daniel Luevanos, and Robert Kaplinsky

JUST BRAIN TEASERS?

Sail away

Two men and two women want to sail to an island.

The boat will only hold
two women **or** one man.



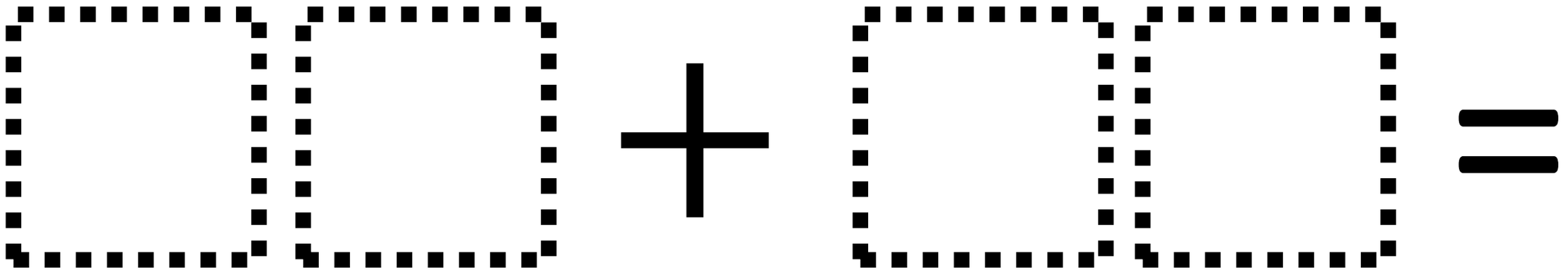
How can all four of them get to the island?

46

Teaching objectives

Solve mathematical problems or puzzles.
Explain methods and reasoning.

Make the largest sum by filling in the boxes below using the whole numbers 1 through 9, no more than one time each.



@robe
tease
more
1:27 P

Mark Chubb
@MarkChubb3

Follow

@robertkaplinsky @openmiddle I think the purpose is the difference. OMP are designed to learn important math. BT are designed to trick!

5:47 PM - 4 May 2015

Mike Flynn
@MikeFlynn55

Follow

@fawnpnguyen @robertkaplinsky @openmiddle I agree OMP allow for multiple approaches and/or solutions where BT seemed closed most of the time

