

# Digging into Depth of Knowledge

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# COMMON CORE

## STATE STANDARDS INITIATIVE

CCSS.MATH.CONTENT.4.MD.A.3

Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

meet the...  
equal intensity, the...  
of each grade: conceptual...  
skills and fluency, and application.

What is the perimeter  
of a rectangle that  
measures 8 units by 4  
units?



# Components of Rigor

Procedural Skill and Fluency

Conceptual Understanding

List the dimensions of  
a rectangle with a  
perimeter of 24 units.



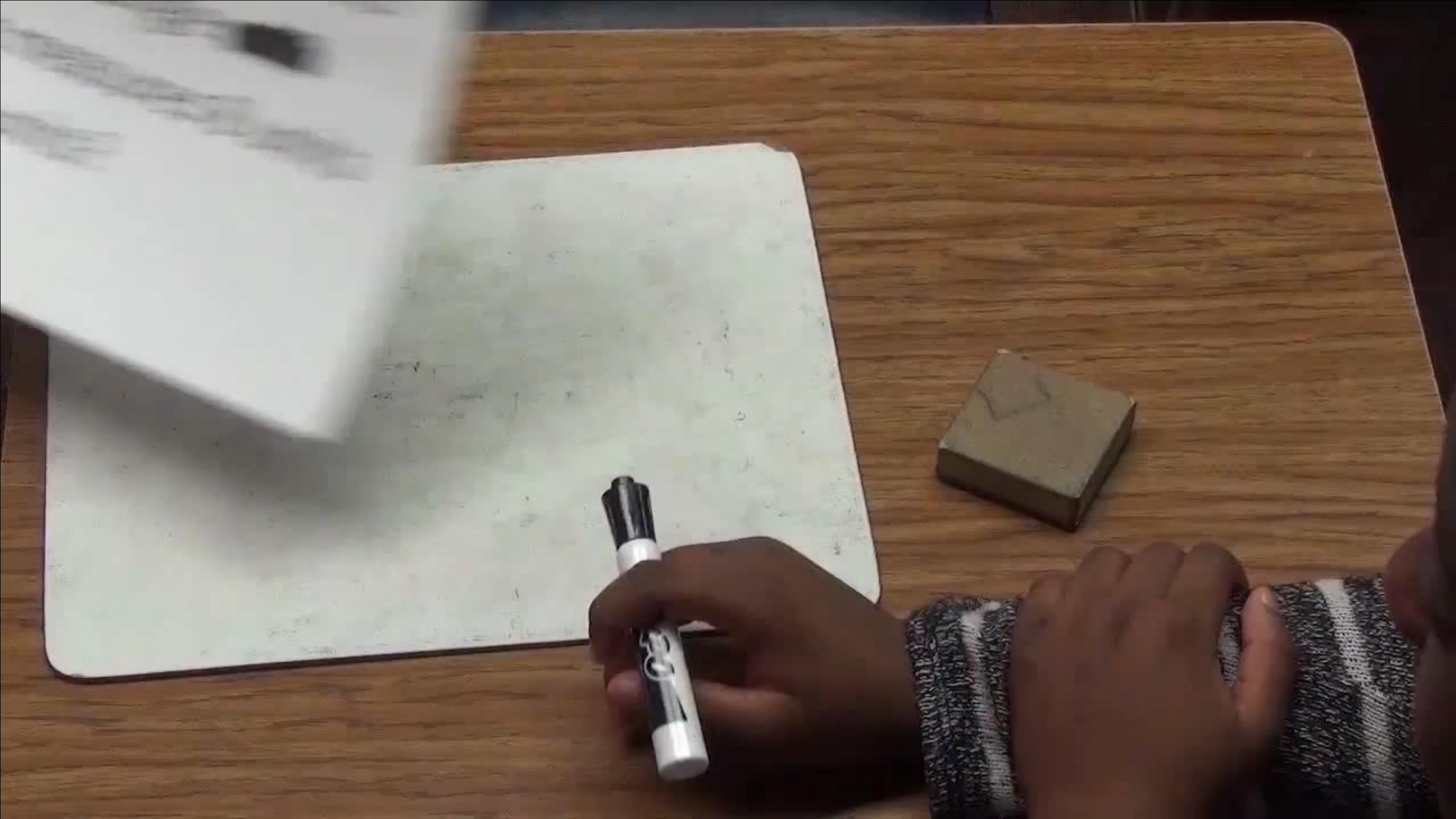
# Components of Rigor

Procedural Skill and Fluency

Conceptual Understanding



**WHY?**



# Components of Rigor

Procedural Skill and Fluency

Conceptual Understanding

List the  
of a rectangle with a  
perimeter of 24  
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# Components of Rigor

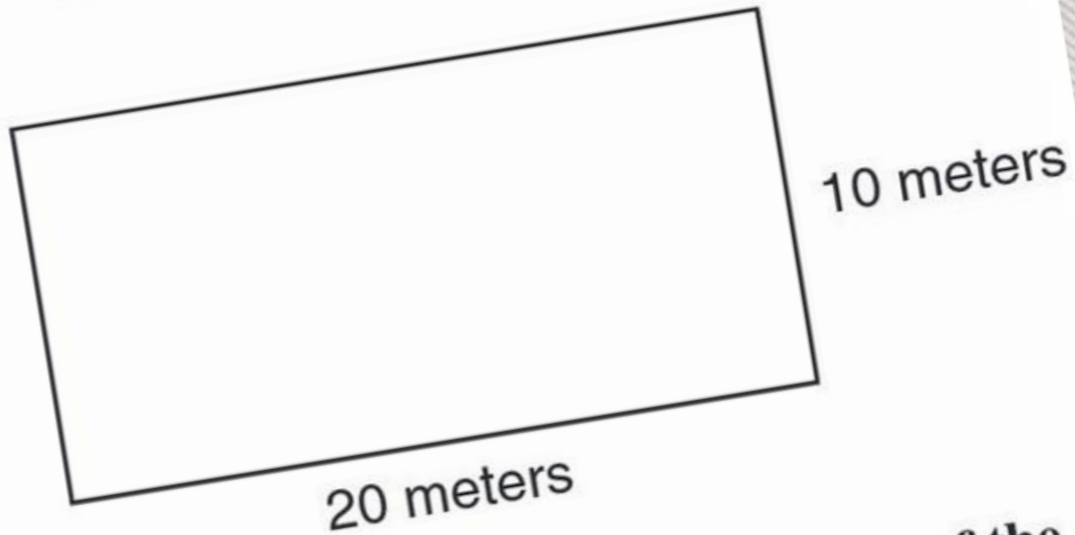
Procedural Skill and Fluency

Conceptual Understanding



71

A basketball court is shaped like a rectangle 20 meters long and 10 meters wide.



What is the perimeter in meters of the court?

- A 30 meters
- B 50 meters
- C 60 meters
- D 200 meters

What is the perimeter  
of a rectangle ~~with~~  
that measures 8 units  
by 4 units?

