# Arizona 

## Department of Education

## ROBERT KAPLINSKY

シ ( $\boldsymbol{f}$ @robertkaplinsky

## Goals

$\square$ Engaging problem solving
$\square$ Real world problem-based learning
$\square$ Higher depth of knowledge problems
$\square$ Better implementation
Improve our ability to ask questions
$\square$ Practice preparing to implement a lesson
$\square$ Figure out how to deal with uncomfortable situations





##   HAMBURGER FRENCH FRIES SHAKES

OPEN 10:30 a.m. to 1:00 a.m. Fri. and Sat. until 1:30 a.m.

## 2004-10-31

YOUR GUEST NUMBER IS
98

$$
\begin{aligned}
& \text { IN-N=OUT BURGER LAS VEGAS EASTERN } \\
& 2004=10-31 \\
& 1651598 \\
& 8: 21 \text { PM }
\end{aligned}
$$

Cashier: SAM

## GLEST <br> \#: 98

## Counter-Eat In

 DblDbl98 Meat Pty KChz
2.65
88.20

Counter-Eat In
TAX 7.50 x
90.85

Amount Due
6.81

CASH TENDEA
Change
$\$ 97.66$
$\$ .00$
$2004-10-31$

## Cashier: SAM <br> GLEST <br> H: 98

## Counter-Eat In

## Dblobl <br> 98 Meat Pty XChz <br> $$
\begin{array}{r} 2.65 \\ 88.20 \end{array}
$$ <br> Counter-Eat In TAX 7.50 x <br> Amount Due <br>  88.20 88.20 <br> 90.85 97.61 97.66

## CASH TENDER Change

$\$ 97,66$ $\$ .00$

## 2004-10-31

$$
8: 21 \mathrm{PM}
$$

|  |  | ¢ |
| :---: | :---: | :---: |
| Hamburger w/Onion | 243 | 390 |
| Cheeseburger w/Onion | 268 | 480 |
| Double-Double w/Onion | 330 | 670 |



|  |  |
| :--- | :--- |
| Layers | Cost |
| 1 | $\$ 1.75$ |
| 2 | $\$ 2.65$ |
| 3 | $\$ 3.55$ |
| 4 | $\$ 4.45$ |
| $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ |
| 20 | $\$ 18.85$ |
| $\cdot$ | $\cdot$ |
| $\cdot$ | $\cdot$ |
| 100 | $\$ 90.85$ |
| $\cdot$ | $\cdot$ |
| $\cdot$ | $\$ 1.75+(\mathrm{N}-1)^{*} \$ 0.90$ |
| N |  |

## Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

## Levels of Convincing



Construct a viable argument

Critique the reasoning of others

Inspired by Connecting Mathematical Ideas by Jo Boaler and Cathy Humphreys
bun + produce + meat + cheese + meat + cheese $=\$ 2.65$
bun + produce + meat + cheese
$=\$ 1.75$
meat + cheese $=\$ 0.90$

## The Reality

- Students needed guidance to figure out a layer's cost
- Not every class is ready to go straight to $100 \times 100$
- Common wrong answers included:
- $\$ 175.00$ ( $\$ 1.75 \times 100$ cheeseburgers)
- \$132.50 (\$2.65 x 50 Double-Doubles)
- Students had equations that had more than X patties
- Students were surprised to see three different equations:
- Starting with a Double-Double
- Starting with a cheeseburger
- Starting with produce and bun only


## STUDENT WORK



The only difference between a double double and a cheeseburger is one patty and one slice of cheese so you subtract the prices of the two to find the price of only one patty $\&$ cheese. You then use that number $(.90)$ \& subtract it from the cost of one whole cheeseburger to find the price of all the extra stuff. Multiply by 100

What is your conclusion?
A $100 \times 100$ at in-n-out cost $\$ 90.85$. To solve that, you start by subtracting the price of a cheeseburger from a double double. The answer (.90) is the price of a patty and cheese slice. You multiply (.90) by one less patty than what you want. $(x-1)$, and you add the price of a cheeseburger $(1.75)$. You end up with the eq. $[y=.90(x-1)+1.75$.].
For the $100 \times 100$, you plug in 100 to the $(x)$ and you end Up with $\$ 90.85$.

$$
\left[\begin{array}{l}
y=.90(100-1)+1.75 \\
y=89.10+1.75 \\
y=90.85
\end{array}\right]
$$

What 15 your conclusion?
Figure the price difference from the Double-Double with a cheeseburger. Then find out the prize for the produce and cheese-bees. get total into 90.85 NHA

$$
\begin{gathered}
\text { There are } 125 \\
\text { sheep and } 5 \text { dogs } \\
\text { in a flock. How old } \\
\text { is the shepherd? }
\end{gathered}
$$

## Of the 32 students I interviewed...

- $75 \%$ of them gave me numerical responses
- 2 students calculated the answer to be $130(125+5)$
- 2 students calculated the answer to be 120 (125-5)
- 12 students calculated the answer to be $25(125 \div 5)$
- 0 students calculated the answer to be 625 ( $125 \times 5$ )
- 4 students stated that they guessed their answer (90, 5,42 , and 50)
- 4 students tried to divide 125 by 5 but could not correctly implement the procedure


## Takeaways

- Making sense of mathematics
- Intellectual autonomy
- Intellectual autonomy is about being able to think for yourself and not being dependent on others for the direction and control of one's thinking.