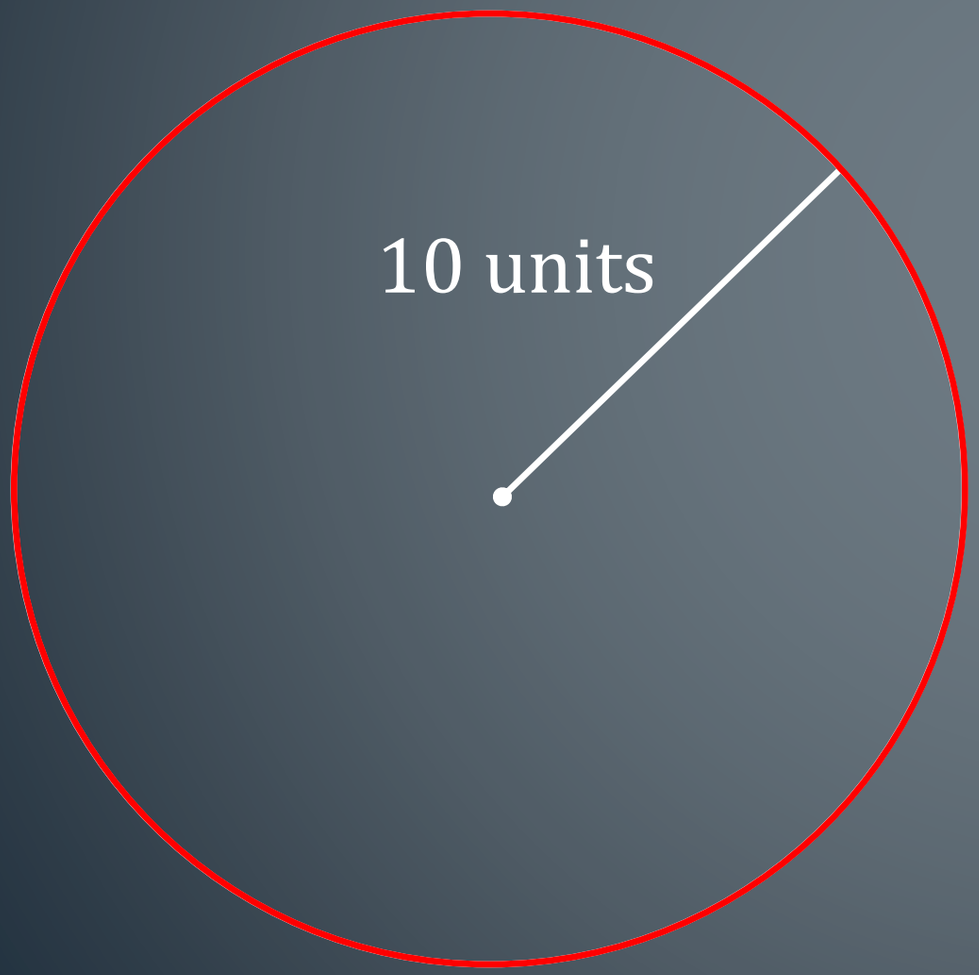
1:44 PM - 4 May 2015

llow

ave

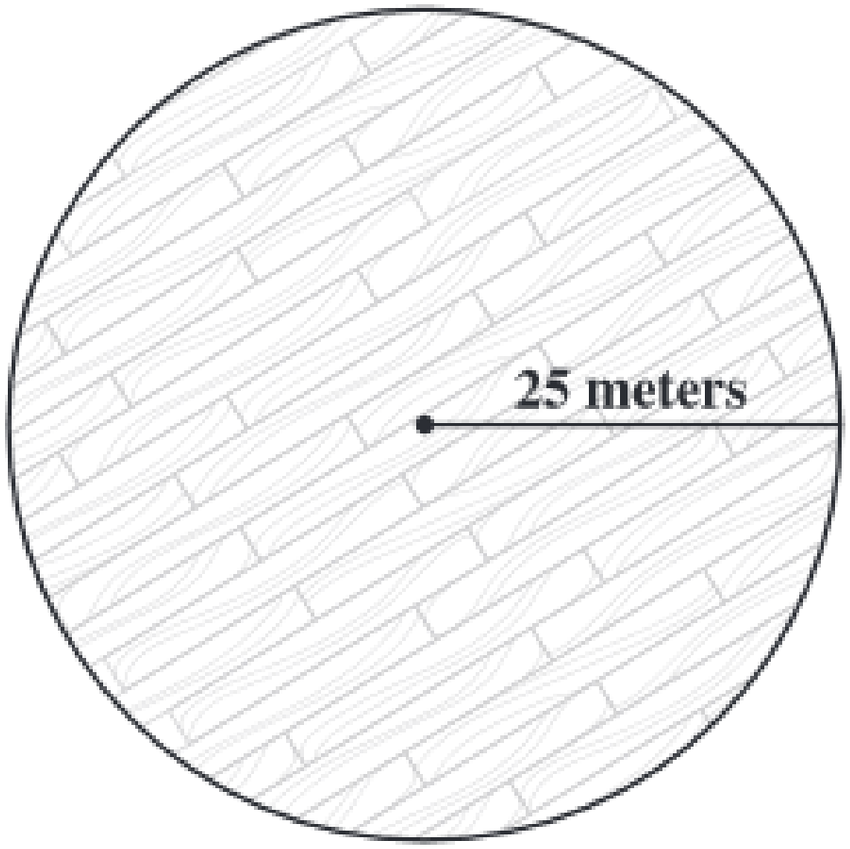
Depth of Knowledge – Level One

What is the circle's circumference? $\pi \approx 3.14$



What is the circle's area? $\pi \approx 3.14$

70 **This circular stage has a radius of 25 meters.**



Which equation could be used to find the area of the stage in square meters?

A $A = 25\pi$

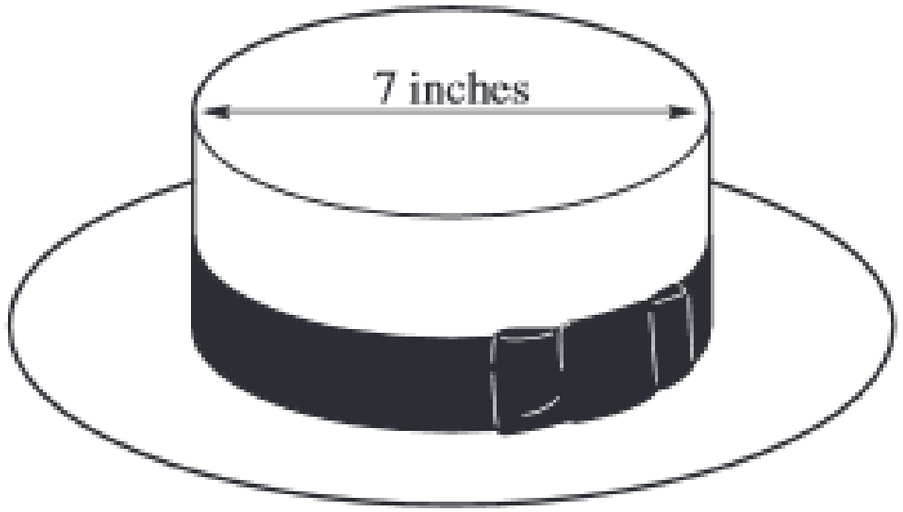
B $A = 50\pi$

C $A = \pi \bullet 25^2$

D $A = \pi \bullet 50^2$

CSM02059

72 **The top part of this hat is shaped like a cylinder with a diameter of 7 inches.**



Which measure is *closest* to the length of the band that goes around the outside of the hat?