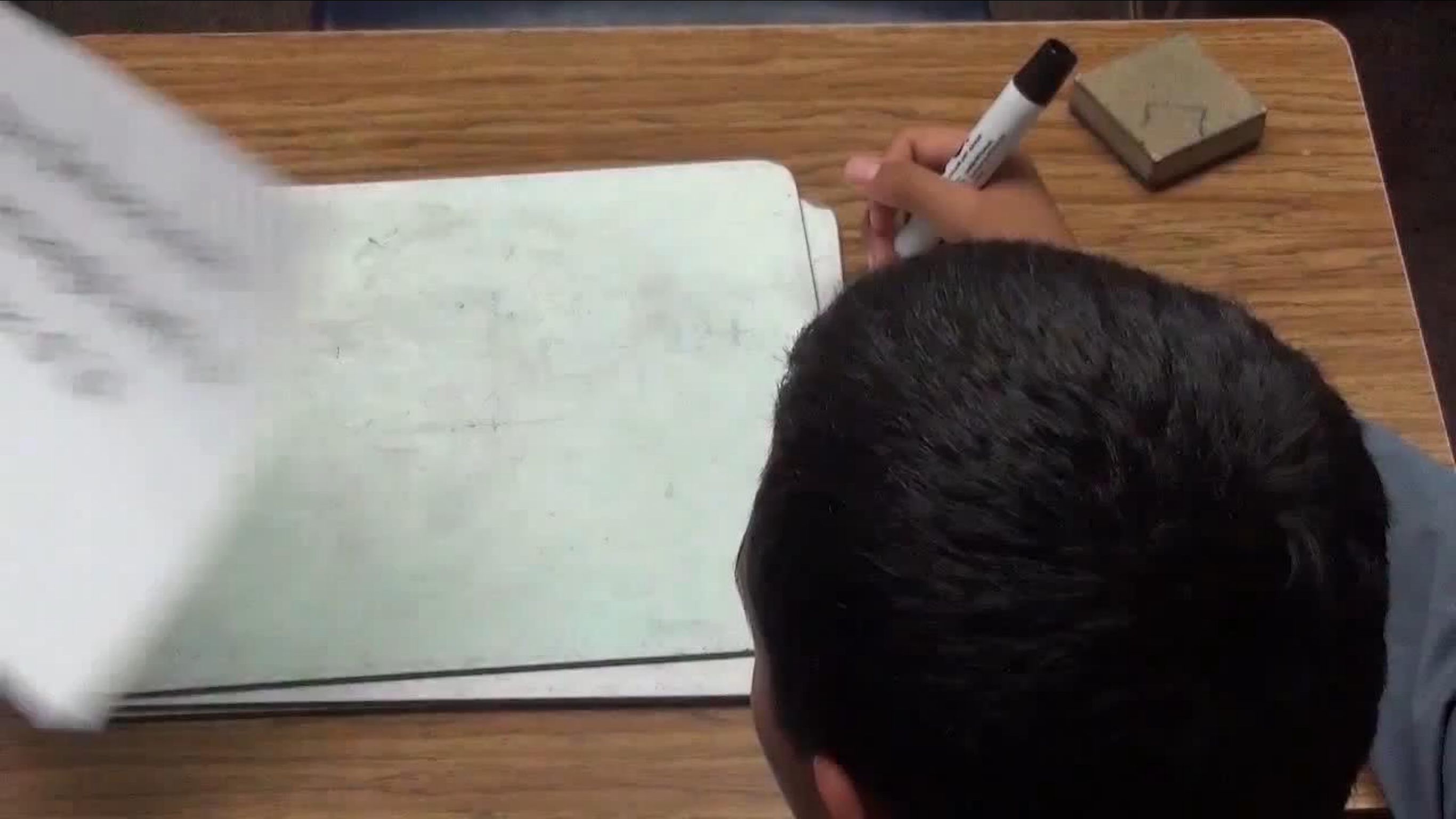


# Components of Rigor

Procedural Skill and Fluency

Conceptual Understanding





# Components of Rigor

Procedural Skill and Fluency

Conceptual Understanding

Of all the rectangles with a perimeter of 24 units, which one has the most area?

Of all the rectangles  
with a perimeter of  
24 units, which one  
has the most area?

00:00:00:00

# Components of Rigor

Procedural Skill and Fluency


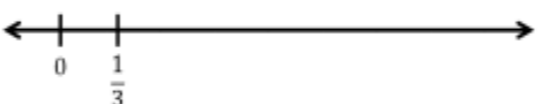
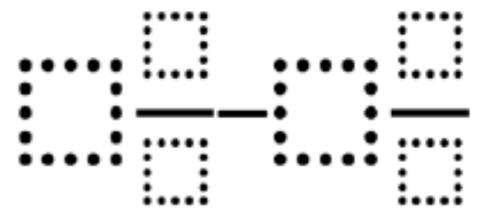
Conceptual Understanding

# Defining the Problem

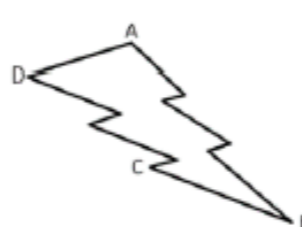
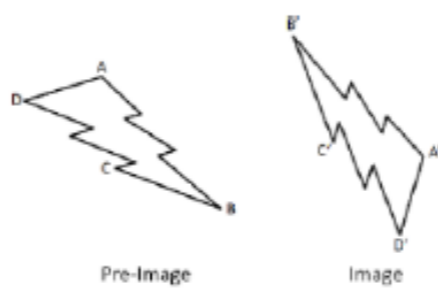
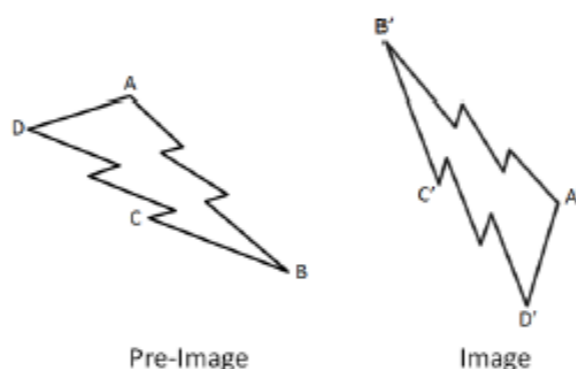
- ▶ Students appear to demonstrate “deep, authentic command of mathematical concepts” when given commonly used problems.
- ▶ However with more challenging problems, the same students seem to no longer demonstrate that command.

# Addressing the Problem

- ▶ First, we must have a clear understanding about why these problems are different from one another.
- ▶ Next, we need to practice using these problems so that we understand how students may react to them.
- ▶ Last, we need a source that can provide us with a variety of free problems.