## What Does the NHTSA Say?

## Kev Statistics and Consumer Insights:

- Motor vehicle crashes are the leading cause of death for children age 1 through 12 years old. ${ }^{1}$


## According to a NHTSA study, 3 out of 4 kids are not as secure in the car as they should be because their car seats are not being used correctly.

${ }^{1}$ Source: Based on the latest mortality data currently available from the CDC's National Center for Health Statistics.


- "because they have their child in the right seat"
- "because their car seats are not being used correctly"

IF YOUR CHILD IS IN THE RIGHT CAR SEAT.
Ad
council
VISIT SAFERCAR.GOV/THERIGHTSEAT


## KNOW FOR SURE

IF YOUR CHILD IS IN THE RIGHT CAR SEAT.

## VISIT SAFERCAR.GOV/THERIGHTSEAT

 NHTSA
## WHAT IS THE PURPOSE OF <br> A K-12 <br> EDUCATION?

- College readiness
- ACT National Curriculum

Survey

- Surveyed 9,937 educators


## "Well" or "Very Well" Prepared for College



Source: http://www.act.org/research/policymakers/pdf/NCS-PolicySummary201 2.pdf

# WHAT IS THE PURPOSE OF <br> A K-12 <br> EDUCATION? 

College readiness

- Career readiness
- Association of American Colleges and Universities survey
- Surveyed over 300 employers with at least 25 employees and many new hires
Critical thinking and
analytical reasoning skills
Analyzing and solving
complex problems
Communicating
effectively orally
and in writing
Applying knowledge and
skills to real-world setting
Working w/ numbers and
understanding statistics






## Sinkhole Dimensions

- Slate: "A sinkhole, 65 feet across and 100 feet deep"



# How To Fix a Giant Sinkhole 



## A sinkhole in Guatamala

It's not clear whether cement is the best option, however. A 6,500-cubic-foot wad of concrete may serve to concentrate water runoff in other areas, leading to more sinkholes. Many engineers prefer the graded-filter technique, in which the hole is filled with a layer of boulders, then a layer of smaller rocks, and, finally, a layer of gravel. This fills the hole, more or less, while permitting water to drain through the area.

## 2010 Guatemalan Sinkhole

Kaplinsky, Robert
To:

Hi Brian,

I am using your "How to Fix a Giant Sinkhole" article for a math lesson on volume of a cylinder. I have one question for you. You mentioned.
"It's not clear whether cement is the best option, however. A 6,500-cubic-foot wad of concrete may serve to concentrate water runoff in other areas, leading to more sinkholes."

Can you please tell me where you got 6500 cubic feet from? Did you do $65 \times 100$ ? We get something closer to 342,000 cubic feet.

Thanks,
Robert

## Reply Reply All Forward v 酔 目

## Re： 2010 Guatemalan Sinkhole

## Brian Palmer

To：
Kaplinsky, Robert

Apparently you picked the wrong article for a math lesson！I apologize．It appears you are correct．I can＇t find anything in my notes to save myself－－I think I just screwed up．Dunce cap for me．



## Student Reflections

- "I didn't say his answer was wrong since he is supposed to know more than an average $8^{\text {th }}$ grader."
- "Even though Brian was wrong, no one corrected him, because of fear of being wrong and lack of confidence in ourselves."
- "I didn't say anything when we were shown the 'right' answer because I thought that it must be right because he's the author, but I knew in my mind he was actually wrong."
- "I think that I should be the one who argues for my opinion, not just listening to others and accepting that my answer is wrong all the time."





 4 what net now hat -





OLD
(Boring)

## NEW

Diamond

Shreddies

Cereal

## NEW <br> (Exciting!)




## "Kraft Foods saw an immediate 18\% increase in baseline sales of Shreddies within the first month alone, and for months thereafter."



## The Reality

- Some students felt anxious about not having enough information to take a guess.
- However, guessing helped students determine what they knew and needed to know.
- Students modeled the problem differently:
- Cylinder
- Semi-sphere
- Truncated cone
- Students didn't know what units to use.


## STUDENT WORK



## How Do We Assess Student Work?

- Option \#1 - Don't assess the problem
- Option \#2 - Use general purpose rubric
- Option \#3 - Use a problem-specific rubric


## Option \#2 - General Purpose Rubric

- One point for student reaching the correct conclusion.
- One point for student providing sufficient reasoning to support this conclusion.

In order to solve this problem, you need the width and the height of the hole. Once you have it you plus them into the equation $r^{2} \pi \cdot h$ which is to find the volume. once you find the volume you will know how much cement you will need to order so that you could fill that cement hole, which in this case would be 341,119 feet of cement.

This particular sinkhole in Guatemala City, was about 20 meters ( 66 feet) in diameter and about 30 meters ( 100 feet) deep. We are trying to find the volume of the hole to figure cut how much material is needed to fill it. I used the cylinder volume formula $\left(v=\pi r^{2} h\right)$. When you plug in the radius and the height, you get $v=\pi(33)^{2}(100)$. I did not use bb as my radius, because that is my diameter. Radius is half of the diameter. After you solve, you are left with $342,119,44 \mathrm{ft}^{3}$. You don't use $\mathrm{ft}^{2}$ or ft because the hole 133 dimensional. From here on, you just use the material cost and amount to find the price of the job.