A 10.1 inches

B 11.0 inches

C 22.0 inches

D 38.5 inches

CSM00268

Source: 6th Grade CST Released Test Questions - http://www.cde.ca.gov/ta/tg/sr/documents/cstrtqmath6.pdf

Student Data Facts

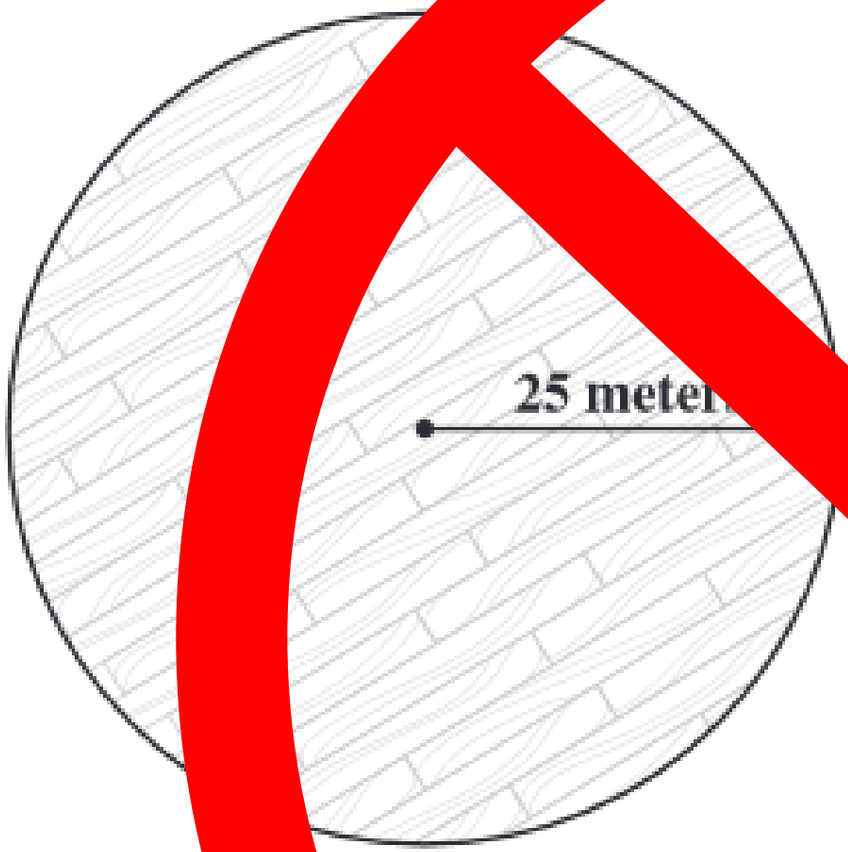
- 396 seventh grade students were assessed
- 68.26% correctly answered the circumference question
- 78.59% correctly answered the area question

Smarter Balanced Assessment Consortium

Mathematics Preliminary Summative Assessment Blueprint
Target Sampling Mathematics Grade 7—Table 6b

Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Min/Max Number of Items
				CAT	PT/ECR	SR	CR	
		A. Analyze proportional relationships and use them to solve real-world and mathematical problems.	1,2	p(9)=1.0				
		E. Draw, construct, and describe geometrical figures and describe the relationship between them.	2,3					
		F. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	1,2					
Supporting	Supporting Cluster	problems involving angle measure, area, surface area, and volume.	1,2	p(2)=1.0	0	2	1	5/8
		G. Use random sampling to draw inferences about a population.	1,2					
		H. Draw informal comparative inferences about two populations.	1,2					
		I. Investigate chance processes and develop, use, and evaluate probability models.	1,2					

70 This circular stage has a radi… meters.



Which equat… ould be used to find the area of the stage in … re meters?