Topic	Adding Whole Numbers	Money	Fractions on a Number Line	Area and Perimeter	Subtracting Mixed Numbers
CCSS Standard(s)	<ul style="list-style-type: none"> <li>1.NBT.4</li> <li>2.NBT.5</li> </ul>	<ul style="list-style-type: none"> <li>2.MD.8</li> </ul>	<ul style="list-style-type: none"> <li>3.NF.2</li> </ul>	<ul style="list-style-type: none"> <li>3.MD.8</li> <li>4.MD.3</li> </ul>	<ul style="list-style-type: none"> <li>5.NF.1</li> </ul>
DOK 1 Example	<p>Find the sum.</p> $44 + 27 =$	<p>If you have 2 dimes and 3 pennies, how many cents do you have</p>	<p>Which point is located at <math>\frac{7}{12}</math> below?</p> 	<p>Find the perimeter of a rectangle that measures 4 units by 8 units.</p>	<p>Find the difference.</p> $5\frac{1}{2} - 4\frac{2}{3} =$
DOK 2 Example	<p>Fill in the boxes below using the whole numbers 1 through 9, no more than one time each, so that you make a true equation.</p> $\square\square + 53 = \square\square$	<p>Make 47¢ in three different ways with either quarters, dimes, nickels, or pennies.</p>	<p>Label the point where <math>\frac{3}{4}</math> belongs on the number line below. Be as precise as possible.</p> 	<p>List the measurements of three different rectangles that each has a perimeter of 20 units.</p>	<p>Create three different mixed numbers that will make the equation true by using the whole numbers 1 through 9, no more than one time each,. You may reuse the same whole numbers for each of the three mixed numbers.</p> $5\frac{4}{5} - \square\frac{\square}{\square} = 3\frac{1}{20}$
DOK 3 Example	<p>Make the largest sum by filling in the boxes below using the whole numbers 1 through 9, no more than one time each.</p> $\square\square + \square\square =$	<p>Make 47¢ using exactly 6 coins with either quarters, dimes, nickels, or pennies.</p>	<p>Create 5 fractions using the whole numbers 0 through 9, no more than one time each, as numerators and denominators and correctly place them all on a number line.</p>	<p>What is the greatest area you can make with a rectangle that has a perimeter of 24 units?</p>	<p>Make the smallest difference by filling in the boxes below using the whole numbers 1 through 9, no more than one time each.</p> 



Topic	Surface Area and Volume	Probability	Transformations	Factoring Quadratics	Quadratics in Vertex Form
CCSS Standard(s)	<ul style="list-style-type: none"> <li>6.G.4</li> <li>7.G.6</li> </ul>	<ul style="list-style-type: none"> <li>7.SP.5</li> <li>7.SP.7</li> </ul>	<ul style="list-style-type: none"> <li>8.G.1</li> <li>G-CO.5</li> </ul>	<ul style="list-style-type: none"> <li>A-SSE.3a</li> </ul>	<ul style="list-style-type: none"> <li>F-IF.7a</li> </ul>
DOK 1 Example	Find the surface area of a rectangular prism that measures 3 units by 4 units by 5 units.	What is the probability of rolling a sum of 5 using two 6-sided dice?	Rotate the image below 90° counterclockwise and reflect it across a horizontal line. 	Find the factors: $2x^2 + 7x + 3$	Find the roots and maximum of the quadratic equation below. $y = -3(x - 4)^2 - 3$
DOK 2 Example	List the measurements of three different rectangular prisms that each has a surface area of 20 square units.	What value(s) have a 1/12 probability of being rolled as the sum of two 6-sided dice?	List three sequences of transformations that take pre-image ABCD to image A'B'C'D'. 	Fill in the blank with integers so that the quadratic expression is factorable. $x^2 + \_\_x + 4$	Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values.
DOK 3 Example	What is the greatest volume you can make with a rectangular prism that has a surface area of 20 square units?	Fill in the blanks to complete this sentence using the whole numbers 1 through 9, no more than one time each.  Rolling a sum of $\_\_$ on two $\_\_$ -sided dice is the same probability as rolling a sum of $\_\_$ on two $\_\_$ -sided dice.	What is the fewest number of transformations needed to take pre-image ABCD to image A'B'C'D'? 	Fill the blank by finding the largest and smallest integers that will make the quadratic expression factorable. $2x^2 + 3x + \_\_$	Create a quadratic equation with the largest maximum value using the whole numbers 1 through 9, no more than one time each. $y = -\square(x - \square)^2 + \square$



Complicated  
or Complex?



# Cookie Monster Cupcakes

## method

---

1. Using an electric mixer, whip the butter until it is pale. This will take at least 5 minutes on high.
2. Gradually add in the icing mixture and vanilla until well combined.
3. With the mixer running, add in food colouring until you get to the Cookie Monster colour. This may be a lot if you are using liquid food colouring or a little if using gel food colouring.
4. Add in the milk and mix until the frosting puffs up.
5. Fill a piping bag with a fluted nozzle and pipe on icing.
6. With the writing icing, place black spots on the marshmallows for pupils.
7. Place on each cupcake.
8. Cut cookies in half and place in 'mouth'.



