DIGGING INTO DEPTH OF KNOWLEDGE

ROBERT KAPLINSKY

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GOALS

- ❑ WHY DO WE NEED THEM?
- ❑ WHY ARE THEY DIFFERENT?
- ❑ HOW DO YOU IMPLEMENT THEM?
- ❑ HOW DO YOU CREATE YOUR OWN?
- ❑ WHERE DO YOU GET OTHERS?

Student Name	ID Number	Perf. Level	Scaled Score	Mathematics Clusters (Clusters where the percent correct is shown in bold represent proficiency for that cluster.)									
				Rational numbers		Exponents, powers, and roots		Quantitative relationships and evaluating expressions		Multi-step problems, graphing, and functions		Measurement a geometry	
				Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Perc Corr
[redacted]	[redacted]	ADV	476	13	93%	8	100%	8	80%	14	93%	12	92%
[redacted]	[redacted]	ADV	464	13	93%	7	88%	8	80%	15	100%	11	85%
[redacted]	[redacted]	ADV	453	10	71%	8	100%	10	100%	14	93%	11	85%
[redacted]	[redacted]	ADV	453	13	93%	8	100%	9	90%	12	80%	11	85%
[redacted]	[redacted]	ADV	444	14	100%	7	88%	8	80%	13	87%	10	77%
[redacted]	[redacted]	ADV	444	12	86%	8	100%	8	80%	15	100%	10	77%
[redacted]	[redacted]	ADV	444	13	93%	8	100%	8	80%	14	93%	9	69%