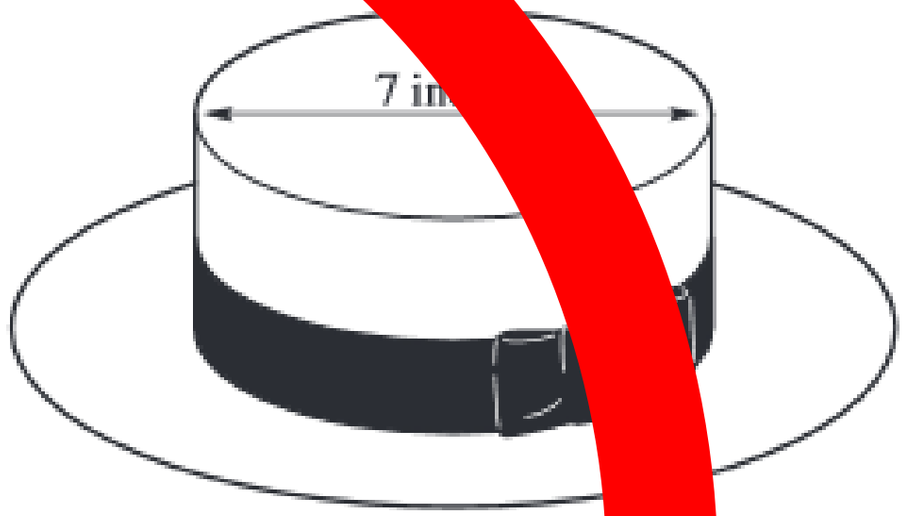
A $A = 25\pi$

B $A = 50\pi$

C $A = \pi \cdot 25^2$

D $A = \pi \cdot 50^2$

72 The … of this hat is shaped like a cylinder with a dia… 7 inches.



Which measure is *closest* to … ength of the band that goes around the c… le of the hat?

A …0.1 inches

B … inches

C 22… s

D 38.5 in…

CSM00268

Depth of Knowledge – Level Two

Which circle is bigger? How do you know?

Circle A

Area = 36 units^2

$$C = \pi \cdot 2 \cdot r$$

$$36 \approx 6.28 \cdot r$$

$$\frac{36}{6.28} \approx r$$

$$5.73\ units \approx r$$

Circle B

Circumference = 36 units

$$A = \pi \cdot r^2$$

$$A \approx 3.14 \cdot 5.73^2$$

$$A \approx 3.14 \cdot 32.83$$

$$A \approx 103.15\ units^2$$

Student Data Facts

- Of the 396 seventh grade students who were assessed, 12.12% earned two points on the DOK 2 question.
- 97.92% of the students who correctly answered the DOK 2 question also correctly answered both of the two DOK 1 questions.
- 10.61% of the students who correctly answered both of the two DOK 1 questions also correctly answered the DOK 2 question.

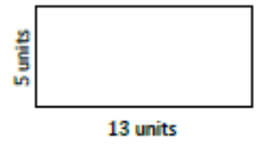
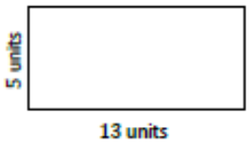
More Student Data Facts

- 28.28% of the students earned only one point.
- All of them earned one point by choosing Circle B and providing insufficient reasoning.
- 59.59% of the students earned no points.

Mathematics Preliminary Summative Assessment Blueprint
Target Sampling Mathematics Grade 7—Table 6b

Claim	Content Category	Assessment Targets	DOK	Minimum # Scored Tasks		Minimum # Items per Item Type		Min/Max Number of Items
				CAT	PT/ECR	SR	CR	
1. Concepts and Procedures	Priority Cluster	A. Analyze proportional relationships and use them to solve real-world and mathematical problems.	1,2	p(9)=1.0	0	7	4	15/20
		D. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	1,2					
		B. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	1,2	p(6)=1.0				
		C. Use properties of operations to generate equivalent expressions.	1					
	Supporting Cluster	E. Draw, construct, and describe geometrical figures and describe the relationship between them.	2,3	p(3)=1.0	0	2	1	5/8
		F. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	1,2					
		G. Use random sampling to draw inferences about a population.	1,2	p(2)=1.0				
		H. Draw informal comparative inferences about two populations.	1,2					
		I. Investigate chance processes and develop, use, and evaluate probability models.	1,2					

DEPTH OF KNOWLEDGE EXTENSIONS MENU

Question #1 3.MD.8 : DOK 2 Draw three different rectangles with a perimeter of 20 units. **2 points**	**Question #2** 3.MD.7 : DOK 1 Find the rectangle's area.  **1 point**	**Question #3** 3.MD.5 : DOK 2 The length of one side of a rectangle is 6 cm and its perimeter is 16 cm. What is the area of the rectangle in square centimeters? **2 points**
Question #4 4.MD.3 : DOK 2 Which square is bigger: a square with a perimeter of 36 units or a square with an area of 36 square units? **2 points**	**Instructions** You must earn **at least 8 points** by doing the problems of your choice. You may work by yourself or in pairs but each person needs to turn in separate work. Circle the questions you have answered.	**Question #5** 4.MD.3 : DOK 3 What is the greatest area you can make with a rectangle that has a perimeter of 24 units? **3 points**
Question #6 4.MD.3 : DOK 3 What is the greatest perimeter you can make on a rectangle with an area of 24 square units? **3 points**	**Question #7** 3.MD.8 : DOK 2 What is the area of a square that has a perimeter of 20 units? **2 point**	**Question #8** 3.MD.8 : DOK 1 Find the rectangle's perimeter.  **1 point**

