Digging into Depth of Knowledge

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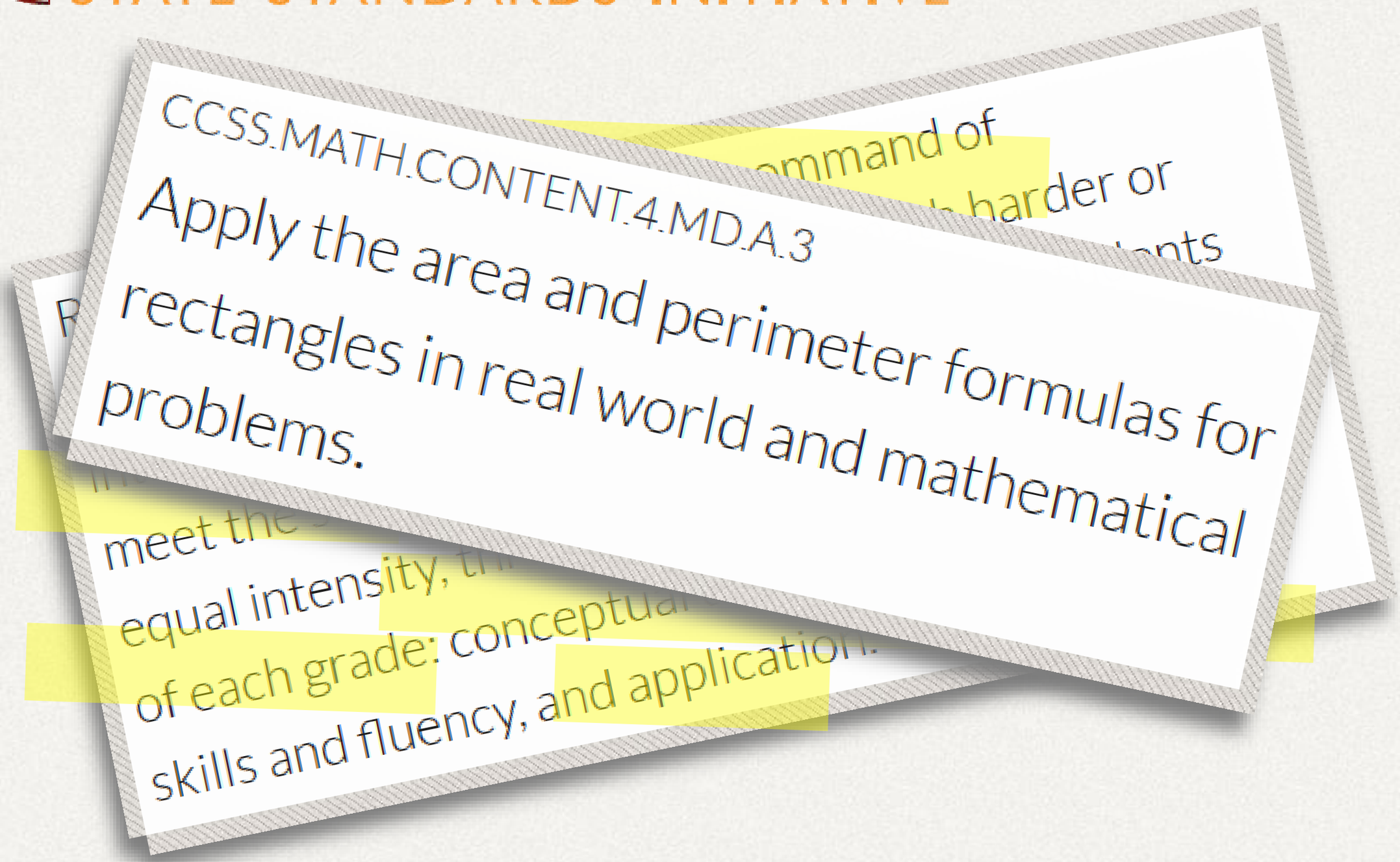
COMMON CORE
STATE STANDARDS INITIATIVE



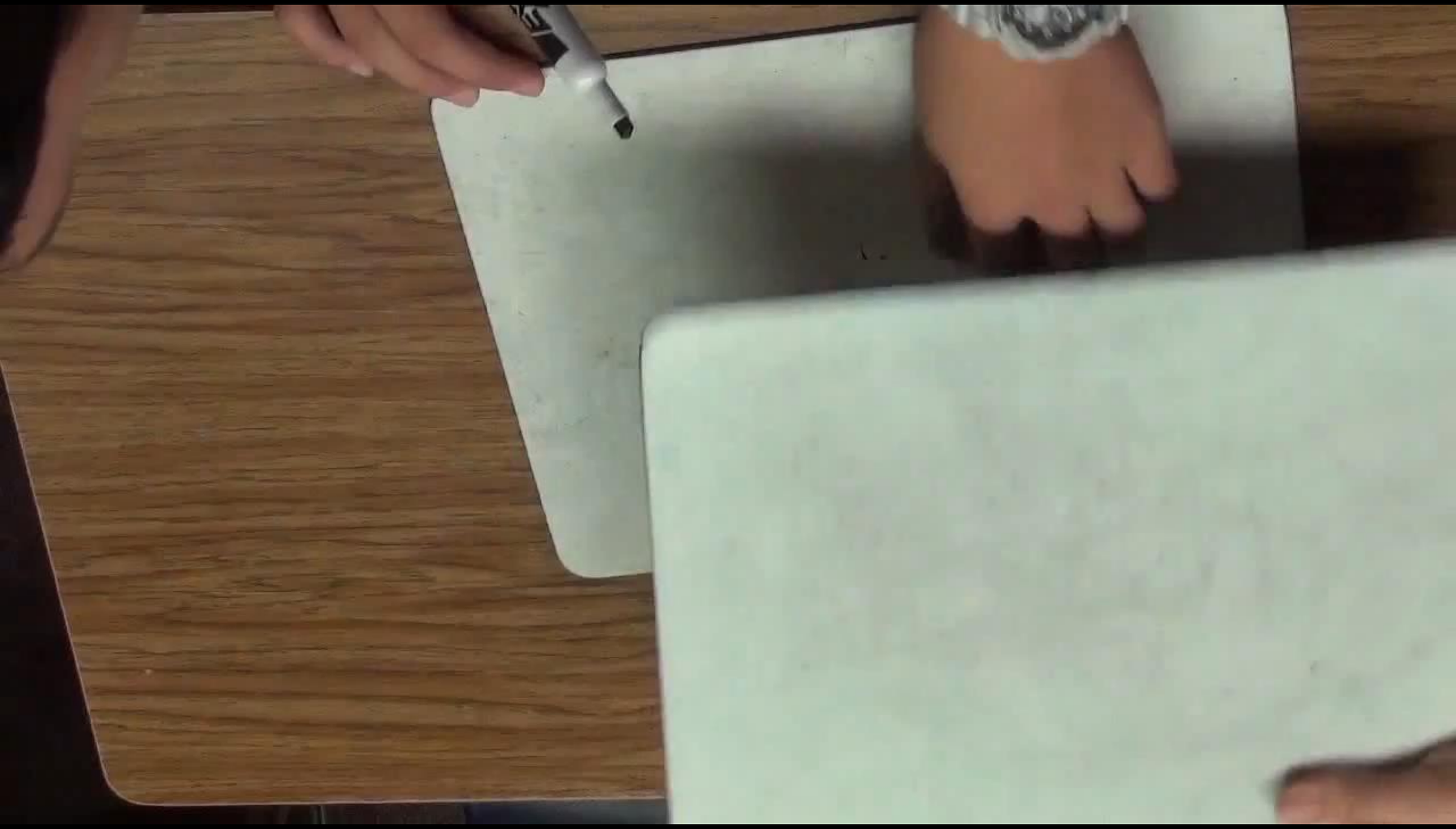
Source: http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

What is the perimeter of a rectangle that measures 8 units by 4 units?