[redacted]	[redacted]	ADV	435	12	86%	6	75%	9	90%	14	93%	10	77%
[redacted]	[redacted]	ADV	435	12	86%	6	75%	8	80%	14	93%	11	85%
[redacted]	[redacted]	ADV	435	13	93%	7	88%	9	90%	12	80%	10	77%
[redacted]	[redacted]	ADV	427	13	93%	6	75%	9	90%	12	80%	10	77%
[redacted]	[redacted]	ADV	427	13	93%	7	88%	6	60%	13	87%	11	85%
[redacted]	[redacted]	ADV	427	14	100%	5	63%	7	70%	14	93%	10	77%
[redacted]	[redacted]	ADV	421	13	93%	6	75%	6	60%	14	93%	10	77%
[redacted]	[redacted]	ADV	421	11	79%	5	63%	9	90%	13	87%	11	85%
[redacted]	[redacted]	ADV	414	12	86%	6	75%	8	80%	11	73%	11	85%
[redacted]	[redacted]	ADV	414	12	86%	8	100%	8	80%	13	87%	8	62%
[redacted]	[redacted]	PRO	408	11	79%	6	75%	9	90%	11	73%	10	77%
[redacted]	[redacted]	PRO	402	12	86%	8	100%	9	90%	8	53%	11	85%
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[redacted]	[redacted]	PRO	380	10	71%	7	88%	8	80%	11	73%	7	54%
[redacted]	[redacted]	PRO	375	14	100%	5	63%	6	60%	10	67%	6	46%
[redacted]	[redacted]	PRO	375	8	57%	7	88%	8	80%	11	73%	8	62%
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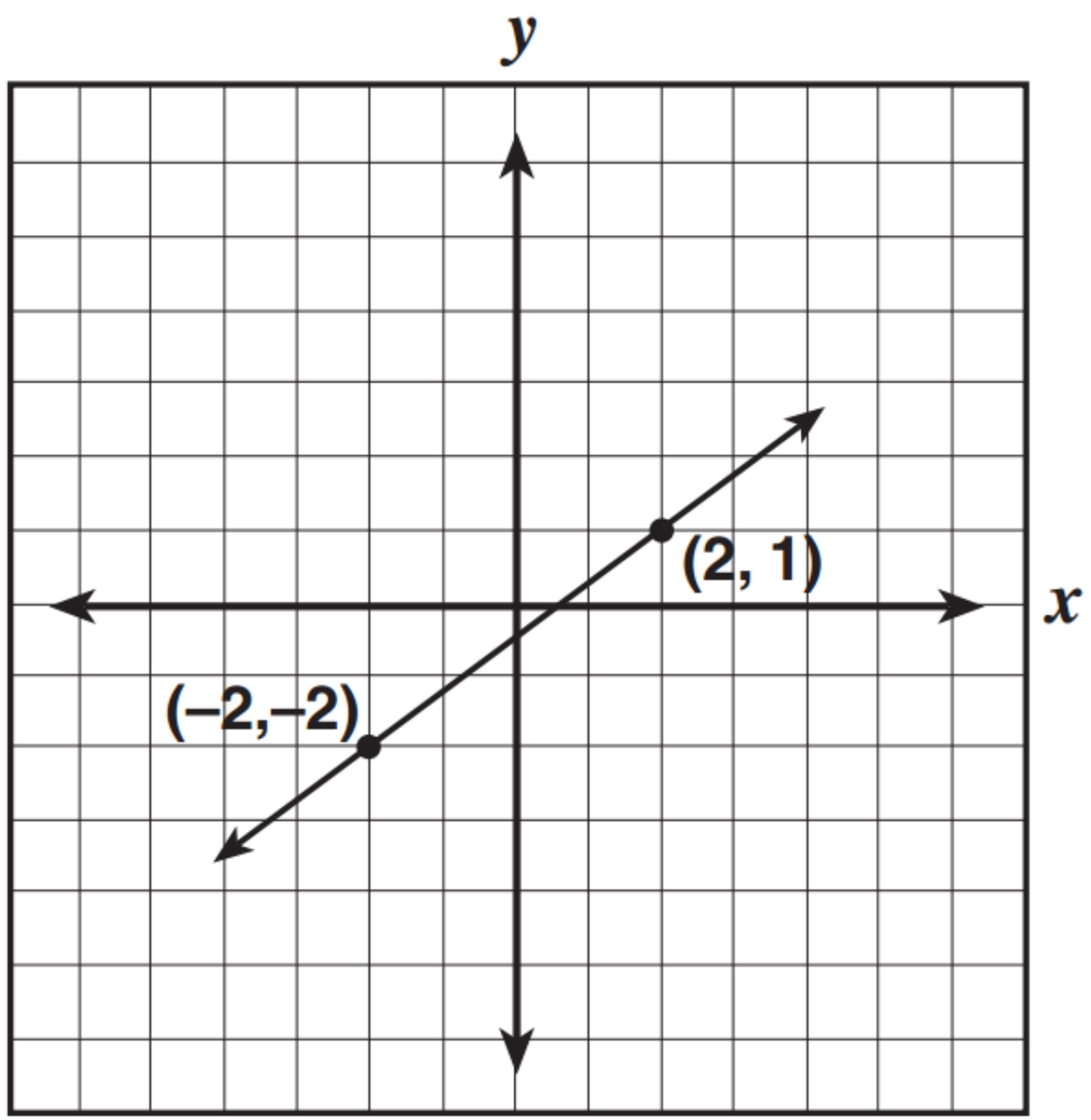
[redacted]	[redacted]	PRO	375	12	86%	4	50%	6	60%	12	80%	7	54%