Lessons Learned

- Strangely little collaboration
  - Students could pick their own problems.
  - Few neighbors were working on the same problem.
  - Next time had kids pair up and pick the same problem to work on.
- The fraction sheet was chaos
  - Just because a problem is below grade level, doesn't mean they can do it.
  - Make sure students can do a DOK 1 before giving them DOK 2 and 3 problems.
- Some problems weren't chosen
  - Problem wording wasn't always as clear for students as it was to me.
  - Point values need fine tuning

Open Middle

Challenging math problems worth solving

NEW OPEN M

Exponents a f Operations

February 10, 2015 Leave

Directions: Find 3 positive i t add up to 10. Place each number into one of the blanks to find the largest possible result. Source: Zack iller (@zmill415) **Read More »**

Create Squares

February 10, 2015 2 Comments

Directions: Create a square with one of the vertices at (2,3). Fill in the blanks with whole numbers 0 through 9, using each number at most once, to show the rest of the vertices of the square. Bonus: Find more than one set of vertices. Source: John Mahlstedt (@jdmahlstedt) **Read More »**

Solution of Two Linear Equations

February 10, 2015 Leave a comment

Directions: Using the Integers 0-9 (without duplication), provide four sets of points that represent two distinct lines. These lines can be written as two linear equations. Then provide a fifth point that represents the intersection (or solution) of those equations. Line 1: (__, __) and (__, __) Line 2: (__, __) and (__, __) Solution (__, __) Source: Bryan Anderson **Read More »**

Bingo card

February 5, 2015 1 Comment

Directions: In a standard game of BINGO, the cards are labeled with numbers 1 through 75. If it was possible, which card would you choose: a card with all of the same number or a standard bingo card? Source: Nanette

OPEN MIDDLE WORKSHEET

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Version 1.1

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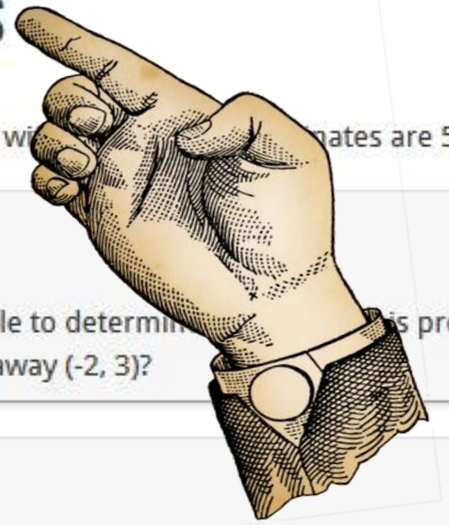
Open Middle

Challenging math problems worth solving

EQUIDISTANT POINTS



Directions: How many points wi nates are 5 units away from (-2, 3)?

Hint

Which methods are available to determi is problem? What shape is defined by *all* of the points that are 5 units away (-2, 3)?

Answer

12 points: (-5, 7), (-7, 3), (-5, -1), (-2, -2), (3, 3), (1, -1), (-2, 8), (1, 7), (2,6), (-6, -6), (-6, 0), and (2, 0)

Source: Dylan Kane

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Tagged with: 8.G.8 | DOK 2: SKILL / CONCEPT | DYLAN KANE | G-GPE.1