Components of Rigor

- ☑ Procedural Skill and Fluency
- ☑ Conceptual Understanding

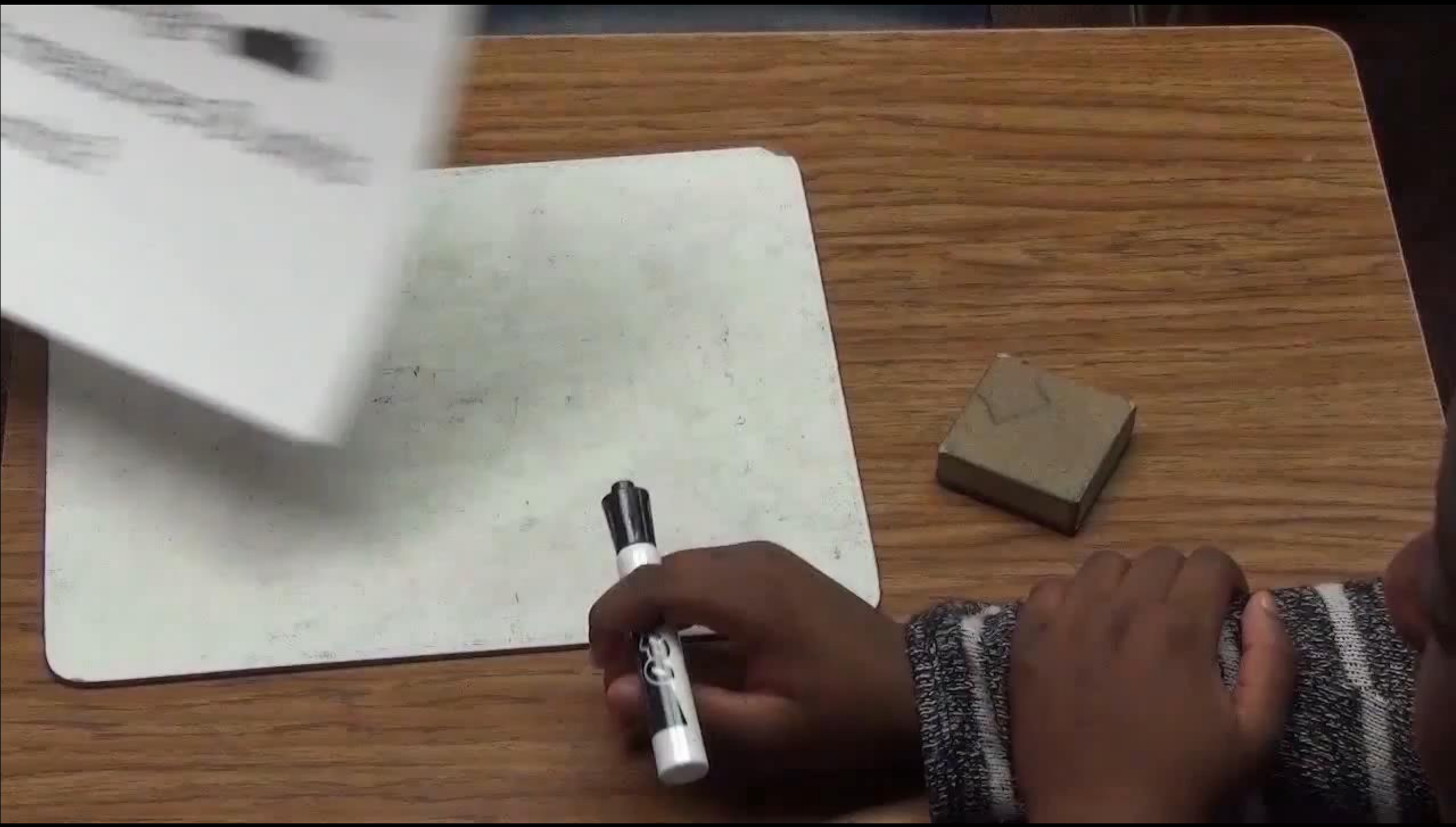
List the dimensions of a rectangle with a perimeter of 24 units.



Components of Rigor

- ☒ Procedural Skill and Fluency
- ☒ Conceptual Understanding

WHY?



Components of Rigor

- ☑ Procedural Skill and Fluency
- ☑ Conceptual Understanding

List the

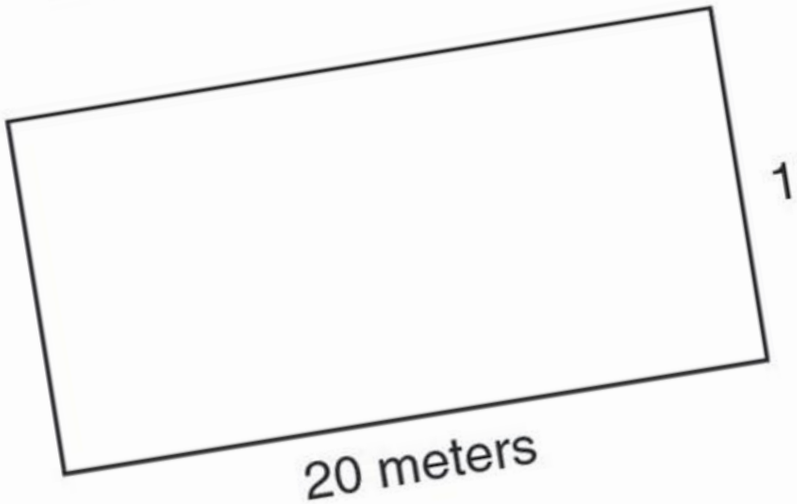
of a rectangle with a perimeter of 24 units.

Components of Rigor

- ☒ Procedural Skill and Fluency
- ☒ Conceptual Understanding

STAR

71 **A basketball court is shaped like a rectangle 20 meters long and 10 meters wide.**



What is the perimeter in meters of the court?

A 30 meters

B 50 meters

C 60 meters

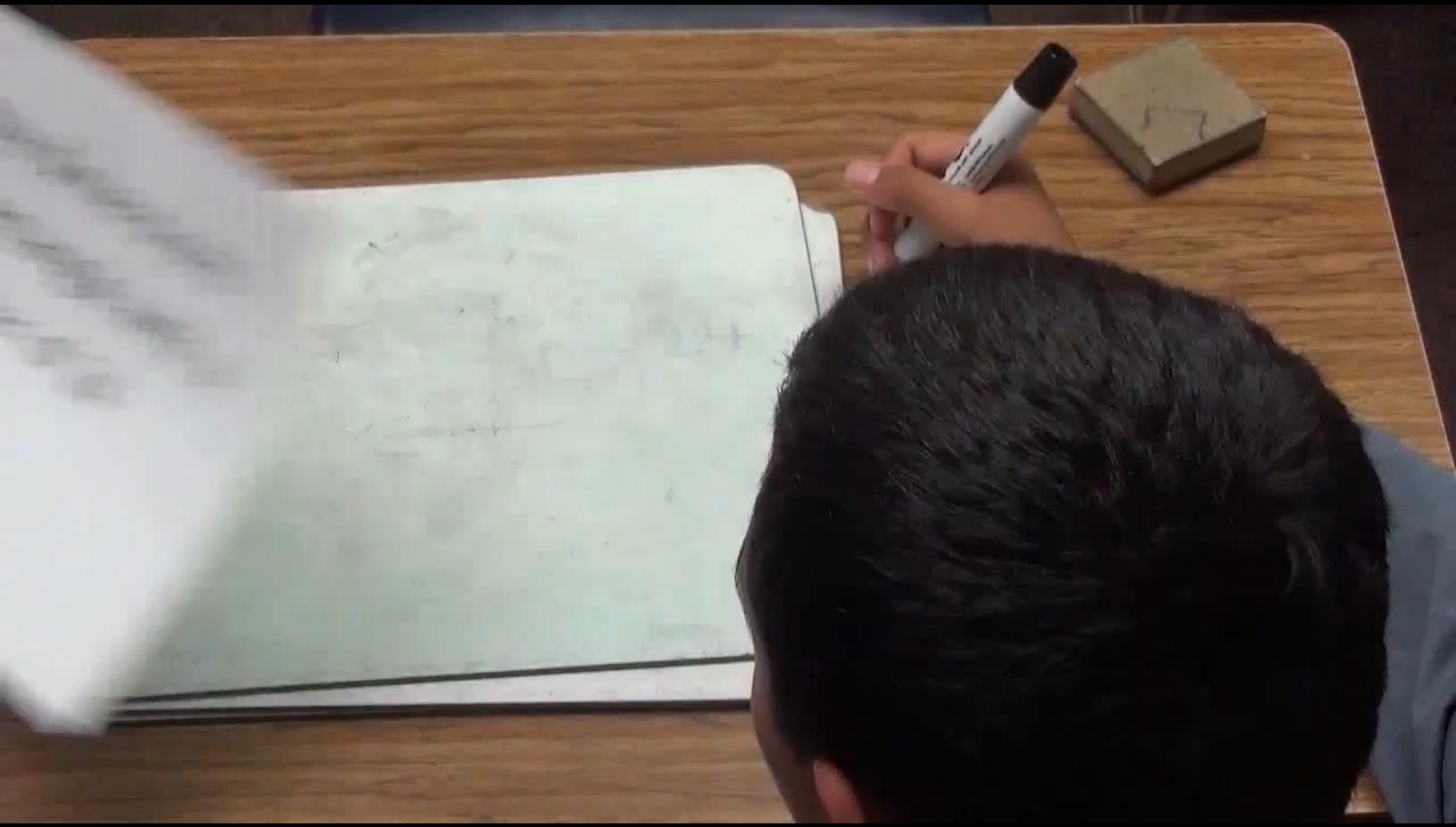
D 200 meters

Source: http://www.cde.ca.gov/ta/tg/sr/documents/cstrtqmath3.pdf

What is the perimeter of a rectangle [redacted] that measures 8 units by 4 units?