In order to fill the smkhide with cement. They will need 342,119 $\mathrm{ft}^{3}$ of cement. How is this possible?
Diameter $=66$ feet, but we are looking for radius.
$66 / 2=33$ Now we got our radius which is 33.
$r=33$
So we have a radius and height.
Depth $=100$ feet. we can use the volume of a cyunder formula. which is $v=\pi r^{2} h$

$$
\begin{aligned}
& V=\pi(33)^{2} \cdot 100 \\
& V=\pi(1089) \cdot 100 \\
& V=3421.20 \\
& V=342119.44
\end{aligned}
$$

## Option \#3 - Problem-Specific Rubric

| Requirement | Possible <br> Points | Points <br> Earned |
| :--- | :--- | :--- |
| Student finds the correct answer based on <br> the dimensions used. | 3 |  |
| Student uses the correct units (i.e., cubic <br> feet/meters for volume and feet/meters <br> for length) | 1 |  |
| Student correctly uses half the diameter <br> for the radius and explains why. | 2 |  |
| Student creates a narrative using sentences <br> to explain his or her reasoning. | 2 |  |


| Correct answer | $\ldots$ | $/ 3$ | Explains $\frac{d}{2}=r$ |
| :--- | :--- | :--- | :--- |
| Correct units | $\ldots$ | $/ 1$ | Narrative w/ sentences |

What is your conclusion?
In order to solve this problem, you neal the width and the height of the hole. Once you have it you plug them into the equation $r^{2} \pi \cdot h$ which is to find the volume. Once you find the volume you will know how much cement you will need to order so that you could fill that cement hole, which in this case would be 342, 119 feet of cement.


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| :--- | :--- | :--- | :--- |
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What is your conclusion?
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& V=3421.20 \\
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$$

$$
\begin{aligned}
& \text { Complicated } \\
& \text { or Complex? }
\end{aligned}
$$

## Gookie Monster Gupcakes



## method

1. Using an electric mixer, whip the butter until it is pale. This will take at least 5 minutes on high.
2. Gradually add in the icing mixture and vanilla until well combined.
3. With the mixer running, add in food colouring until you get to the Cookie Monster colour. This may be a lot if you are using liquid food colouring or a little if using gel food colouring.
4. Add in the milk and mix until the frosting puffs up.
5. Fill a piping bag with a fluted nozzle and pipe on icing.
6. With the writing icing, place black spots on the marshmallows for pupils.
7. Place on each cupcake.
8. Cut cookies in half and place in 'mouth'.

| Landing Page ? |  | Pages / <br> Session <br> Avg. Session <br> Duration |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

How much is one third of a cup of butter?
All News Shopping Maps Images More v Search tools

# WHAT ISN'T MATHEMATICAL MODELING? 

- It is not modeling in the sense of, "I do; now you do."
- It is not modeling in the sense of using manipulatives to represent mathematical concepts.
- It is not modeling in the sense of a "model" being just a graph, equation, or function.
- It is not just starting with a real world situation and solving a math problem.
- It is not beginning with the mathematics and then moving to the real world.


## Content and Language Objectives using

Content Objective Example:
SWBAT apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. (MP4)

Language Objective Example:
SWBAT understand and use stated assumptions, definitions, and previously established results in constructing arguments. (MP3)

- In early grades, this might be as simple as writing an addition equation to describe a situation. (MP4)
- In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. (MP4)
- By high school, a student might use geometry to solve a design problem or use a function to describe how one


## Get Updates!

- Receive the latest news, blog posts, and lessons by email.
- To get them:
- Scan this QR code or
- Go to
tinyurl.com/RKupdates



# WHAT DOES IT LOOK LIKE. 

- when students have procedural skill but not conceptual understanding or the ability to apply mathematics?
- when students can work with numbers but cannot:
- critically think
- applying knowledge and skills to real-world settings
- analyze and solve complex problems


# How far apart are the exits on this freeway: Jct 90 and Jefferson Blvd? 





## 1 1 <br> 1


l’

$$
\geq 1
$$



# The Four C's 

- Communication - Curiosity
- 6.G. 4 - Represent threedimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures.
7.G.6 - Solve real-world and mathematical problems involving area, volume and surface area.
- 8.G.9 - Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.
- G-GMD.3-Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
: 8.G.3 Describe the effect of dilations, translations, rotations, and reflections on twodimensional figures using coordinates.
- G-C0.6 Use geometric descriptions of rigid motions to transform figures and to predictt the effect of a given rigid motion on a given figưre.
- A-CED. 1 - Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- F-IF.7a - Graph linear and quadratic functions and show intercepts, maxima, and minima.
6.RP.2 - Understand the concept of a unit rate


## The Four C's

- Communication
- Curiosity
- Critical Thinking


## Problem Solving Framework

- Inspired by Geoff Krall's resources at emergentmath.com



## The Four C's

- Communication
- Curiosity
- Critical Thinking
- Content Knowledge


## Goals

$\square$ Engaging problem solving
$\square$ Real world problem-based learning
$\square$ Higher depth of knowledge problems
$\square$ Better implementation
Improve our ability to ask questions
$\square$ Practice preparing to implement a lesson
$\square$ Figure out how to deal with uncomfortable situations

## Questioning Scenarios

- The activity begins with teachers in groups of three taking the roles of teacher, student, or observer.
- The individuals playing the role of teacher and student each receive a slip of paper describing their scenario.
- The individual playing the role of observer waits to record all of the teacher's questions to the student.
- Once the activity begins, the teacher will talk to the student in the context of the scenario they read about on the slips of paper.

What did you get for the area of the circle with a radius of 2 units?

Great. Do you have any questions?

What did you get for the area of the circle with a radius of 2 units?

Great. How did you get your answer?

The radius is 2 so I plugged it into 2 pi $r$ and got 4 pi.