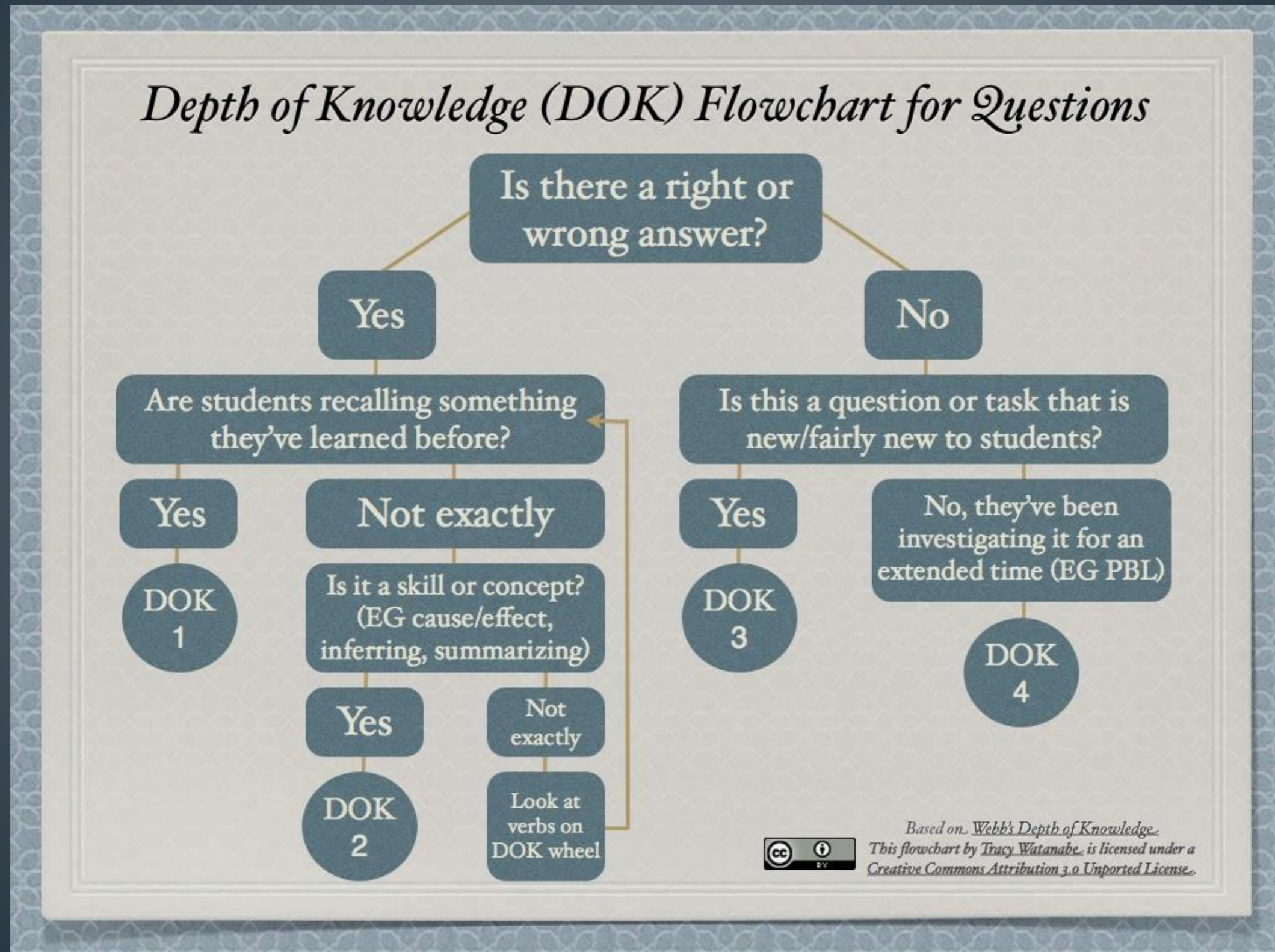
**Nailed it**

# DOK Verb Wheel



Source: Unknown

# DOK Flowchart for Questions



# DOK Posters

## DOK 1

### Routine Thinking

- Can you recall \_\_\_?
- Can you identify \_\_\_?
- How would you describe \_\_\_?
- What might you include on a list about \_\_\_?
- Can you select \_\_\_?
- How can you find the meaning of \_\_\_?

arrange    calculate    memorize  
measure    name    recognize  
recall    repeat    identify  
illustrate    match    label  
state    list    state

## DOK 2

### Conceptual Thinking

- Can you explain how \_\_\_ affected \_\_\_?
- How would you apply what you learned to develop \_\_\_?
- How would you summarize \_\_\_?
- What do you notice about \_\_\_?
- How would you estimate \_\_\_?
- How could you organize \_\_\_?

compare    classify    categorize  
measure    graph    distinguish  
predict    modify    construct  
organize    infer    summarize  
interpret    make observations

## DOK 3

### Strategic Reasoning

- How is \_\_\_ related to \_\_\_?
- What conclusions can be drawn?
- Can you elaborate on \_\_\_?
- How would you test \_\_\_?
- What evidence supports \_\_\_?
- What would happen if \_\_\_?
- Why is that the best answer?

assess    compare    construct  
apprise    revise    hypothesize  
critique    investigate  
draw conclusions  
develop a logical argument

## DOK 4

### Extended Reasoning

- Write a research paper.
- What information can you gather to support your idea about \_\_\_?
- Write a thesis, drawing conclusions from multiple sources.
- Apply information from one text to another to develop an persuasive argument.

design    connect    prove  
analyze    critique    synthesize  
create    apply concepts



# DOK Level Differences



## ▶ **Level 1: Recall & Reproduction**

- ▶ Often a trivial application of facts.
- ▶ Generally requires little to no cognitive effort beyond remembering the right formula.
- ▶ Usually only one answer.

## ▶ **Level 2: Skills & Concepts**

- ▶ Usually requires more than one step to solve.
- ▶ Often multiple answers.

## ▶ **Level 3: Strategic Thinking**

- ▶ Usually requires critical thinking about the best way to approach a problem.
- ▶ May be multiple answers or a single optimal answer.
- ▶ Often challenging enough to make your head hurt.

## ▶ **Level 4: Extended Thinking**

- ▶ In mathematics these are generally represented by performance tasks or problem-based lessons.

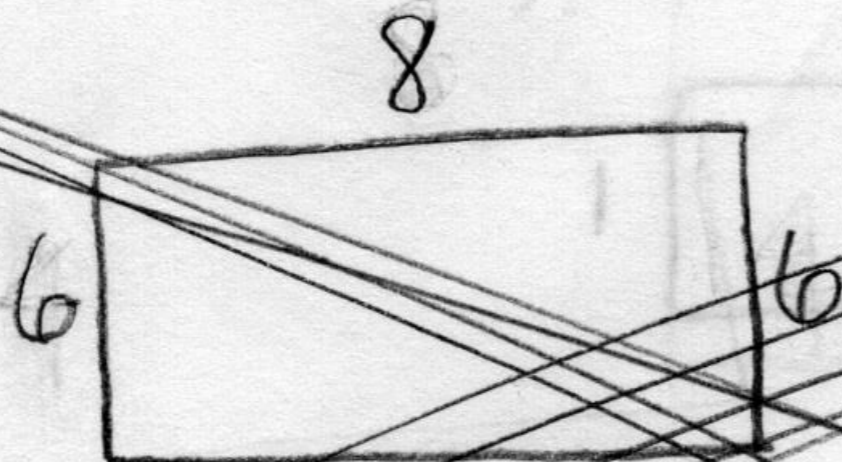
# Adding Decimals

Use the numbers 1 through 9, exactly one time each, to fill in the boxes and make three decimals whose sum is as close to 1 as possible.

$$\begin{array}{r} 0.\square\square\square \\ 0.\square\square\square \\ + 0.\square\square\square \\ \hline \end{array}$$

First attempt:

Points: \_\_\_/2 attempt \_\_\_/2 explanation



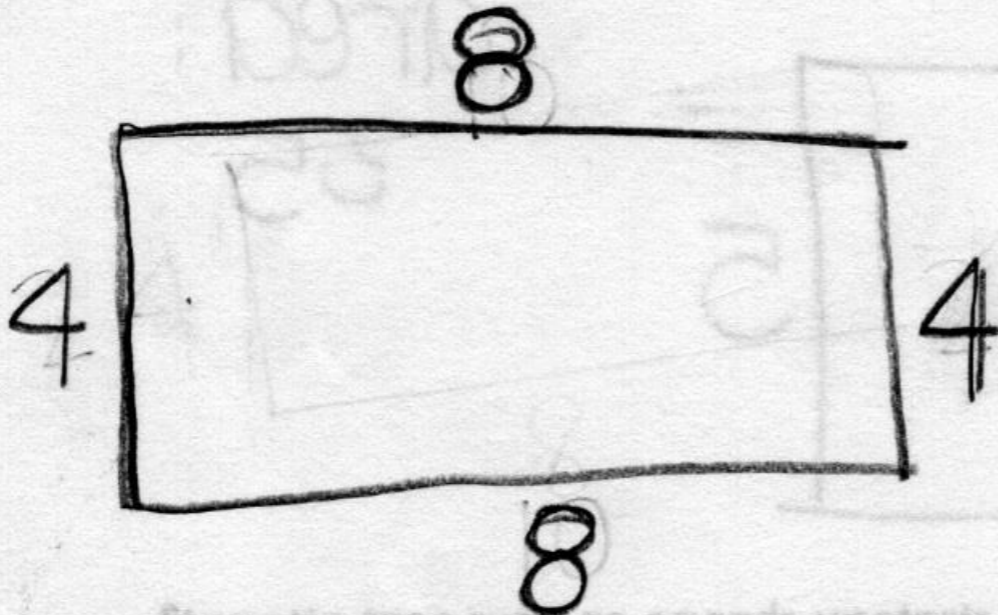
area:  
48

What did you learn from this attempt? How will your strategy change on your next attempt?