Components of Rigor

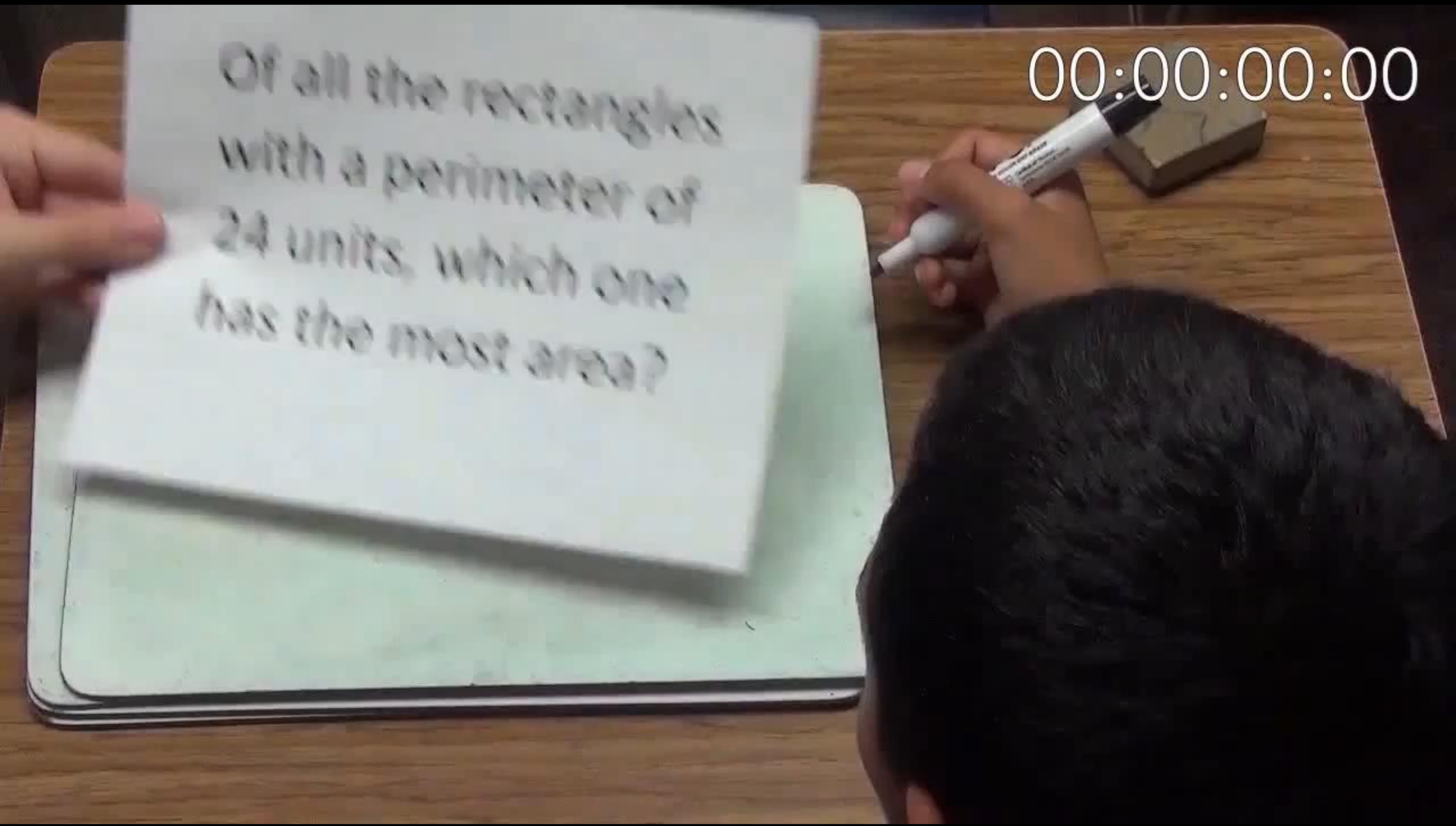
- ☑ Procedural Skill and Fluency
- ☑ Conceptual Understanding



Components of Rigor

- ☑ Procedural Skill and Fluency
- ☑ Conceptual Understanding

Of all the rectangles with a perimeter of 24 units, which one has the most area?


Of all the rectangles with a perimeter of 24 units, which one has the most area?
00:00:00:00

Components of Rigor

- ☒ Procedural Skill and Fluency
- ☒ Conceptual Understanding

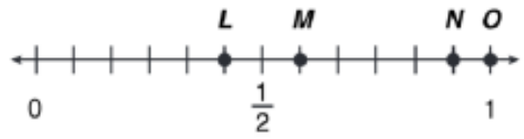
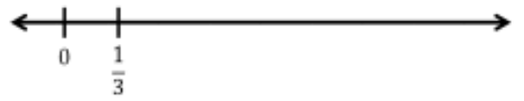
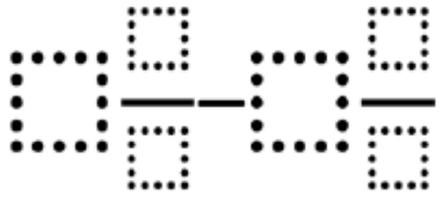
Defining the Problem

- Students appear to demonstrate "deep, authentic command of mathematical concepts" when given commonly used problems.
- However with more challenging problems, the same students seem to no longer demonstrate that command.

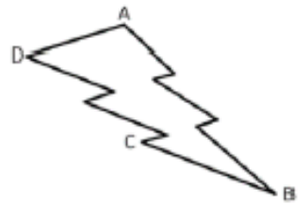
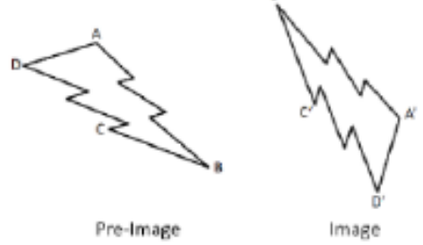
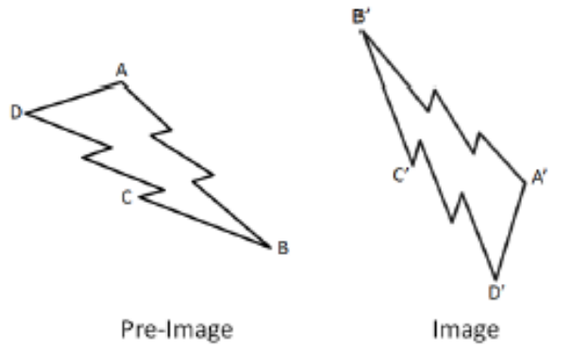
Addressing the Problem

- First, we must have a clear understanding about why these problems are different from one another.
- Next, we need to practice using these problems so that we understand how students may react to them.
- Last, we need a source that can provide us with a variety of free problems.