## Goals

$\square$ Engaging problem solving
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I Improve our ability to ask questions
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## COMMON CORE STATE STANDARDS INITIATIVE

CCSS.MATH.CONTENT demand of Apply the NIENT.4.MD.A. 3 mana harder or problems. meetth
equal intensity, of each grade: con ct skills and fluency, and application.

Source: http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

# What is the perimeter 

 of a rectangle that measures 8 units by 4 units?
## Components of Rigor

## $\square$ Procedural Skill and Fluency

$\square$ Conceptual Understanding

## List the dimensions of

a rectangle with a perimeter of 24 units.

## Components of Rigor

 [-] Procedural Skill and Fluency[I Conceptual Understanding



## Components of Rigor

## $\square$ Procedural Skill and Fluency

$\square$ Conceptual Understanding

LIst ur angle with a
of a rectangle with perimeter of 24 units.

## Components of Rigor

 [-] Procedural Skill and Fluency[I Conceptual Understanding

71 A basketball court is shaped like a rectangle 20 meters long and 10 meters wide.


What is the perimeter in meters of the court?

A 30 meters
B 50 meters
C 60 meters
D 200 meters

Source: http://www.cde.ca.gov/ta/tg/sr/documents/cstrtqmath3.pdf

## What is the perimeter

 of a rectanglethat measures 8 units by 4 units?

## Components of Rigor

## $\square$ Procedural Skill and Fluency

$\square$ Conceptual Understanding


## Components of Rigor

## $\square$ Procedural Skill and Fluency

$\square$ Conceptual Understanding

Of all the rectangles with a perimeter of 24 units, which one has the most area?

Of all the rectangles with a perimeter of 24 units, which one
has the most area?

## Components of Rigor

 [-] Procedural Skill and Fluency[I Conceptual Understanding

## Defining the Problem

- Students appear to demonstrate "deep, authentic command of mathematical concepts" when given commonly used problems.
> However with more challenging problems, the same students seem to no longer demonstrate that command.


## Addressing the Problem

- First, we must have a clear understanding about why these problems are different from one another.
- Next, we need to practice using these problems so that we understand how students may react to them.
$>$ Last, we need a source that can provide us with a variety of free problems.

DOK
Distinguishing Between Depth of Knowledge Levels in Mathematics

| Topic | Adding Whole Numbers | Money | Fractions on a Number Line | Area and Perimeter | Subtracting Mixed Numbers |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline \text { CCSS } \\ & \text { Standard(s) } \end{aligned}$ | - 1.NBT. 4 <br> - 2.NBT. 5 | - 2.MD. 8 | - 3.NF. 2 | - 3.MD. 8 | - 5.NF. 1 |
| DOK 1 <br> Example | Find the sum. $44+27=$ | If you have 2 dimes and 3 pennies, how many cents do you have | Which point is located at $\frac{7}{12}$ below? | Find the perimeter of a rectangle that measures 4 units by 8 units. | Find the difference. $5 \frac{1}{2}-4 \frac{2}{3}=$ |
| DOK 2 <br> Example | Fill in the boxes below using the whole numbers 1 through 9, no more than one time each, so that you make a true equation. $\square$ $+53=$ $\square$ | Make 47\$ in three different ways with either quarters, dimes, nickels, or pennies. | Label the point where $\frac{3}{4}$ belongs on the number line below. Be as precise as possible. | List the measurements of three different rectangles that each has a perimeter of 20 units. | Create three different mixed numbers that will make the equation true by using the whole numbers 1 through 9, no more than one time each. You may reuse the same whole numbers for each of the three mixed numbers. $5 \frac{4}{5}-\square \square=3 \frac{1}{20}$ |
| DOK 3 <br> Example | Make the largest sum by filling in the boxes below using the whole numbers 1 through 9, no more than one time each. $\square$ $+$ $\square$ $=$ | Make 47\$ using exactly 6 coins with either quarters, dimes, nickels, or pennies. | Create 5 fractions using the whole numbers 0 through 9 , exactly one time each as numerators and denominators, and place them all on a number line. | What is the greatest area you can make with a rectangle that has a perimeter of 24 units? | Make the smallest difference by filling in the boxes below using the whole numbers 1 through 9, no more than one time each. |

## DOK | <br> Distinguishing Between Depth of Knowledge Levels in Mathematics

| Topic | Surface Area and Volume | Probability | Transformations | Factoring Quadratics | Quadratics in Vertex Form |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \hline \text { CCSS } \\ & \text { Standard(s) } \end{aligned}$ | $\begin{array}{ll} \hline- & 6 . \mathrm{G} .4 \\ - & 7 . \mathrm{G.} 6 \end{array}$ | $\begin{aligned} & \hline \text { - 7.SP. } 5 \\ & \text { - 7.SP. } 7 \end{aligned}$ | - 8.G. 1 <br> - G-CO. 5 | - A-SSE.3a | - F-IF.7a |
| DOK 1 <br> Example | Find the surface area of a rectangular prism that measures 3 units by 4 units by 5 units. | What is the probability of rolling a sum of 5 using two 6-sided dice? | Rotate the image below $90^{\circ}$ counterclockwise about point D and reflect it across a horizontal line. | Find the factors: $2 x^{2}+7 x+3$ | Find the roots and maximum of the quadratic equation below. $y=-3(x-4)^{2}-3$ |
| DOK 2 <br> Example | List the measurements of three different rectangular prisms that each have a surface area of 20 square units. | What value(s) have a $1 / 12$ probability of being rolled as the sum of two 6 -sided dice? | List three sequences of transformations that take preimage ABCD to image $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$. | Fill in the blank with integers so that the quadratic expression is factorable. $x^{2}+\ldots x+4$ | Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values. |
| DOK 3 Example | What is the greatest volume you can make with a rectangular prism that has a surface area of 20 square units? | Fill in the blanks to complete this sentence using the whole numbers 1 through 9, no more than one time each. <br> Rolling a sum of $\qquad$ on two $\qquad$ -sided dice is the same probability as rolling a sum of $\qquad$ on two $\qquad$ sided dice. | What is the fewest number of transformations needed to take pre-image $A B C D$ to image $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ ? <br> Pre-Image <br> Image | Fill the blank by finding the largest and smallest integers that will make the quadratic expression factorable. $2 x^{2}+3 x+$ | Create a quadratic equation with the largest maximum value using the whole numbers 1 through 9 , no more than one time each. $y=-\square(x-\square)^{2}+\square$ |