~~This attempt doesn't equal 24.~~

Second attempt:

Points: \_\_\_/2 attempt \_\_\_/2 explanation



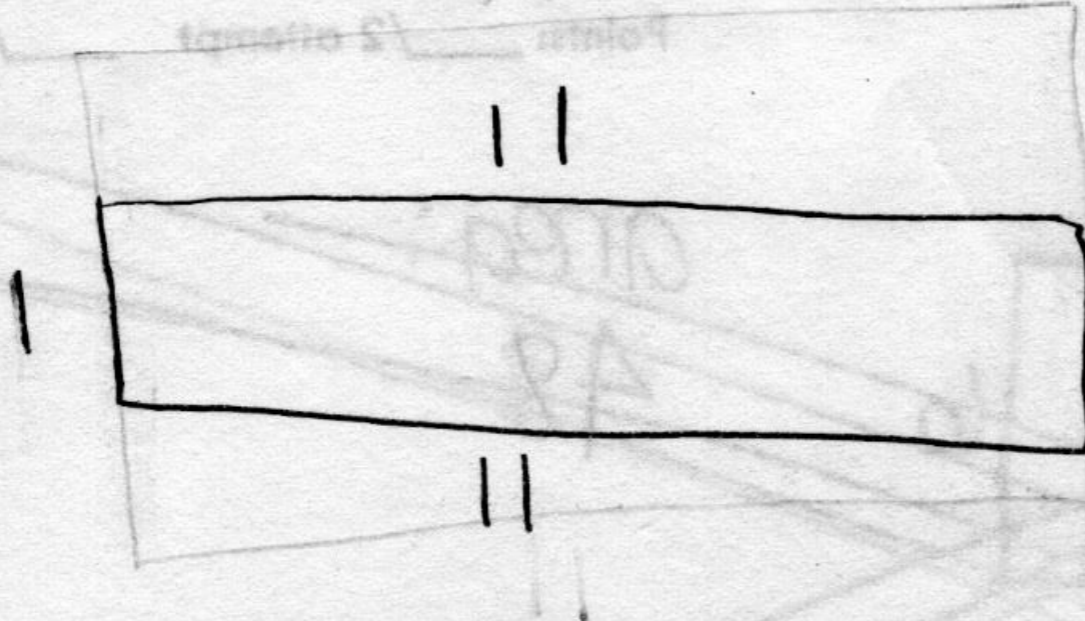
area:  
32

What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter was 24, and the area was 32  
but I think there's a blader #

Fourth attempt:

Points: \_\_\_/2 attempt \_\_\_/2 explanation

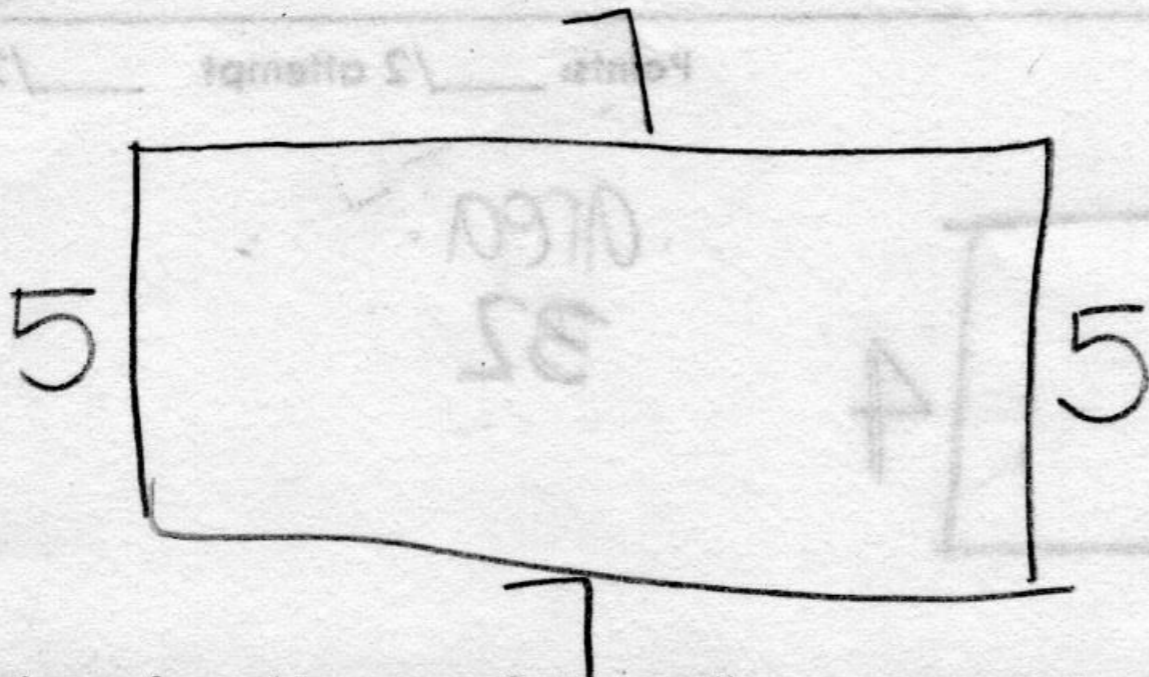


What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter is 24, but the area is 11 and attempt #2 the area is 32  
Strategy: Use #'s with more than one row.

Fifth attempt:

Points: \_\_\_/2 attempt \_\_\_/2 explanation



area:  
35

What did you learn from this attempt? How will your strategy change on your next attempt?

# DOK FAQ

- *When will students ever use this?*
- *What DOK level should I start students off with?*
- *How do teachers fit these problems into their pacing?*
- *How do I help prevent students from giving up after trying the problem once or twice?*
- *Where can I find other DOK 2 and DOK 3 problems or submit ones I've made?*



## NEW OPEN MIDDLE

### Exponents and Order of Operations

February 10, 2015 Leave a comment

Directions: Find 3 positive integers that add up to 10. Place each number into one of the blanks to find the largest possible result. Source: Zack Miller (@zmill415) [Read More »](#)

### Create Squares

February 10, 2015 2 Comments

Directions: Create a square with one of the vertices at (2,3). Fill in the blanks with whole numbers 0 through 9, using each number at most once, to show the rest of the vertices of the square. Bonus: Find more than one set of vertices. Source: John Mahlstedt (@jdmahlstedt) [Read More »](#)

### Solution of Two Linear Equations

February 10, 2015 Leave a comment

Directions: Using the Integers 0-9 (without duplication), provide four sets of points that represent two distinct lines. These lines can be written as two linear equations. Then provide a fifth point that represents the intersection (or solution) of those equations. Line 1: ( , ) and ( , ) Line 2: ( , ) and ( , ) Solution ( , ) Source: Bryan Anderson [Read More »](#)

### Bingo card

February 5, 2015 1 Comment

Directions: In a standard game of BINGO, the cards are labeled with numbers 1 through 75. If it was possible, which card would you choose: a card with all of the same number or a standard bingo card? Source: Nanette

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## OPEN MIDDLE WORKSHEET

Download the Open Middle Worksheet:  
Version 1.1

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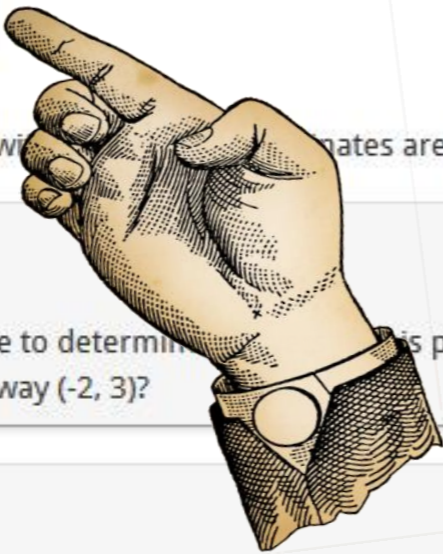
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## COMMON CORE STATE STANDARDS

- Grade 1 (6)
  - Number & Operations in Base Ten (3)
  - Operations & Algebraic Thinking (3)
- Grade 2 (6)
  - Measurement & Data (2)
  - Number & Operations in Base Ten (4)
- Grade 3 (11)
  - Measurement & Data (6)
  - Number & Operations in Base Ten (3)
  - Number & Operations—Fractions (2)

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## EQUIDISTANT POINTS



Directions: How many points with integer coordinates are 5 units away from  $(-2, 3)$ ?