52 **What is the slope of this line?**



A $\frac{1}{2}$

B $\frac{3}{4}$

C 1

D $\frac{4}{3}$

Source: California Released Test Questions (7th Grade Math)


X-RAY
VISION

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[redacted]	[redacted]	ADV	[illegible]	[illegible]	86%	8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]%	10	77%
[redacted]	[redacted]	ADV	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]%	9	69%
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[redacted]	[redacted]	ADV	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	93%	11	85%
[redacted]	[redacted]	ADV	[illegible]	[illegible]	[illegible]%	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	80%	10	77%
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[redacted]	[redacted]	ADV	[illegible]	[illegible]	100%	5	[illegible]	[illegible]	[illegible]	[illegible]	93%	10	77%
[redacted]	[redacted]	ADV	[illegible]	[illegible]	93%	6	[illegible]	[illegible]	[illegible]	[illegible]	93%	10	77%
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PROBLEM ONE

Solve for x.

$$21 + x = 70$$

PROBLEM TWO

Using the digits 1 to 9, at most one time each, create two equations: one where x has a positive value and one where x has a negative value.

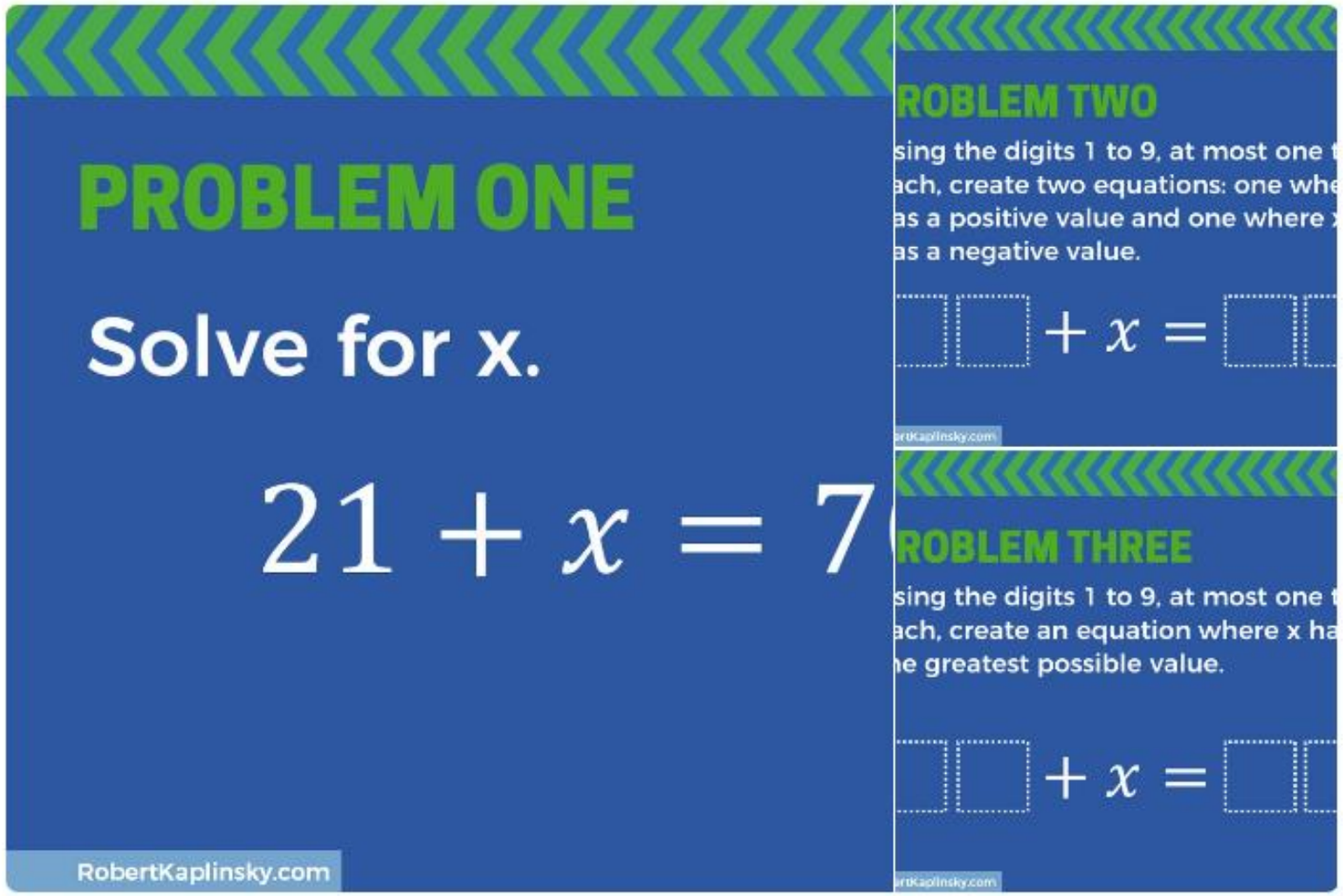
$$\square\square + x = \square\square$$

PROBLEM THREE

Using the digits 1 to 9, at most one time each, create an equation where x has the greatest possible value.