$$
\begin{aligned}
& \text { Complicated } \\
& \text { or Complex? }
\end{aligned}
$$



## DOK Flowchart for Questions



Source: Tracy Watanabe - @tracywatanabe


## Routine Thinking

- Can you recall ?
- Can you identify ?
- How would you describe
- What might you include on a list about
- Can you select $?$
- How can you find the meaning of ?

| arrange | calculate memorize |  |
| :--- | :--- | :--- |
| measure | nane | recosnize |
| recall | repeat | ideneity |
| llustrate | match | labed |
| state | list | state |


surateric Reasoning
-How is
related to ?

- What conclusions can be drawn?
- Can you elaborate on ? - How would you test $p^{?}$
- What evidence supports $?$
- What would happen if $\qquad$ - Why is that the best answer?

2ssess compape consuruct appris: Pevise hypothosize cribique innesutate drew conclusions develop \& logical argument

DOK 2
Conceptual Thinking

- Can you explain how $\qquad$ affected?
- How would you apply what you learned to develop ?
- How would you summarize ?
- What do you notice about ?
- How would you estimate
- How could you organize



## Extended Reasoning

- Write a research paper. - What information can you gather to support your idea about ?
- Write a thesis, drawing conclusions from multiple sources.
- Apply information from one text to another to develop an persuasive argument.



## DOK Level Differences

- Level 1: Recall \& Reproduction
- Often a trivial application of facts.
- Generally requires little to no cognitive effort beyond remembering the right formula.
- Usually only one answer.
- Level 2: Skills \& Concepts
- Usually requires more than one step to solve.
- Often multiple answers.

Level 3: Strategic Thinking

- Usually requires critical thinking about the best way to approach a problem.
- May be multiple answers or a single optimal answer.
- Often challenging enough to make your head hurt.
> Level 4: Exłended Thinking
- In mathematics these are generally represented by performance tasks or problem-based lessons.


## Adding Decimals

## Use the numbers 1

 through 9, exactly one time each, to fill in the boxes and make three decimals whose sum is as close to 1 as possible.



Fifth attempt:


Points: $\qquad$ /2 attempt /2 explanation


Aquatic SI
Point

$\qquad$


$$
35
$$

What did you learn from this attempt? How will your strategy change on your next attempt?

- When will students ever use this?
- What DOK level should I start students off with?
- How do teachers fit these problems into their pacing?
- How do I help prevent students from giving up after trying the problem once or twice?
- Where can I find other DOK 2 and DOK 3 problems or submit ones l've made?



## Goals

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U Higher depth of knowledge problems
$\square$ Better implementation
IImprove our ability to ask questions
$\square$ Practice preparing to implement a lesson
$\square$ Figure out how to deal with uncomfortable situations

## Open Middle <br> Challenging math problems worth solving



WHAT ARE PEOPLE SAYING ABOUT OPEN MIDDLE?

Brian Marks
@Yummymath
Have you checked out openmiddle.com @openmiddle Should be on your short list of math ed resources \#MTBoS \#mathchat \#maths \#elemchat

BROWSE BY COMMON CORE STATE STANDARDS

- .Kindergarten (6)

Counting \& Cardinality (2)
$\square$ Number \& Operations in Base Ten (1)
$\square$ Operations \& Algebraic Thinking (3)
Grade 1 (12)


Source: Dylan Kane


## COMMON CORE STATE STANDARDS

- Grade 1 (6)
- Number \& Operations in Base Ten (3)
- Operations \& Algebraic Thinking (3)
- Grade 2 (6)
- Measurement \& Data (2)
- Number \& Operations in Base Ten (4)
- Grade 3 (11)
- Measurement \& Data (6)
- Number \& Operations in Base Ten (3)
- Number \& Operations-Fractions (2)


## Problem-Based Lesson Resources

- Problem-based lesson search engine:


## http://robertkaplinsky.com/prbl-search-engine/

- My lessons: http://www.robertkaplinsky.com/lessons
- Dan Meyer: http://threeacts.mrmeyer.com
- Andrew Stadel: http://tinyurl.com/mrstadel
- Graham Fletcher: http://gfletchy.com/3-act-lessons/
- Geoff Krall: http://tinyurl.com/PrBLmaps
- Dan Meyer's TED talk: http://tinyurl.com/meyer-TED

Home

## Search



## Facebook

## Robert Kaplinsky <br> flike Page 837 likes

Be the first of your friends to like this


## How I Can Help You



Real World Problems
My workshops help teachers implement problem-based lessons by helping them experience them from both student and teacher perspective, leading to


Depth of Knowledge
Problems at higher depth of knowledge levels have the potential to challenge the most gifted students yet remain accessible to struggling students. I can help teachers

## Lessons



How Many Chip Bags Will There Be?