### Hint

Which methods are available to determine the answer to this problem? What shape is defined by *all* of the points that are 5 units away  $(-2, 3)$ ?

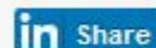
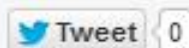
### Answer

12 points:  $(-5, 7)$ ,  $(-7, 3)$ ,  $(-5, -1)$ ,  $(-2, -2)$ ,  $(3, 3)$ ,  $(1, -1)$ ,  $(-2, 8)$ ,  $(1, 7)$ ,  $(2, 6)$ ,  $(-6, -6)$ ,  $(-6, 0)$ , and  $(2, 0)$

Source: [Dylan Kane](#)



SHARE !



Tagged with: [8.G.8](#) [DOK 2: SKILL / CONCEPT](#) [DYLAN KANE](#) [G-GPE.1](#)

◀ Previous: [Cone and Cylinder Volumes](#)

Next: [Pythagorean Shell](#) ▶

## LEAVE A REPLY

## OPEN MIDDLE WORKSHEET

Download the Open Middle Worksheet:  
Version 1.1

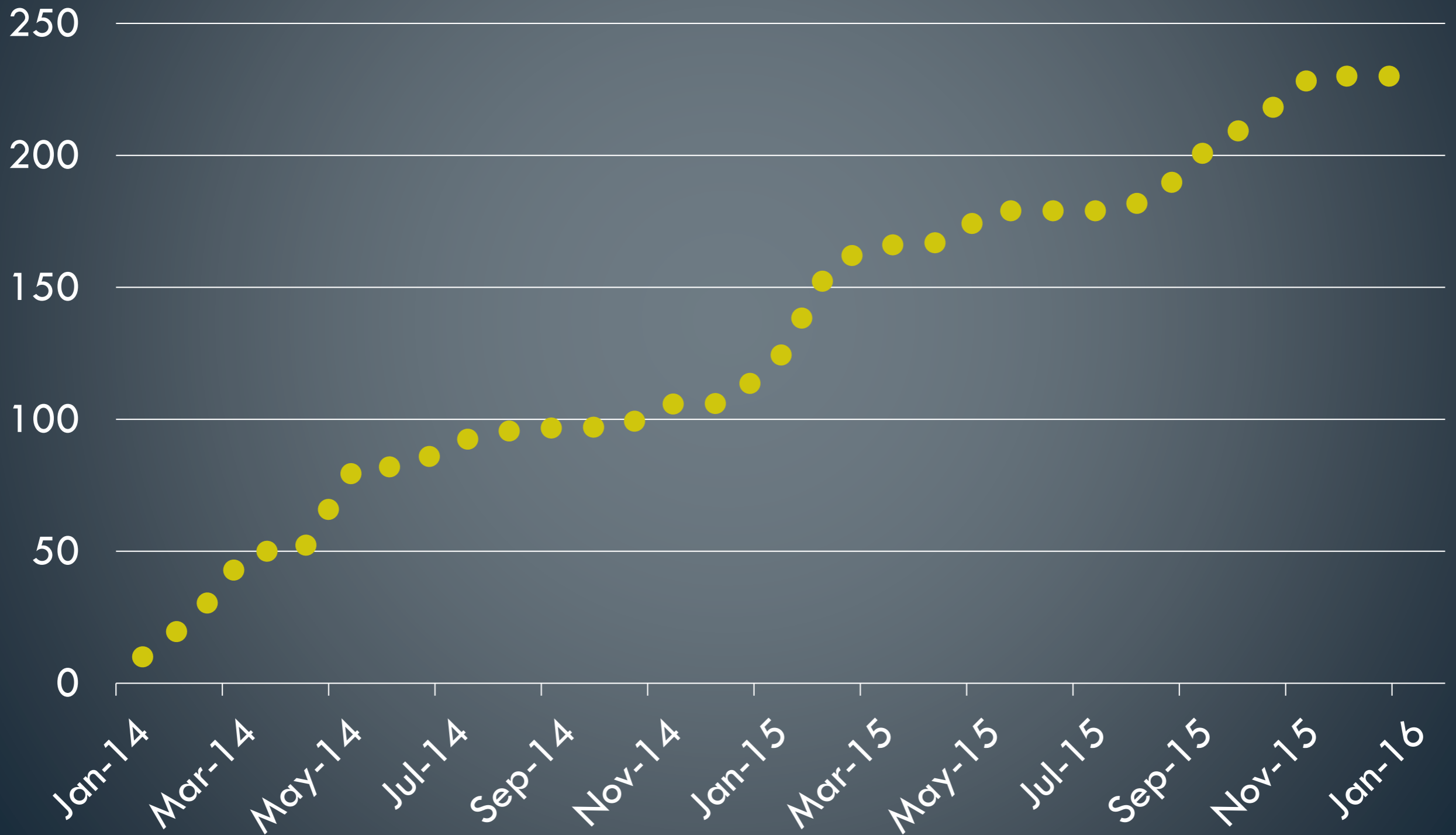
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## COMMON CORE STATE STANDARDS