DOK | Distinguishing Between Depth of Knowledge Levels in Mathematics

Topic	Adding Whole Numbers	Money	Fractions on a Number Line	Area and Perimeter	Subtracting Mixed Numbers
CCSS Standard(s)	• 1.NBT.4 • 2.NBT.5	• 2.MD.8	• 3.NF.2	• 3.MD.8 • 4.MD.3	• 5.NF.1
DOK 1 Example	Find the sum. $44 + 27 =$	If you have 2 dimes and 3 pennies, how many cents do you have	Which point is located at $\frac{7}{12}$ below? 	Find the perimeter of a rectangle that measures 4 units by 8 units.	Find the difference. $5\frac{1}{2} - 4\frac{2}{3} =$
DOK 2 Example	Fill in the boxes below using the whole numbers 1 through 9, no more than one time each, so that you make a true equation. $\square\square + 53 = \square\square$	Make 47¢ in three different ways with either quarters, dimes, nickels, or pennies.	Label the point where $\frac{3}{4}$ belongs on the number line below. Be as precise as possible. 	List the measurements of three different rectangles that each has a perimeter of 20 units.	Create three different mixed numbers that will make the equation true by using the whole numbers 1 through 9, no more than one time each,. You may reuse the same whole numbers for each of the three mixed numbers. $5\frac{4}{5} - \square\frac{\square}{\square} = 3\frac{1}{20}$
DOK 3 Example	Make the largest sum by filling in the boxes below using the whole numbers 1 through 9, no more than one time each. $\square\square + \square\square =$	Make 47¢ using exactly 6 coins with either quarters, dimes, nickels, or pennies.	Create 5 fractions using the whole numbers 0 through 9, no more than one time each, as numerators and denominators and correctly place them all on a number line.	What is the greatest area you can make with a rectangle that has a perimeter of 24 units?	Make the smallest difference by filling in the boxes below using the whole numbers 1 through 9, no more than one time each. 

DOK | Distinguishing Between Depth of Knowledge Levels in Mathematics

Topic	Surface Area and Volume	Probability	Transformations	Factoring Quadratics	Quadratics in Vertex Form
CCSS Standard(s)	• 6.G.4 • 7.G.6	• 7.SP.5 • 7.SP.7	• 8.G.1 • G-CO.5	• A-SSE.3a	• F-IF.7a
DOK 1 Example	Find the surface area of a rectangular prism that measures 3 units by 4 units by 5 units.	What is the probability of rolling a sum of 5 using two 6-sided dice?	Rotate the image below 90° counterclockwise and reflect it across a horizontal line. 	Find the factors: $2x^2 + 7x + 3$	Find the roots and maximum of the quadratic equation below. $y = -3(x-4)^2 - 3$
DOK 2 Example	List the measurements of three different rectangular prisms that each has a surface area of 20 square units.	What value(s) have a 1/12 probability of being rolled as the sum of two 6-sided dice?	List three sequences of transformations that take pre-image ABCD to image A′B′C′D′.  Pre-Image Image	Fill in the blank with integers so that the quadratic expression is factorable. $x^2 + \_\_x + 4$	Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values.
DOK 3 Example	What is the greatest volume you can make with a rectangular prism that has a surface area of 20 square units?	Fill in the blanks to complete this sentence using the whole numbers 1 through 9, no more than one time each. Rolling a sum of ___ on two ___-sided dice is the same probability as rolling a sum of ___ on two ___-sided dice.	What is the fewest number of transformations needed to take pre-image ABCD to image A′B′C′D′?  Pre-Image Image	Fill the blank by finding the largest and smallest integers that will make the quadratic expression factorable. $2x^2 + 3x + \_\_$	Create a quadratic equation with the largest maximum value using the whole numbers 1 through 9, no more than one time each. $y = -\square(x-\square)^2 + \square$

Version 1.10

Complicated or Complex?

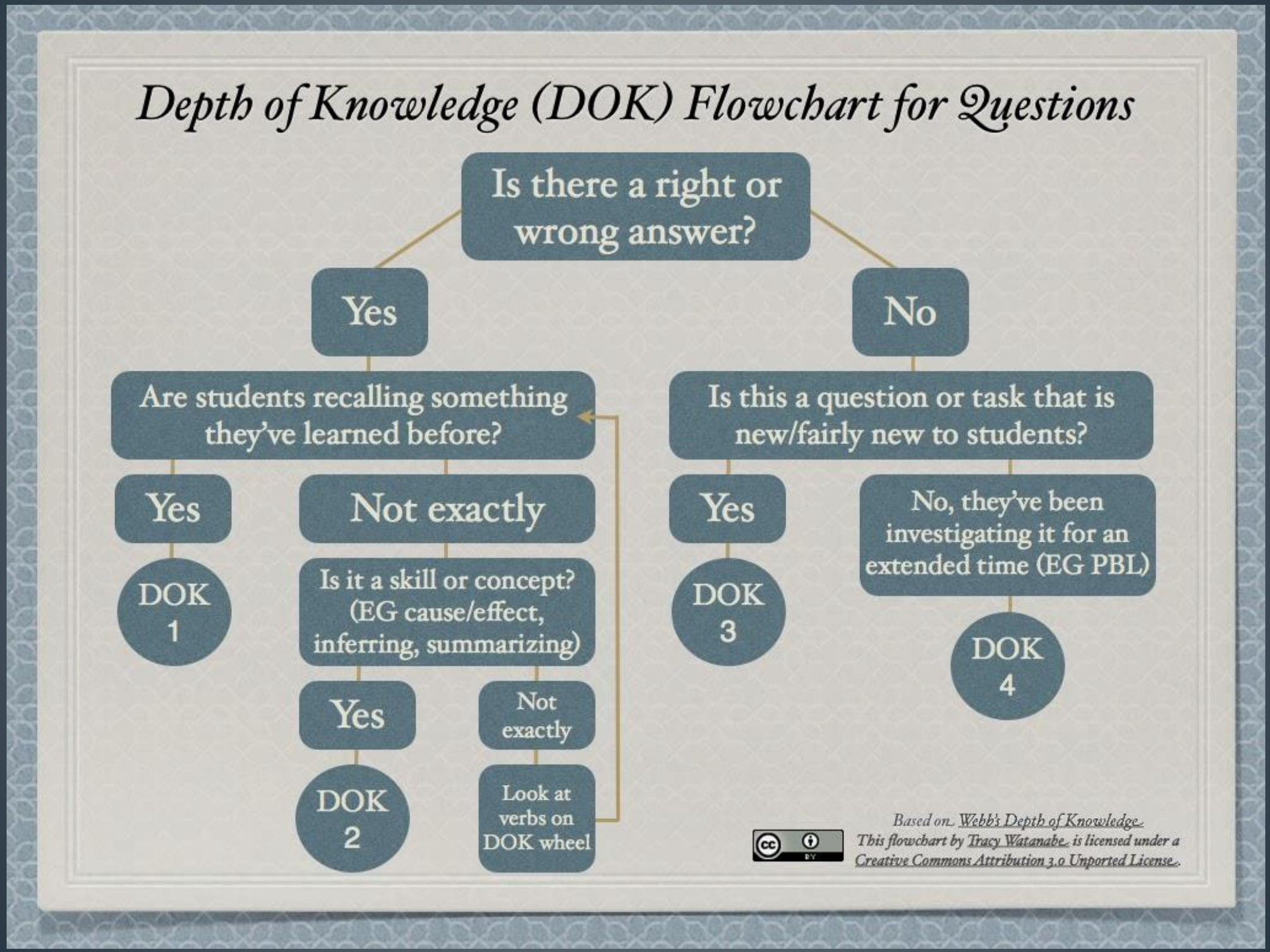

Cookie Monster Cupcakes


Nailed it

DOK Verb Wheel

Source: Unknown

DOK Flowchart for Questions



Source: Tracy Watanabe - @tracywatanabe

DOK Posters

DOK 1

Routine Thinking

- Can you recall ____?
- Can you identify ____?
- How would you describe ____?
- What might you include on a list about____?
- Can you select____?
- How can you find the meaning of____?

arrange	calculate	memorize
measure	name	recognize
recall	repeat	identify
illustrate	match	label
state	list	state

DOK 2

Conceptual Thinking

- Can you explain how _____ affected____?
- How would you apply what you learned to develop____?
- How would you summarize____?
- What do you notice about____?
- How would you estimate____?
- How could you organize____?

compare	classify	categorize
measure	graph	distinguish
predict	modify	construct
organize	infer	summarize
interpret	make observations	

DOK 3

Strategic Reasoning

- How is____related to____?
- What conclusions can be drawn?
- Can you elaborate on____?
- How would you test____?
- What evidence supports____?
- What would happen if____?
- Why is that the best answer?

assess	compare	construct
apprise	revise	hypothesize
critique	investigate	
draw conclusions		
develop a logical argument		

DOK 4

Extended Reasoning

- Write a research paper.
- What information can you gather to support your idea about____?
- Write a thesis, drawing conclusions from multiple sources.
- Apply information from one text to another to develop an persuasive argument.

design	connect	prove
analyze	critique	synthesize
create	apply concepts	

Created by Penny Lund 2013

Source: Penny Lund
http://isntitelementary.blogspot.com/

DOK Level Differences

- **Level 1: Recall & Reproduction**
  - Often a trivial application of facts.
  - Generally requires little to no cognitive effort beyond remembering the right formula.
  - Usually only one answer.
- **Level 2: Skills & Concepts**
  - Usually requires more than one step to solve.
  - Often multiple answers.
- **Level 3: Strategic Thinking**
  - Usually requires critical thinking about the best way to approach a problem.
  - May be multiple answers or a single optimal answer.
  - Often challenging enough to make your head hurt.
- **Level 4: Extended Thinking**
  - In mathematics these are generally represented by performance tasks or problem-based lessons.