How Many Hot Dogs And Buns Should He Buy?

## Search

## Facebook



## What dOES 20 Calories LÔOK LIKEZ?

What Does 2000 Calories Look Like?

## Subscribe for Updates

Enter your information below so you don't miss out on news, blog posts, and lessons! If you live in the United States, enter your zip code and I'll use it to let you know about events near you.

# Robert Kaplinsky's Problem-Based Lessons 

File Edit View Insert Format Data Tools Add-ons Help

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## Lesson

| How Many Chip Bags Will There Be? |
| :--- |
| How Can We Make Stronger Passwords? |
| How Many Hot Dogs And Buns Should He Buy? |
| What Does 2000 Calories Look Like? |
| How Much Money Are The Coins Worth? |
| How Many Times Will A Case of Paper Jam? |
| How Many Soda Combinations Are There On A Coke Freestyle? |
| What Should The Freeway Sign Show? |
| How Fast Was The Fastest Motorcycle Speeding Ticket Ever? |
| How Much Did Patrick Peterson Lose By Not Cashing His Check? |
| How Many Biscuits Can You Make? |
| How Much Bigger Should They Make Zoolander's School? |
| Where Is The Freeway Sign Located? |
| How Far Apart Are Exits On A Ring Road? |
| How Much Is One Third Of A Cup Of Butter? |
| How Do Skytypers Write Messages? |
| How Big Is The Bermuda Triangle? |
| What Fraction Of Children Are In The Right Car Seat? |
| How Much Did The Temperature Drop? |
| How Much Shorter Are Staggered Pipe Stacks? |
| How Do You Write A Check To Pay For Something? |
| How Can We Correct The Scarecrow? |
| How Much Does A 100×100 In-N-Out Cheeseburger Cost? |
| How Can We Water All Of The Grass? |
| How Much Money IS That?! |
| How Much Money Should Dr. Evil Demand? |
| How Tall Is Mini-Me? |
| How Did They Make Ms. Pac-Man? |
| Which Ticket Option Is The Best Deal? |
| How Far Apart Are The Freeway Exits? |
| Do We Have Enough Paint? |
| How Many Stars Are There In The Universe? |
| What Rides Can You Go On? |
| Do You Have Enough Money? |
| Which Bed Bath \& Beyond Coupon Should You Use? |
| Is Gas Cheaper With Cash Or Credit Card? |
| Where's The Nearest Toys R Us? |

## Concept / Skill

Ratio and Proportions, Population Sampling
Permutations, Combinations, Probability, Exponents, Exponential Growth Least Common Multiple (LCM)
Unit Rates, Ratios, Solving Equations, and Solving Inequalities
Decimal Operations and Coin Counting
Interpreting Percentages
Counting, Composing, and Decomposing Numbers
Fractions on Number Lines, Converting Units, Decimal and Fraction Operations
Converting Units and Unit Rates
Compound and/or Simple Interest
Dividing Fractions and Mixed Numbers
Scale and Proportions
Identifying Fractions on a Number Line
Arc length measures
Identifying Fractions on a Number Line
Transformations (Rotations, Reflections, Dilations, and Translations)
Coordinate Geometry: Area of Triangle
Representing and Comparing Fractions
Absolute Value
Circles, Pythagorean Theorem, trigonometric ratios, and linear functions
Expanded Form
Pythagorean Theorem
Building and Interpretting Linear Functions
Circles, Pythagorean Theorem, trigonometric ratios
Volume of rectangular prism
Exponential Growth
Scale and Dividing Decimals
Transformations (Rotations, Reflections, and Translations)
Unit Rates and Ratios
Fractions on a Number Line and Subtracting Fractions
Area
Scientific Notation
Inequalities and Measurement
Money
Percent Discount
Percent Discount
Pythagorean Theorem (Distance in coordinate system)

Standard 1
RP 3
6.NS. 4
6.EE. 3
MD. 8
6.RP.3c
K.CC. 5
3.NF. 1
5.MD. 1
7.RP. 3

5NF 7
5.NF.5A
3.NF. 1

G-C. 5
3.NF. 1 3.NF. 2 3.NF
8.G. 1

G-GPE. 7
3.NF. 1 3.NF
6.NS.7c
8.G. 7
2.NBT. 3
8.G. 6
$8 . F$
7.G. 4
5.MD. 3

N-RN. 2
5.NF. 5
8. G.
6.RP. 2

3NF 2
3.MD. 5
8.EE. 3
2.MD. 1
2.MD. 8
7.RP. 3
7.RP. 3
8.G. 8
3.NF. 2
7.NS.1c A-CED. 1 4.NBT. 2 G-SRT. 4
8.F. 3
8.G. 7
5.MD. 4

A-SSE. 1
5.NF.5a
8.G. 2
6.RP. 3
3.NF.2b
3.MD. 6
8.EE. 4 6.NS.7a


# Arizona 

## Department of Education

## ROBERT KAPLINSKY

シ ( $\boldsymbol{f}$ @robertkaplinsky

## Goals

Engaging problem solving
$\triangle$ Real world problem-based learning
U Higher depth of knowledge problems
$\square$ Better implementation
IImprove our ability to ask questions
$\square$ Practice preparing to implement a lesson
$\square$ Figure out how to deal with uncomfortable situations

## Sin




## Height: 78 inches

Source: Andrew Stadel via www.estimation180.com


## Height: 78 inches




## Heights

 78 inches

Width:
56 inches


Depth: 18 inches


## Sticky note

## Recycled Self Stirk Notes Notas autoadhesivas reciclado Notes autoofllantes recyclés

- 18 pads / blocs
- 100 sheets per pad / hojas por bloc / f
- Total 1800 sheets/hojas / feuillets
- 3 in $\times 3$ in ( $76,2 \mathrm{~mm} \times 76,2 \mathrm{~mm}$ )

Dimensions: $5^{7 n} \times 5^{7 n}$


## PERFORMANCE TASK

## CEREAL BOXES

A cereal company uses cereal boxes that are rectangular prisms The boxes have the dimensions shown.