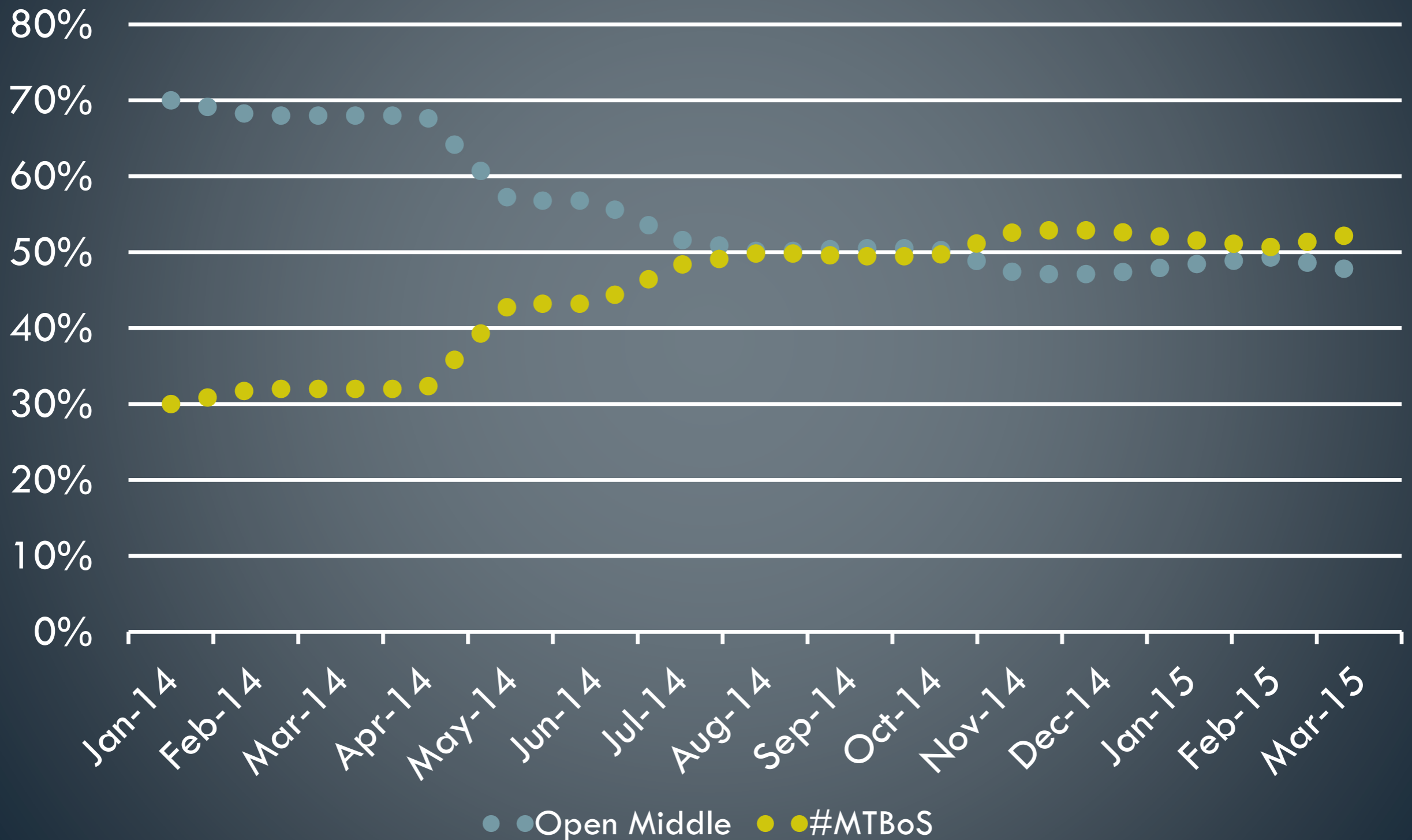
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  - Number & Operations in Base Ten (3)
  - Number & Operations—Fractions (2)