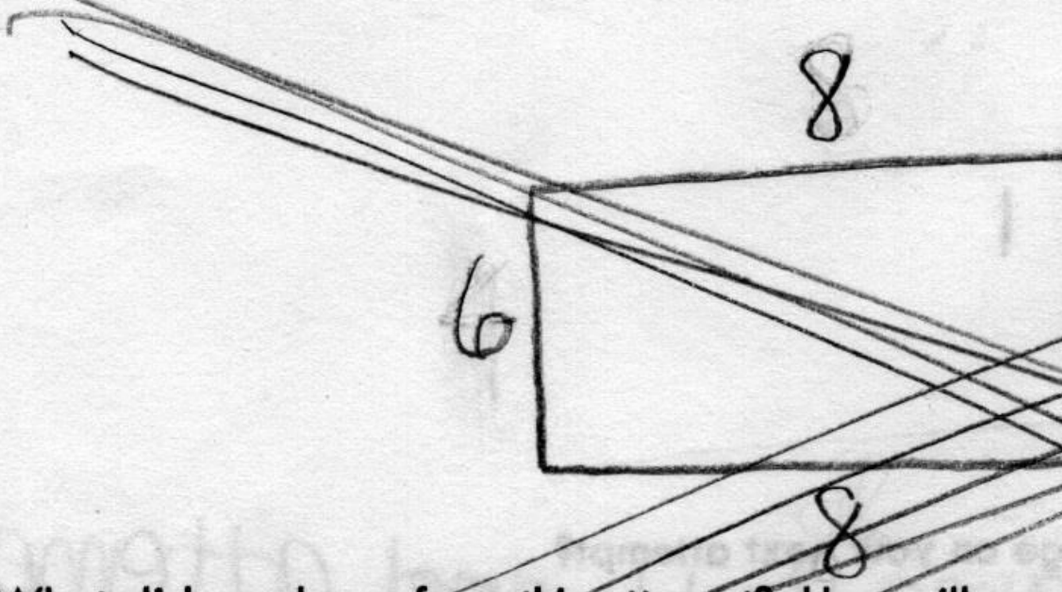
Adding Decimals

Use the numbers 1 through 9, exactly one time each, to fill in the boxes and make three decimals whose sum is as close to 1 as possible.

$$\begin{array}{r} 0.\square\square\square \\ +\,0.\square\square\square \\ \hline \end{array}$$

First attempt:

Points: ____/2 attempt ____/2 explanation



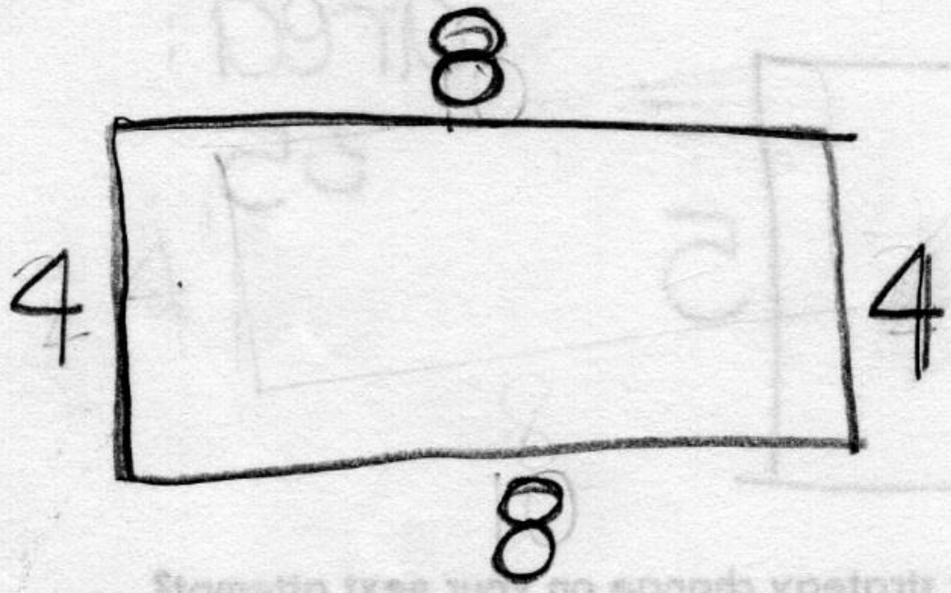
area:
48

What did you learn from this attempt? How will your strategy change on your next attempt?

This attempt doesn't equal 24.

Second attempt:

Points: ____/2 attempt ____/2 explanation



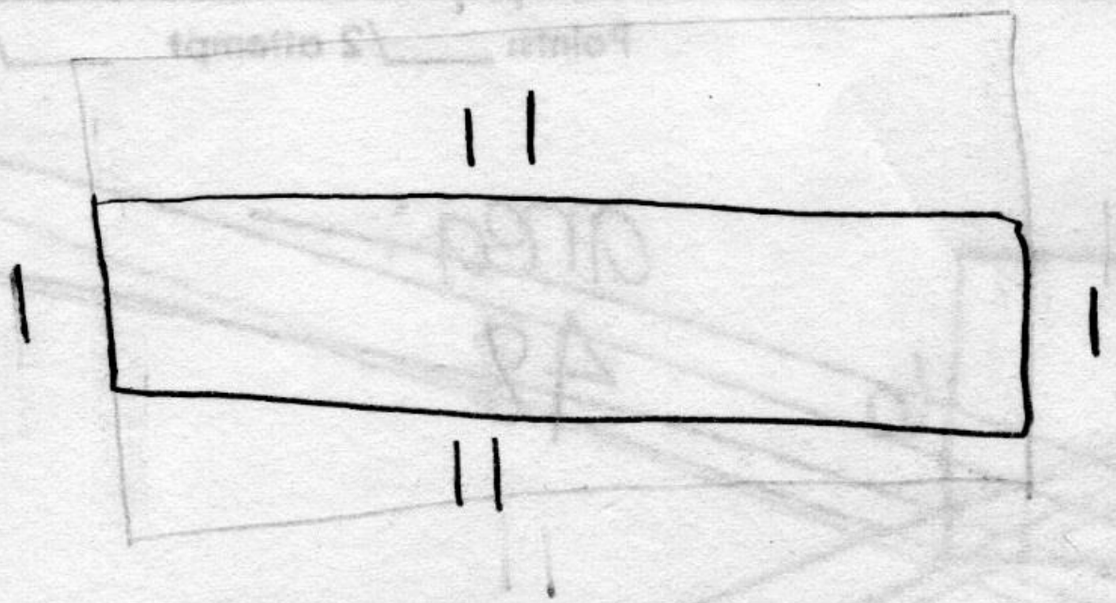
area:
32

What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter was 24, and the area was 32 but I think there's a bigger #

Fourth attempt:

Points: ____/2 attempt ____/2 explanation

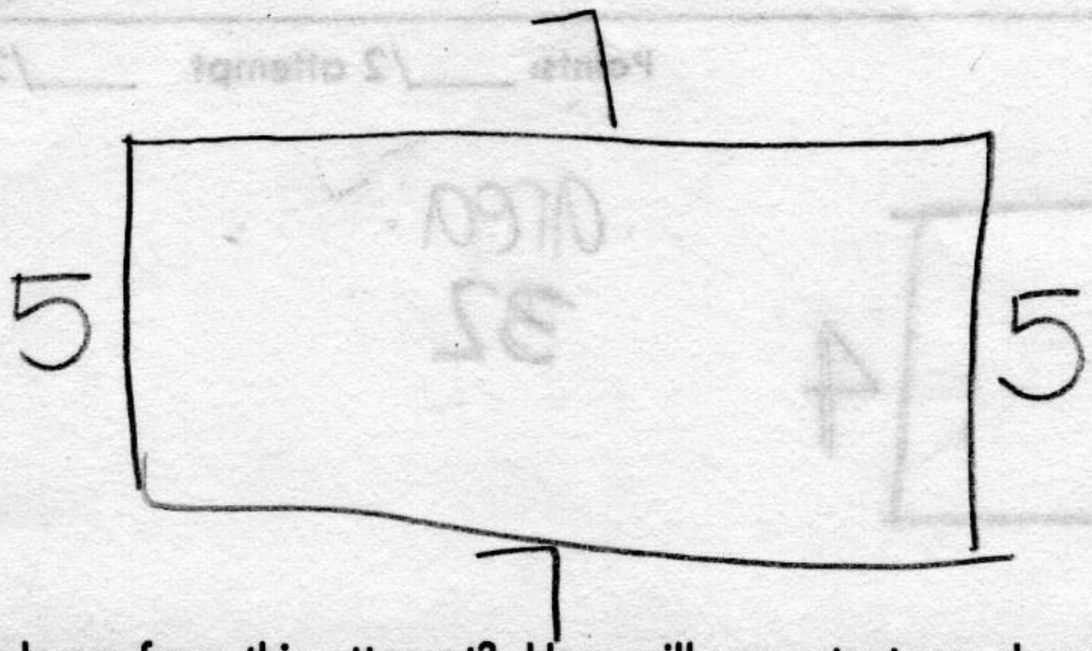


What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter is 24, but the area is 11 and attempt #2 the area is 32
Strategy: Use #'s with more than one raw.

Fifth attempt:

Points: ____/2 attempt ____/2 explanation



area:
35

What did you learn from this attempt? How will your strategy change on your next attempt?

DOK FAQ

- *When will students ever use this?*
- *What DOK level should I start students off with?*
- *How do teachers fit these problems into their pacing?*
- *How do I help prevent students from giving up after trying the problem once or twice?*
- *Where can I find other DOK 2 and DOK 3 problems or submit ones I've made?*

Open Middle

Challenging math problems worth solving

NEW OPEN M

Exponents a f Operations

February 10, 2015 Leave

Directions: Find 3 positive i t add up to 10. Place each number into one of the blanks to find the largest possible result. Source: Zack iller (@zmill415) **Read More »**

Create Squares

February 10, 2015 2 Comments

Directions: Create a square with one of the vertices at (2,3). Fill in the blanks with whole numbers 0 through 9, using each number at most once, to show the rest of the vertices of the square. Bonus: Find more than one set of vertices. Source: John Mahlstedt (@jdmahlstedt) **Read More »**

Solution of Two Linear Equations

February 10, 2015 Leave a comment

Directions: Using the Integers 0-9 (without duplication), provide four sets of points that represent two distinct lines. These lines can be written as two linear equations. Then provide a fifth point that represents the intersection (or solution) of those equations. Line 1: (__, __) and (__, __) Line 2: (__, __) and (__, __) Solution (__, __) Source: Bryan Anderson **Read More »**

Bingo card

February 5, 2015 1 Comment

Directions: In a standard game of BINGO, the cards are labeled with numbers 1 through 75. If it was possible, which card would you choose: a card with all of the same number or a standard bingo card? Source: Nanette

OPEN MIDDLE WORKSHEET

Download the Open Middle Worksheet:
Version 1.1

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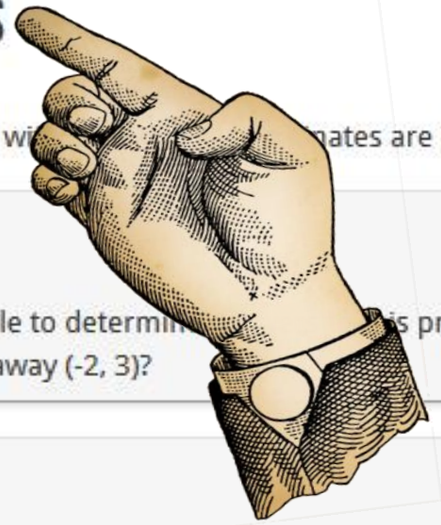
COMMON CORE STATE STANDARDS

- Grade 1 (6)
  - Number & Operations in Base Ten (3)
  - Operations & Algebraic Thinking (3)
- Grade 2 (6)
  - Measurement & Data (2)
  - Number & Operations in Base Ten (4)
- Grade 3 (11)
  - Measurement & Data (6)
  - Number & Operations in Base Ten (3)
  - Number & Operations—Fractions (2)

Open Middle

Challenging math problems worth solving

EQUIDISTANT POINTS



Directions: How many points wi nates are 5 units away from (-2, 3)?

Hint

Which methods are available to determi is problem? What shape is defined by *all* of the points that are 5 units away (-2, 3)?

Answer

12 points: (-5, 7), (-7, 3), (-5, -1), (-2, -2), (3, 3), (1, -1), (-2, 8), (1, 7), (2,6), (-6, -6), (-6, 0), and (2, 0)

Source: Dylan Kane

Print

Tagged with: 8.G.8 | DOK 2: SKILL / CONCEPT | DYLAN KANE | G-GPE.1

« Previous: Cone and Cylinder Volumes

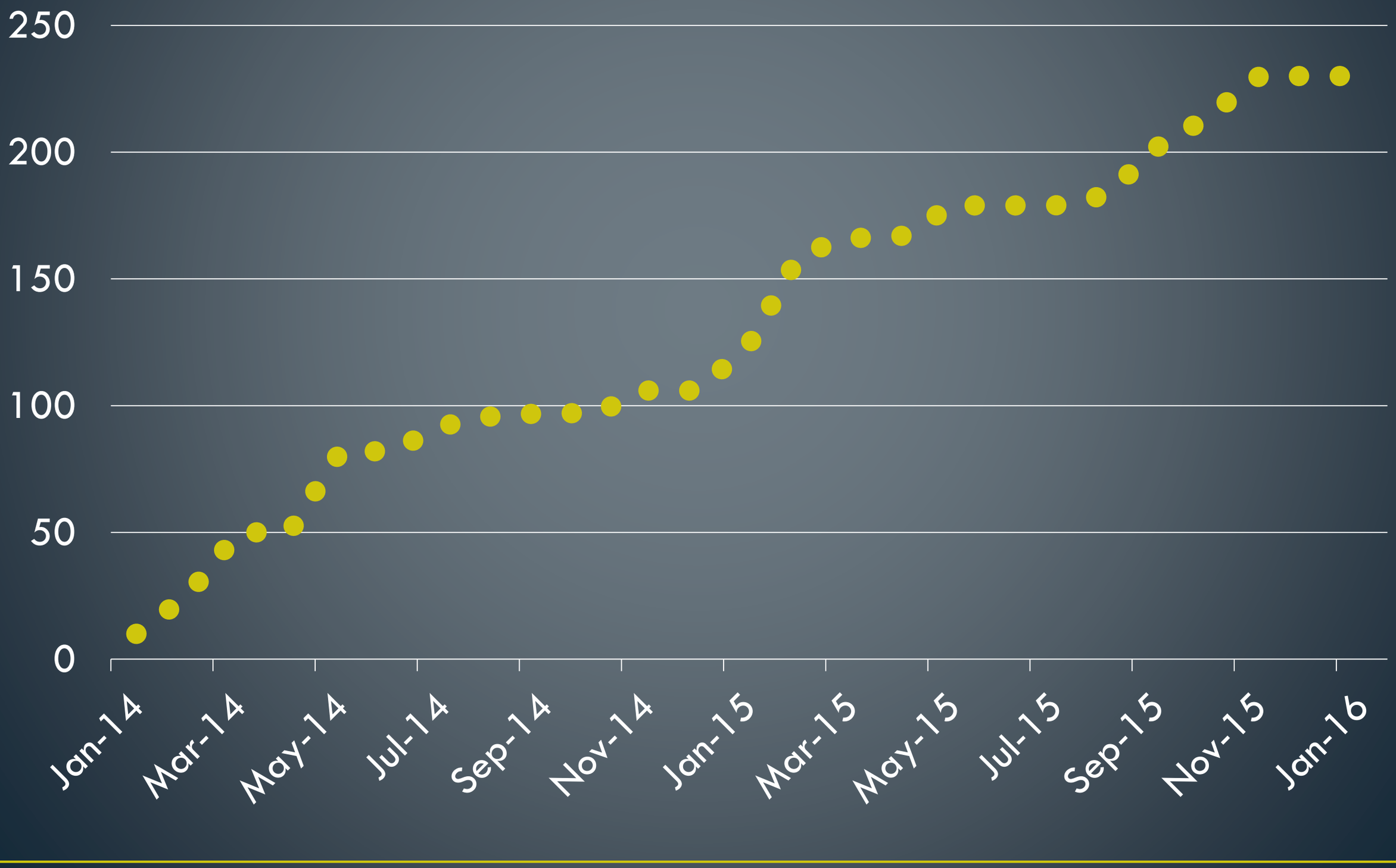
Next: Pythagorean Shell »