- 12 inches high
- 8 inches wide
- 2 inches deep

The managers of the company want a new size for their cereal boxes. The new boxes have to be rectangular prisms. You will evaluate one box design the company proposed. Then you will create and propose your own design for the company.

Requirements for the new boxes:

- The new boxes have to use less cardboard than the

Determine the volume of the current cereal box with the dimensions 12 inches high, 8 inches wide, and 2 inches deep.

Find the volume, $V$, in cubic inches, of each box.
Volume of Original Box: $V=$ $\ldots \mathrm{in}^{3}$

| + | $\rightarrow$ | $\rightarrow$ |
| :--- | :--- | :--- |
| 1 2 3  <br> 4 5 6  <br>     <br> 7 8 9  <br> 0 . -  |  |  |

## 2

Label the dimensions of the net for the current cereal box with dimensions 12 inches high, 8 inches wide, and 2 inches deep.

Distinguishing Between Depth of Knowledge Levels in Mathematics

| Topic | Surface Area and Volume | Probability | Transformations | Factoring Quadratics | Quadratics in Vertex Form |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { CCSS } \\ & \text { Standard(s) } \end{aligned}$ | - 6.G. 4 <br> - 7.G. 6 | $\begin{aligned} & \text { - } \quad \text { 7.SP. } 5 \\ & \text { - } 7 . S P . ~ \end{aligned}$ | - 8.G. 1 <br> - G-CO. 5 | - A-SSE.3a | - F-IF.7a |
| DOK 1 <br> Example | Find the surface area of a rectangular prism that measures 3 units by 4 units by 5 units. | What is the probability of rolling a sum of 5 using two 6-sided dice? | Rotate the image below $90^{\circ}$ counterclockw ise and reflect it across a horizontal line. | Find the factors: $2 x^{2}+7 x+3$ | Find the roots and maximum of the quadratic equation below. $y=-3(x-4)^{2}-3$ |
| DOK 2 <br> Example | List the measurements of three different rectangular prisms that each has a surface area of 20 square units. | What value(s) have a $1 / 12$ probability of being rolled as the sum of two 6 -sided dice? | List three sequences of transformations that take preimage ABCD to image $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$. | Fill in the blank with integers so that the quadratic expression is factorable. $x^{2}+\ldots x+4$ | Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values. |
| DOK 3 <br> Example | What is the greatest volume you can make with a rectangular prism that has a surface area of 20 square units? | Fill in the blanks to complete this sentence using the whole numbers 1 through 9, no more than one time each. <br> Rolling a sum of $\qquad$ on two $\qquad$ -sided dice is the same probability as rolling a sum of $\qquad$ on two $\qquad$ sided dice. | What is the fewest number of transformations needed to take pre-image $A B C D$ to image $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$ ? | Fill the blank by finding the largest and smallest integers that will make the quadratic expression factorable. $2 x^{2}+3 x+$ | Create a quadratic equation with the largest maximum value using the whole numbers 1 through 9, no more than one time each. $y=-\square(x-\square)^{2}+\square$ |

## Discussion Questions

- How will problem-based lessons like these prepare students for a performance task like the Cereal Box?
- What skills might students still be lacking to be successful with a problem like this?


## Why Are You Using That Problem?

- Use the problem to introduce a new concept - Best Case:
- Great context for beginning a unit
- Worst Case:
-What was the purpose of this problem?
- Why didn't you finish it?
- Why didn't you let students struggle through it?
- Did the teacher end the problem because he or she was confused and gave up?


## Why Are You Using Thał Problem?

- Productive struggle
- Best Case:
- Students worked hard and made connections.
- Worst Case:
- Why did the teacher let the students sit there confused instead of telling them what to do?
- Did the students even learn anything because they never figured out the answer?
- Why didn't the teacher finish the problem? Did she lose track of time?


## Why Are You Using Thał Problem?

- Problem completion
- Best Case:
- Everyone experienced a complete problem.
- Worst Case:
- Who really did the work today: the students or the teacher?
- Why did the teacher not see all those great opportunities for students to make their own connections and take advantage of them?
- Why did the teacher give such obvious hints and tell them what to do?


## FIVE PRACTICES



## Discussion Questions

- "Giving students too much or too little support, or too much direction, can result in a decline in the cognitive demands of the task." (p. 550) Why?
- "By making purposeful choices about the order in which students' work is shared, teachers can maximize the chances that their mathematical goals for the discussion will be achieved." (p. 554) What ways do teachers currently select students? How would you suggest they change their selection process after reading this?
- What challenges might teachers have when trying to "connect" student solutions? (p. 554)


## Implementing the Five Practices

1. Pick a selection strategy you anticipate using before looking at the student work.
2. Next, review the student work to simulate the reality that you won't know what students will actually do.
3. Figure out which students you would have share their mathematical work.
4. Determine the order you would have those students present their work.
5. Decide on which connections you would emphasize between the students' work and mathematical ideas.