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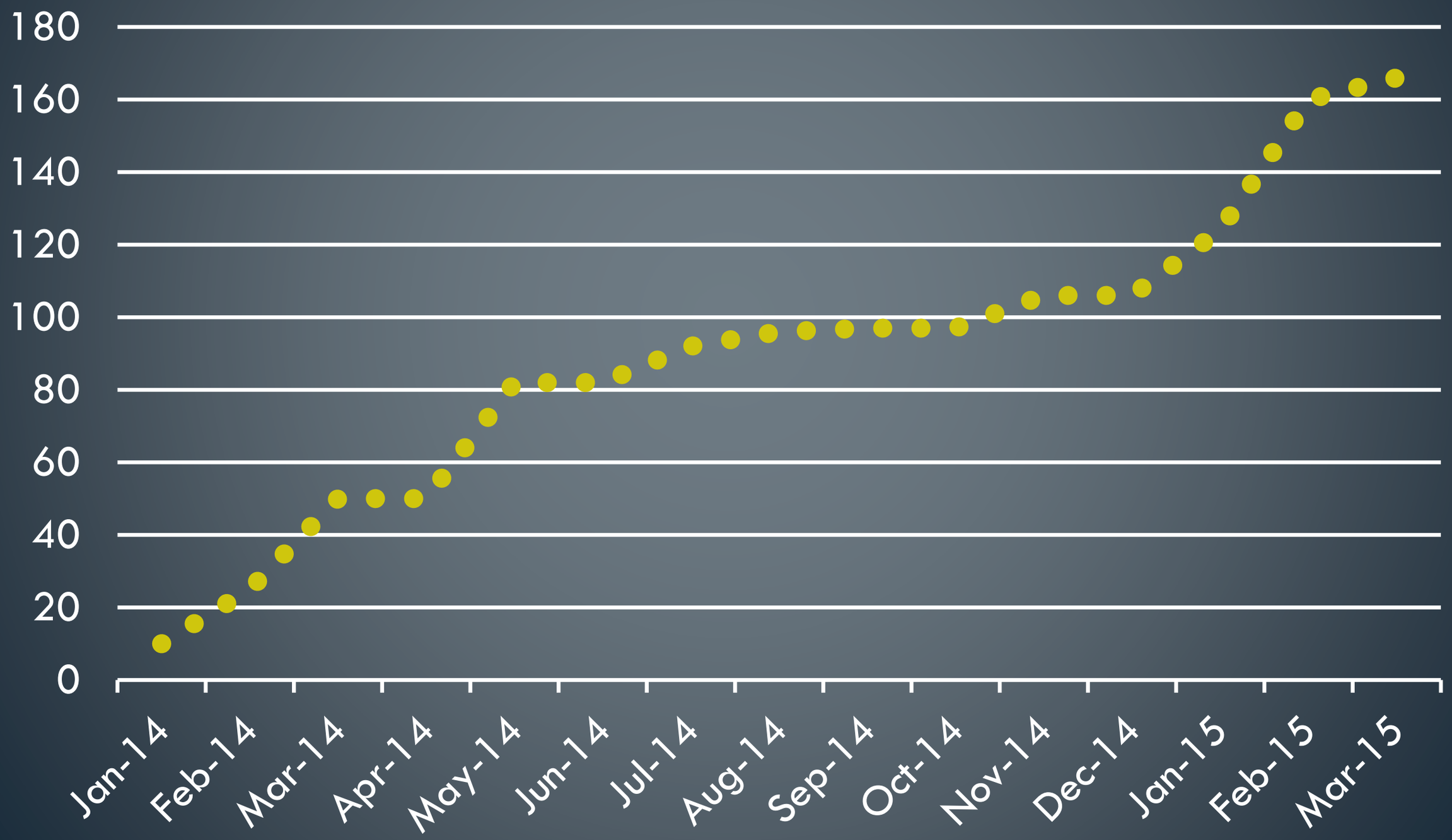