LEAVE A REPLY

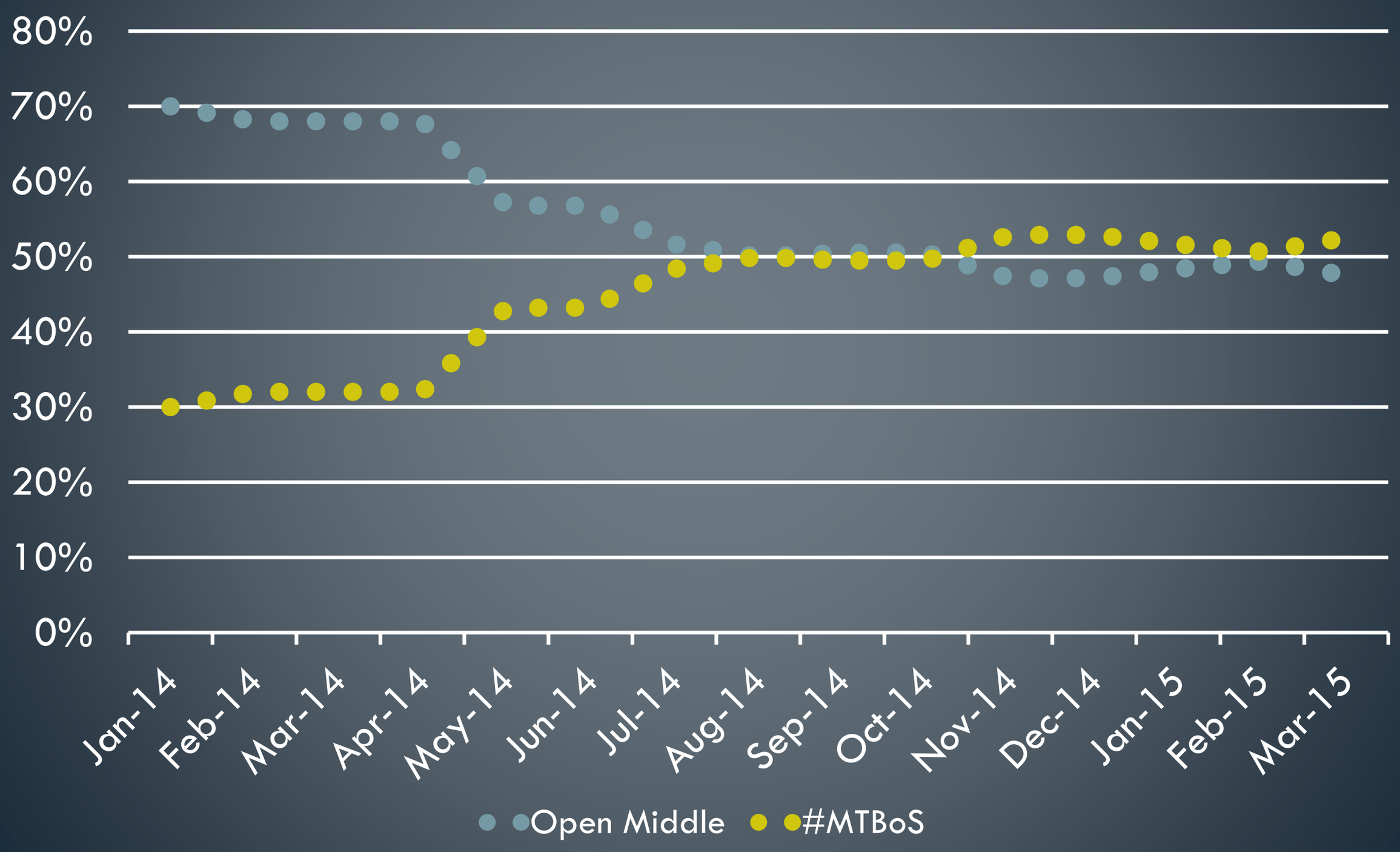
COMMON CORE STATE STANDARDS

- Grade 1 (6)
  - Number & Operations in Base Ten (3)
  - Operations & Algebraic Thinking (3)
- Grade 2 (6)
  - Measurement & Data (2)
  - Number & Operations in Base Ten (4)
- Grade 3 (11)
  - Measurement & Data (6)
  - Number & Operations in Base Ten (3)
  - Number & Operations—Fractions (2)