## Posters

- At the top of the poster, list the selection strategy used by your group. For example:
- Starting with the most commonly used strategy and moving to one that few students used.
- Starting with a strategy that is more concrete and moving to strategies that are more abstract.
- Incorporating wrong answers to address common misconceptions ("Who made the best mistake?')
- Attach those students' work to the poster in the order that you would present it.
- Next to the student work list the questions you would ask the student(s) or ideas that you would want to come out as a result of showing that student's work.






## - Change

## - Transition

- Ending


## - Change

## - Transition

## - Ending

- Neutral Zone


## - Change

## - Transition

## - Ending

- Neutral Zone
- New Beginning


## What does this mean for math education?

## - Change

## - Transition

- Ending
- People may not stop doing anything. They may try to do all the old things and the new things. Soon they burn out with the overload.
- People make their own decisions about what to discard and what to keep, and the result is inconsistency and chaos.
- People toss out everything that was done in the past.


## - Change

## - Transition

## - Ending

- Neutral Zone


## - Change

## - Transition

## - Ending

- Neutral Zone
- New Beginning


## Goals

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$\triangle$ Real world problem-based learning
Higher depth of knowledge problems
$\square$ Better implementation
I Improve our ability to ask questions
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## Get Updates!

- Receive the latest news, blog posts, and lessons by email.
- To get them:
- Scan this QR code or
- Go to
tinyurl.com/RKupdates





## struggle: low feedback: lou reward: high <br> struggle: low feedback: low reward: high <br> struggle: low feedback: lou reward: high

 2


## struggle: high feedback: high reward: high

## Pre-Mortem

- The lesson flopped. What went wrong?
- You have sixty seconds to write down all the reasons the lesson did not go well.
- Create a combined list with your neighbors.
- Then discuss "less helpful" and "more helpful" ways you could address them if they do happen.



## Setting Up The Problem

- What do you do when students ask for data/information you don't have, hadn't considered, or forgot to get?
- What do you do when students ask for information that is probably not important or that they don't actually need?


## TICNET BOOT:

 12TICRETS=\$500 25 TICRETS $=\$ 10.00$ 50TCKEETS: 82500 20 TICRETS: 550.00 HANE FUNY



# Does a hybrid car pay for itself? 






## Setting Up The Problem

- What do you do when students ask for data/information you don't have, hadn't considered, or forgot to get?
- What do you do when students ask for information that is probably not important or that they don't actually need?
- What do you do when students don't know what to write for what they know and don't know?
- What do you do when you ask for a guess and they don't know?
- What do you do when they don't ask you for information that they need to solve the problem?



## Problem Solving Process

- What do you do when students don't use the strategy you anticipated they would use?


## TICNET BOOT:

 12TICRETS=\$500 25 TICRETS $=\$ 10.00$ 50TCKEETS: 82500 20 TICRETS: 550.00 HANE FUNY



## Problem Solving Process

- What do you do when students don't use the strategy you anticipated they would use?
- What do you do when a student comes up with a strategy for solving the problem that you do not understand?







## Problem Solving Process

- What do you do when students don't use the strategy you anticipated they would use?
- What do you do when a student comes up with a strategy for solving the problem that you do not understand?
- What do you do when the answer we calculate does not match with the actual answer?
- What do you do when students get stuck during the problem solving process and are not sure what to do?


## Problem Solving Process

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## Problem Solving Process

- What do you do when students don't use the strategy you anticipated they would use?
- What do you do when a student comes up with a strategy for solving the problem that you do not understand?
- What do you do when the answer we calculate does not match with the actual answer?
- What do you do when students get stuck during the problem solving process and are not sure what to do?
- What do you do when you ask students questions and few to no people are ready to respond?
- What do you do when the student conclusions are low quality and/or effort?


## ||||||||||||||||||||||||||||||||||||||| <br> $1015773283 \quad 9456613028$

Also exchudes Starbucks
Also excludes Dyson vacuums ant Miele



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 of ${ }^{5} 15$ or more.


# BED BATH \& <br> BEYON D 

Beyond any store of its kind:
OFFICES: 650 LIBERTY AVENUE, UNION, NJ 07083

IA conclusion each conclusion Each Item is good for different Items

If the flem is 447 it is better to use the $20 \%$ offcoupon because

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Curry Chicken
Lemon Chicken
Vegetable Chicken
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Shrimp with Lobster Sauce ..... 6.25
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$\checkmark$ Fish Fillet with Black Bean Sauce6.25
Crab meat with Asparagus ..... 6.25
Sweet \& Sour Shrimp ..... 6.25

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You can use the $10 \%$ off when you pay 20-24.99 or more the freechicken Lomein when you pay $25-49.99$ or more and the free orange Chitin whengou pay 50 or more

## Goals

Engaging problem solving
$\triangle$ Real world problem-based learning
Higher depth of knowledge problems
$\triangle$ Better implementation
Improve our ability to ask questions
$\pm$ Practice preparing to implement a lesson
$\pm$ Figure out how to deal with uncomfortable situations

## Construction

- Pick two:



## Family

- Pick two:



## Problem-Based Learning

- Pick two:



## Call to Action

-Implement one problem-based lesson in your classroom in the next two weeks of school.

- Implement one DOK 2 or DOK 3 problem in your classroom in the next two weeks of school.