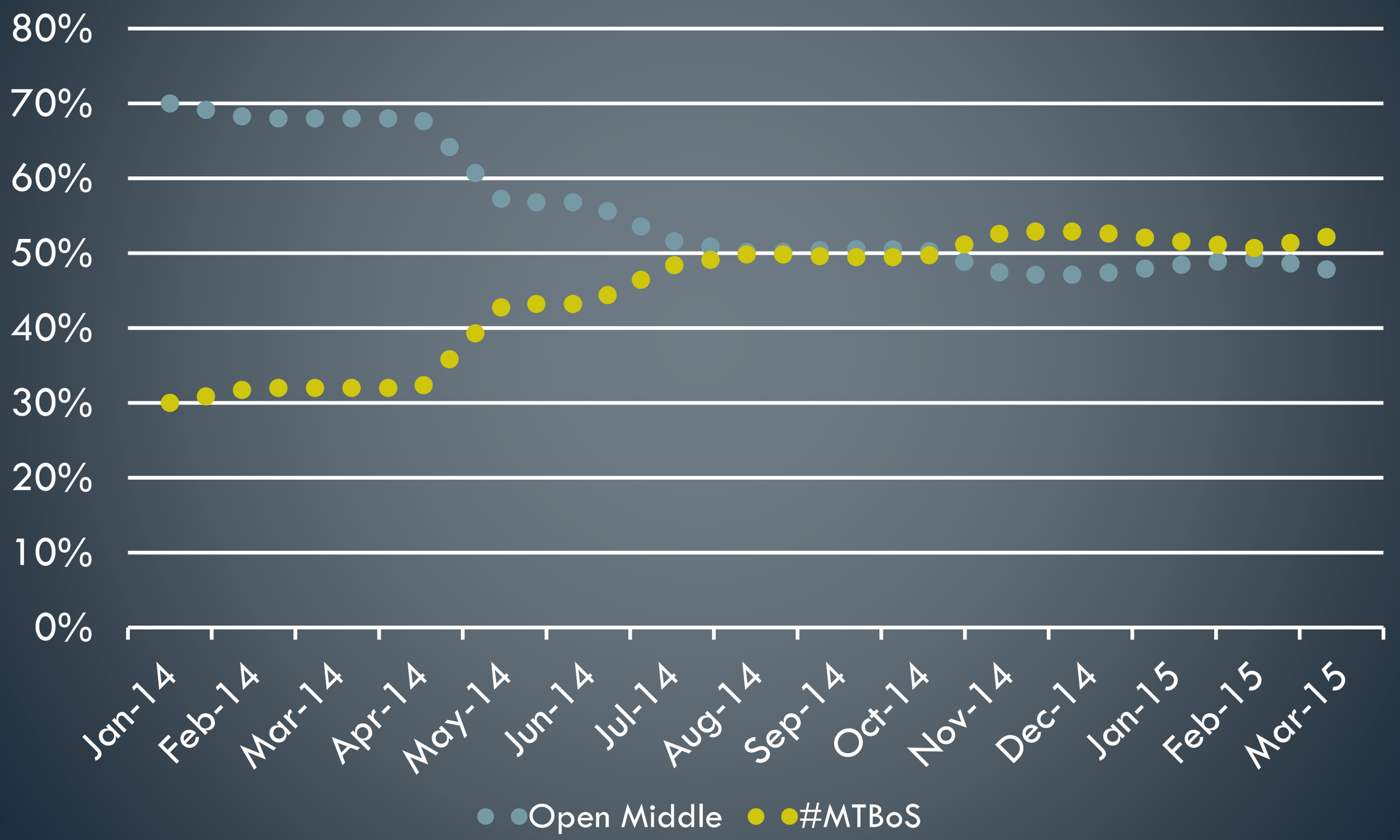
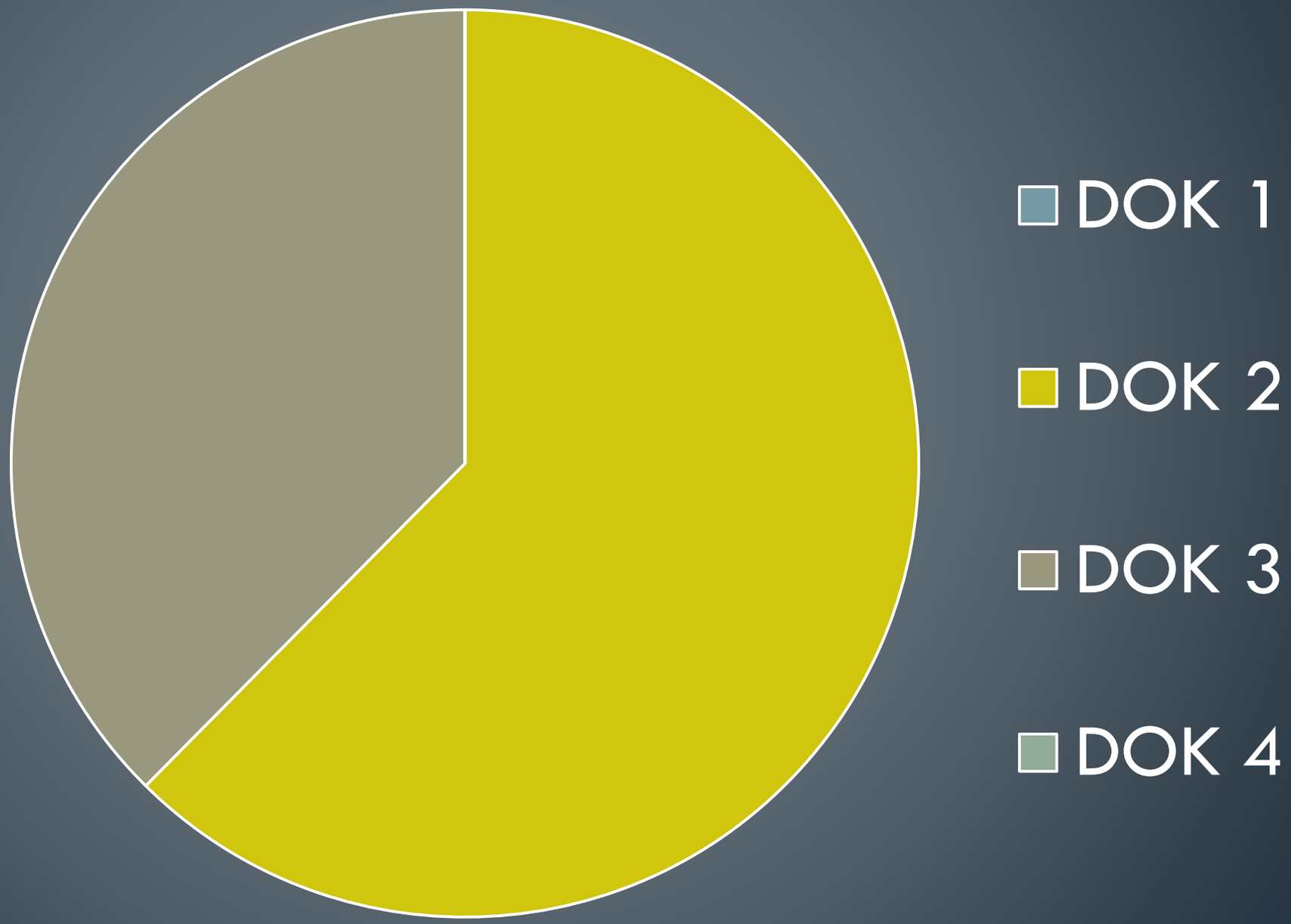
Total Open Middle Problems