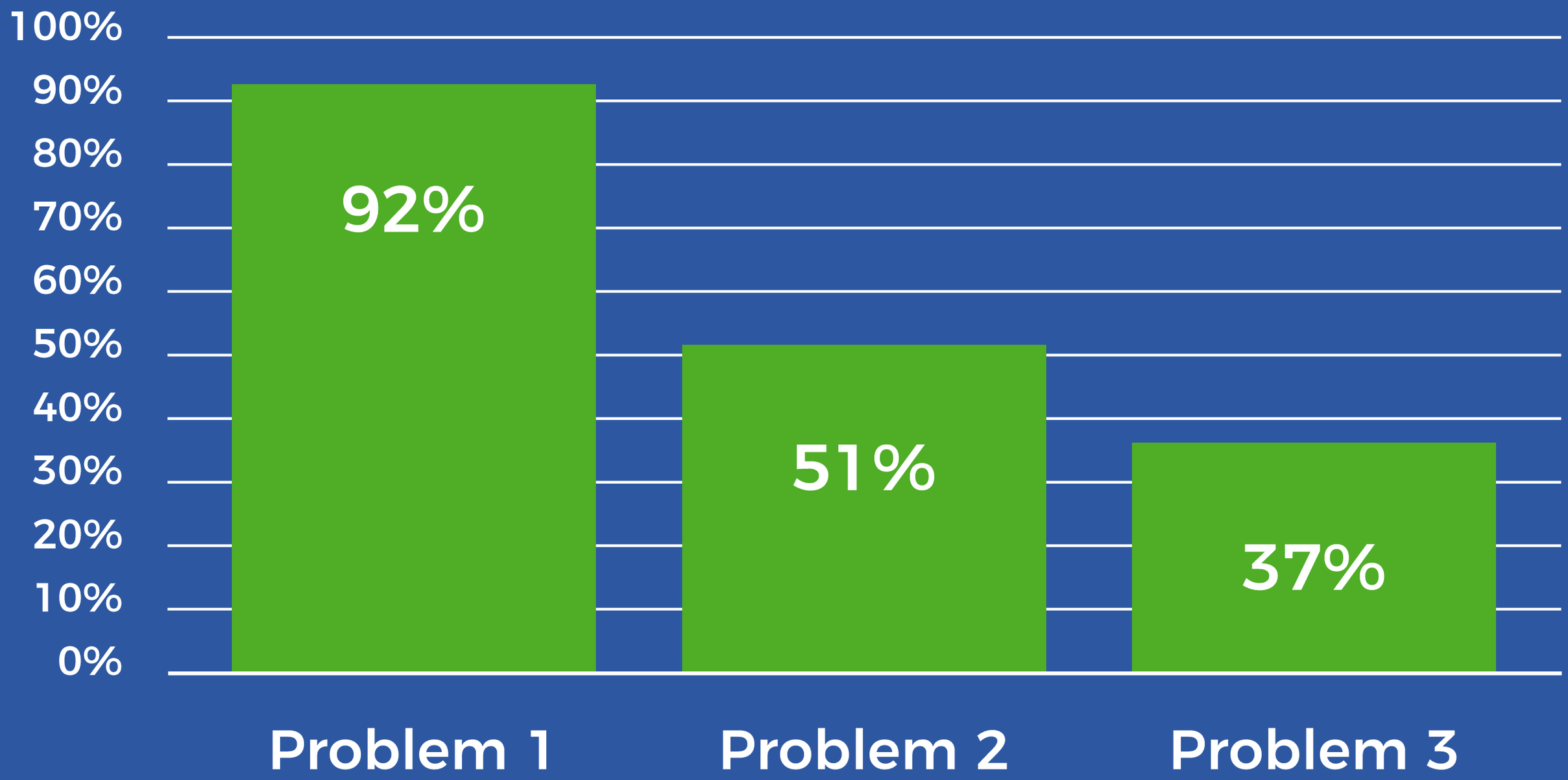
Robert Kaplinsky
@robertkaplinsky

MS & HS #MTBoS Ts, please ask your Ss these 3 ?s and put the % who answered correctly here: docs.google.com/forms/d/e/1FAI …. Answers at top of form.

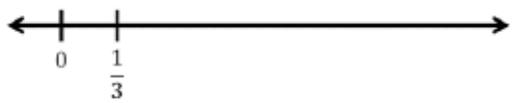


RETWEETS 36 LIKES 54

PROBLEM RESULTS

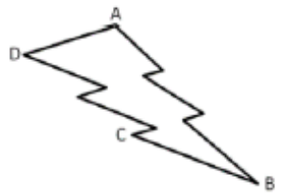
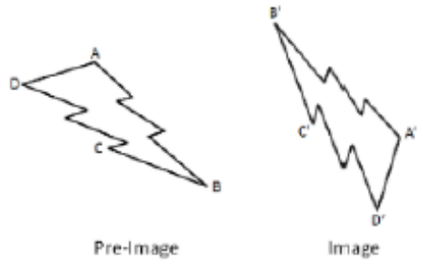
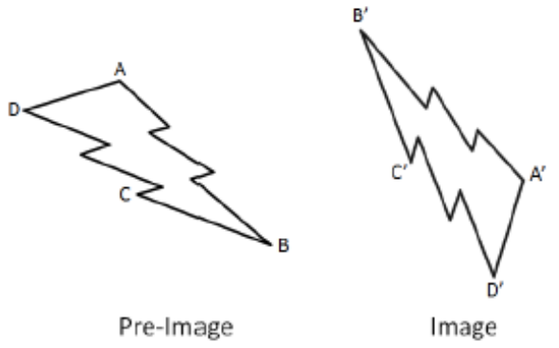