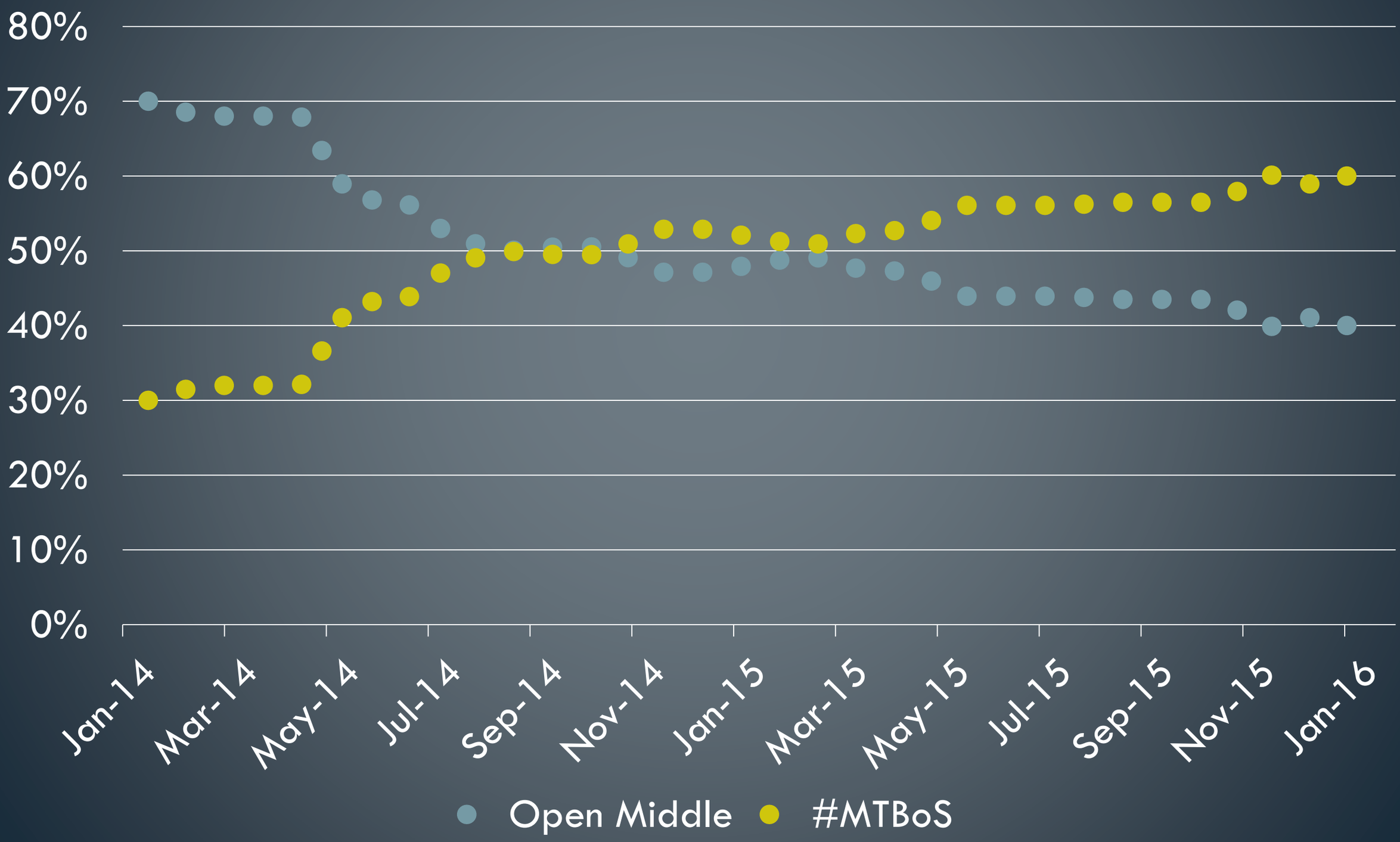
Total Open Middle Problems



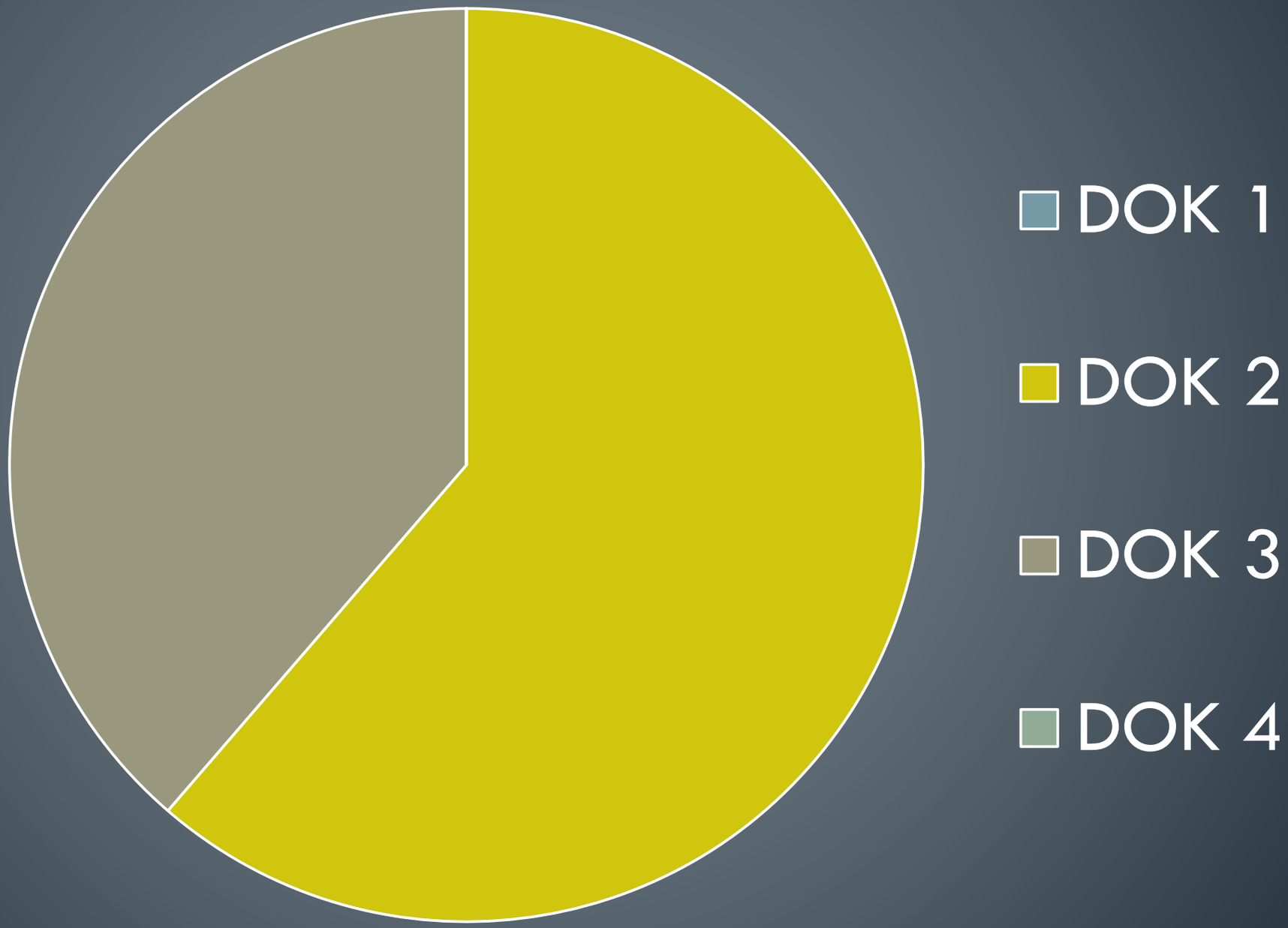
Open Middle Author Percentages



Open Middle Author Percentages

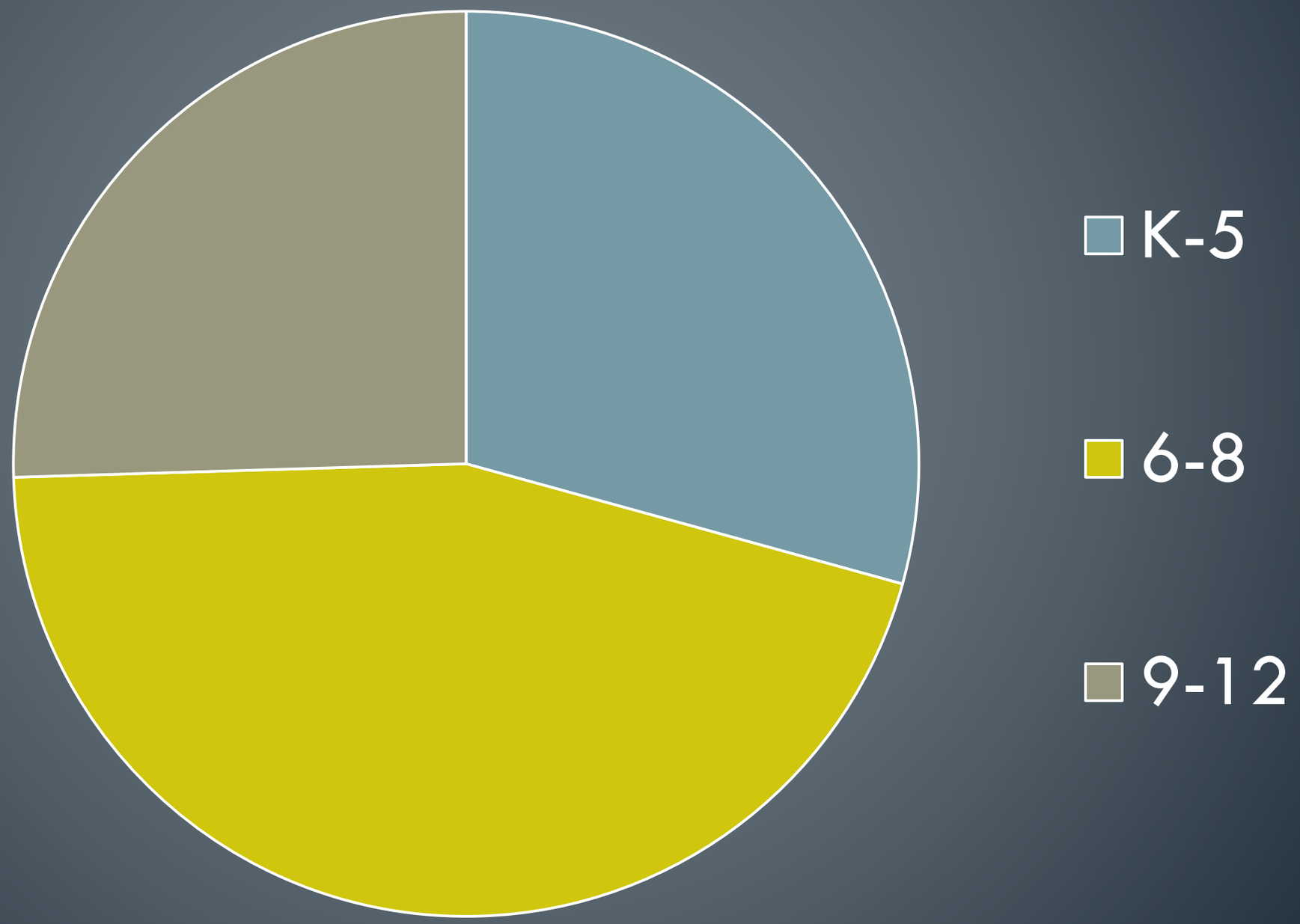


Problems by DOK Level



Note: Data as of February 2016

Problems by Grade Band



Note: Data as of February 2016

COMMON CORE
STATE STANDARDS INITIATIVE

Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades. To help students meet the standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skills and fluency, and application.

Source: http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

Call to Action

- Commit to one of these choices:
  - Implement a single DOK 2 or DOK 3 problem from openmiddle.com in your classes within the week.
  - Put a DOK 2 question from openmiddle.com on your next assessment.

Contact

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nctm16