Open Middle Author Percentages

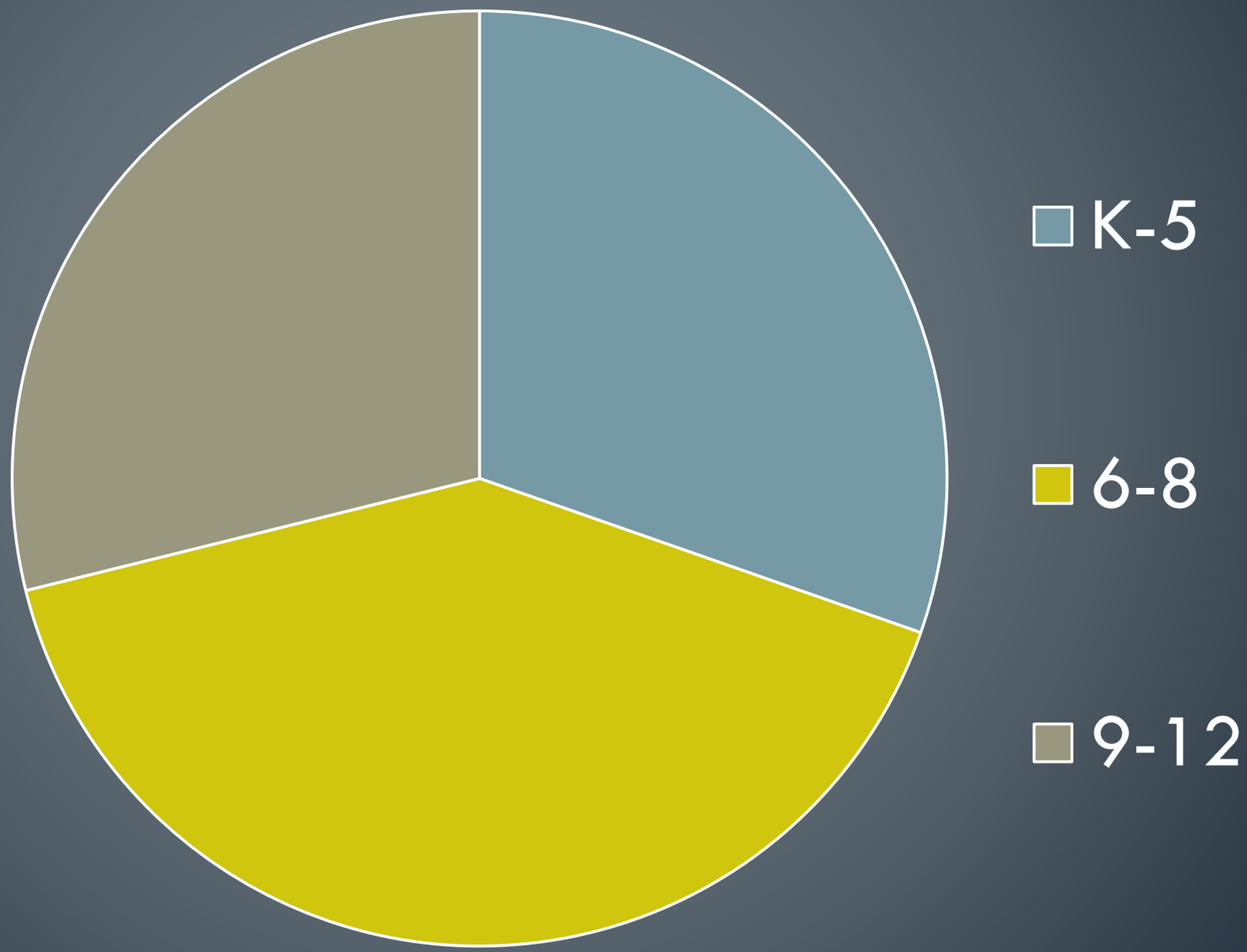


Problems by DOK Level



Note: Data as of March 2015

Problems by Grade Band



Note: Data as of March 2015

COMMON CORE
STATE STANDARDS INITIATIVE

Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades. To help students meet the standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skills and fluency, and application.

Source: http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

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