# Total Open Middle Problems

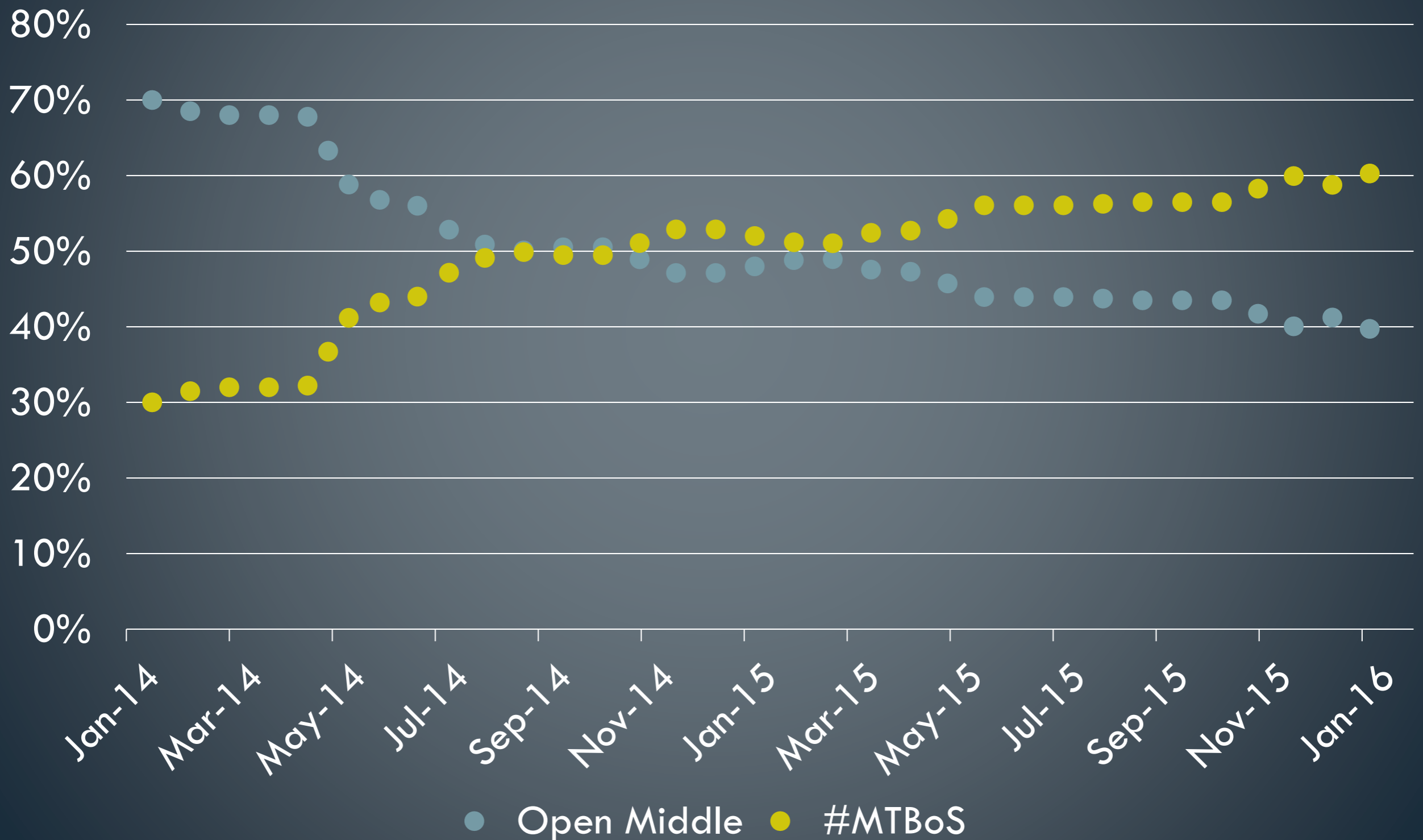




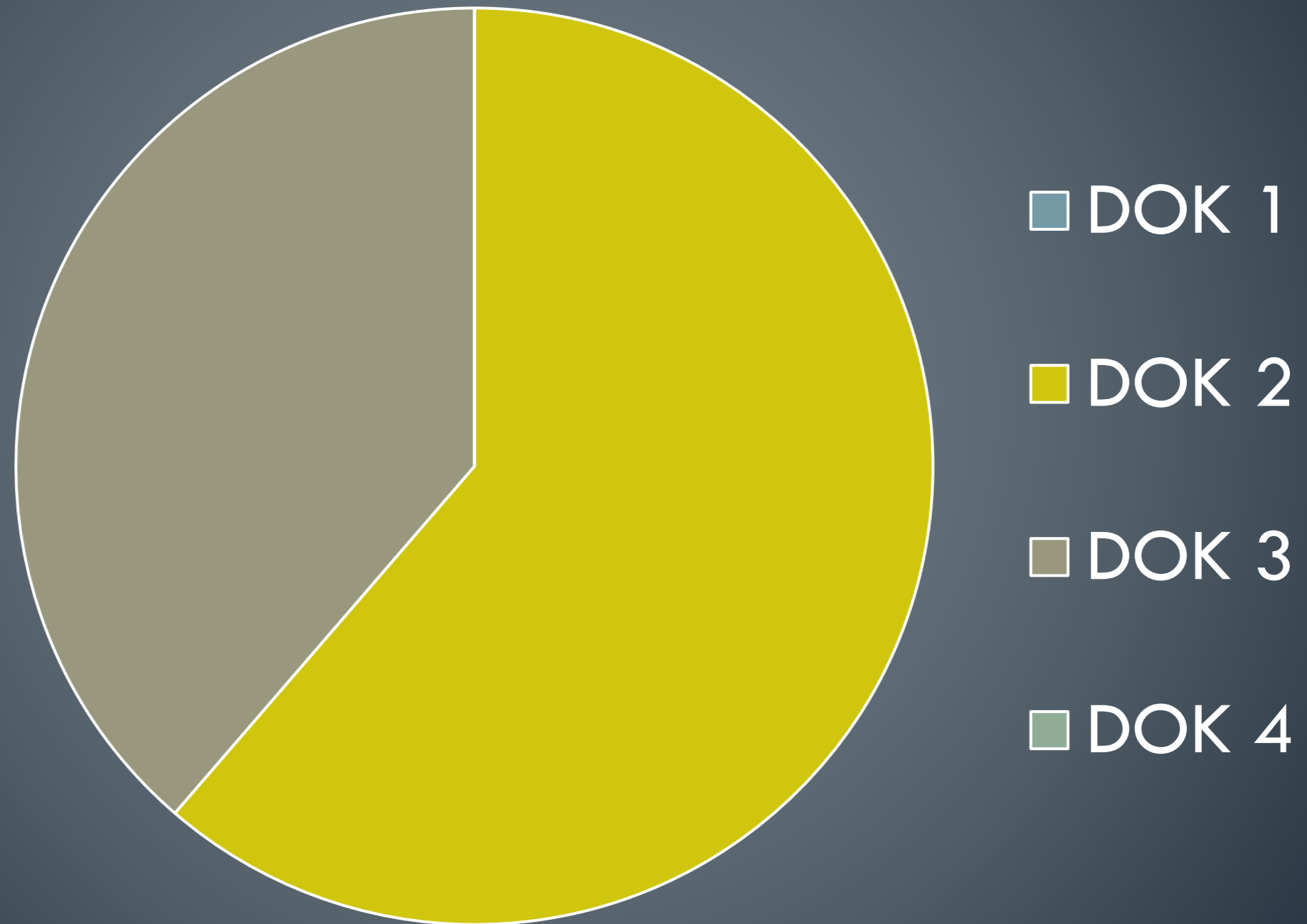
# Open Middle Author Percentages



# Open Middle Author Percentages

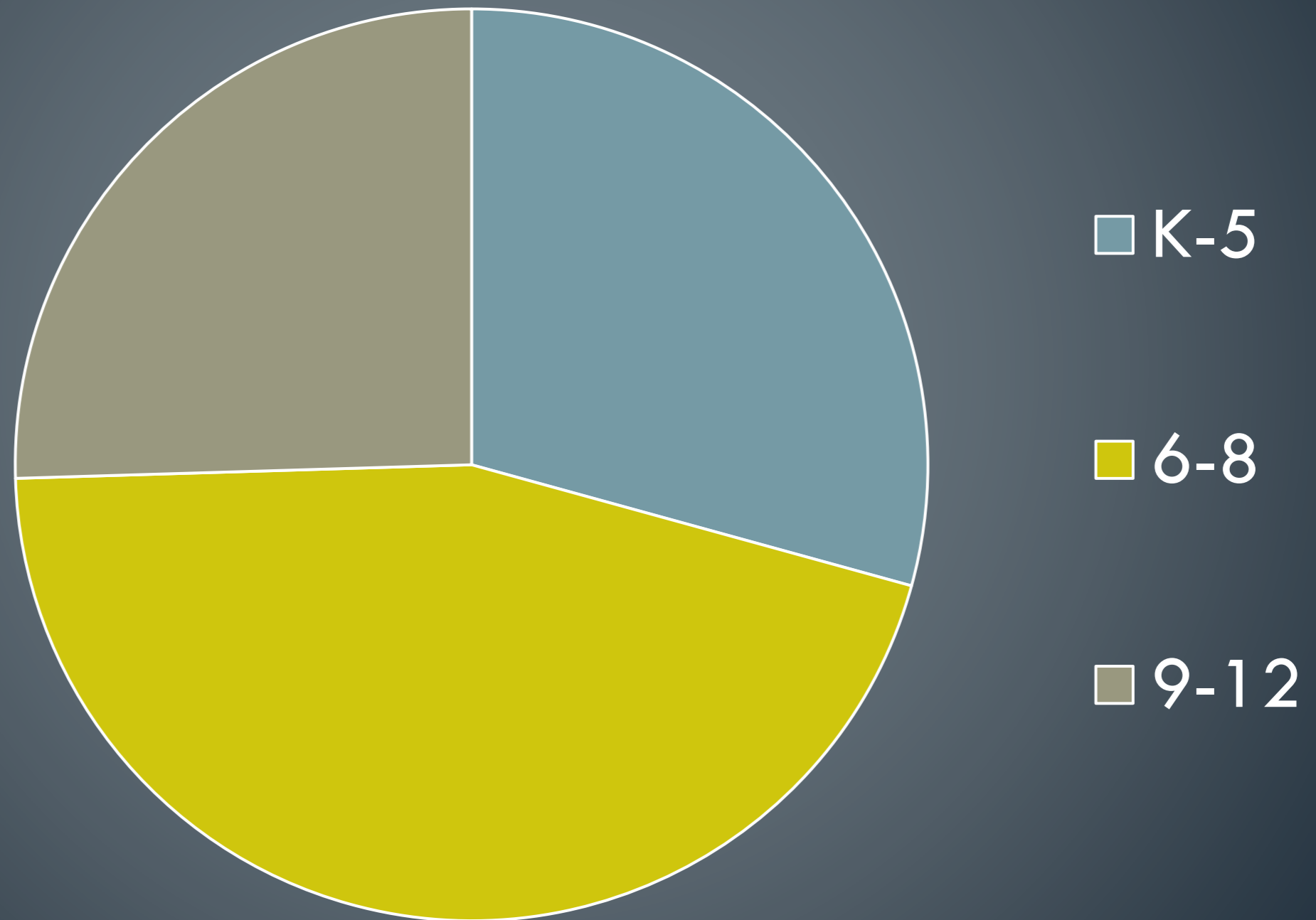


# Problems by DOK Level



Note: Data as of February 2016

# Problems by Grade Band



Note: Data as of February 2016



# COMMON CORE

## STATE STANDARDS INITIATIVE

Rigor refers to deep, authentic command of mathematical concepts, not making math harder or introducing topics at earlier grades. To help students meet the standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skills and fluency, and application.

# Call to Action

- ▶ Commit to one of these choices:
  - ▶ Implement a single DOK 2 or DOK 3 problem from [openmiddle.com](https://openmiddle.com) in your classes within the week.
  - ▶ Put a DOK 2 question from [openmiddle.com](https://openmiddle.com) on your next assessment.

# Contact

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