Depth of Knowledge Matrix - Elementary & Secondary Math

Topic	Adding Whole Numbers	Money	Fractions on a Number Line	Area and Perimeter	Subtracting Mixed Numbers
CCSS Standard(s)	• 1.NBT.4 • 2.NBT.5	• 2.MD.8	• 3.NF.2	• 3.MD.8 • 4.MD.3	• 5.NF.1
DOK 1 Example	Find the sum. $44 + 27 =$	If you have 2 dimes and 3 pennies, how many cents do you have?	Which point is located at $\frac{7}{12}$ below? 	Find the perimeter of a rectangle that measures 4 units by 8 units.	Find the difference. $5\frac{1}{2} - 4\frac{2}{3} =$
DOK 2 Example	Fill in the boxes below using the whole numbers 1 through 9, no more than one time each, so that you make a true equation. $\square\square + 53 = \square\square$	Make 47¢ in three different ways with either quarters, dimes, nickels, or pennies.	Label the point where $\frac{3}{4}$ belongs on the number line below. Be as precise as possible. 	List the measurements of three different rectangles that each has a perimeter of 20 units.	Create three different mixed numbers that will make the equation true by using the whole numbers 1 through 9, no more than one time each. You may reuse the same whole numbers for each of the three mixed numbers. $5\frac{4}{5} - \square\frac{\square}{\square} = 3\frac{1}{20}$
DOK 3 Example	Make the largest sum by filling in the boxes below using the whole numbers 1 through 9, no more than one time each. $\square\square + \square\square =$	Make 47¢ using exactly 6 coins with either quarters, dimes, nickels, or pennies.	Create 5 fractions using the whole numbers 0 through 9, exactly one time each as numerators and denominators, and place them all on a number line.	What is the greatest area you can make with a rectangle that has a perimeter of 24 units?	Make the smallest difference by filling in the boxes below using the whole numbers 1 through 9, no more than one time each. 

Version 1.17

Depth of Knowledge Matrix - Elementary & Secondary Math

Topic	Surface Area and Volume	Probability	Transformations	Factoring Quadratics	Quadratics in Vertex Form
CCSS Standard(s)	• 6.G.4 • 7.G.6	• 7.SP.5 • 7.SP.7	• 8.G.1 • G-CO.5	• A-SSE.3a	• F-IF.7a
DOK 1 Example	Find the surface area of a rectangular prism that measures 3 units by 4 units by 5 units.	What is the probability of rolling a sum of 5 using two 6-sided dice?	Rotate the image below 90° counterclockwise about point D and reflect it across a horizontal line. 	Find the factors: $2x^2 + 7x + 3$	Find the roots and maximum of the quadratic equation below. $y = -3(x-4)^2 - 3$
DOK 2 Example	List the measurements of three different rectangular prisms that each have a surface area of 20 square units.	What value(s) have a 1/12 probability of being rolled as the sum of two 6-sided dice?	List three sequences of transformations that take pre-image ABCD to image A′B′C′D′.  Pre-Image Image	Find three different integers to put in the blank that will make the quadratic expression factorable. $x^2 + \_\_x + 4$	Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values.
DOK 3 Example	What is the greatest volume you can make with a rectangular prism that has a surface area of 20 square units?	Fill in the blanks to complete this sentence using the whole numbers 1 through 9, no more than one time each. Rolling a sum of ___ on two ___-sided dice is the same probability as rolling a sum of ___ on two ___-sided dice.	What is the fewest number of transformations needed to take pre-image ABCD to image A′B′C′D′?  Pre-Image Image	Fill the blank by finding the largest and smallest integers that will make the quadratic expression factorable. $2x^2 + 3x + \_\_$	Create a quadratic equation with the largest maximum value using the whole numbers 1 through 9, no more than one time each. $y = -\square(x-\square)^2 + \square$

GOALS

- [x] WHY DO WE NEED THEM?
- [x] WHY ARE THEY DIFFERENT?
- [ ] HOW DO YOU IMPLEMENT THEM?
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IMPLEMENTATION

- Open Middle Worksheet

First attempt: Points: _____/2 attempt _____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

IMPLEMENTATION

- Open Middle Worksheet
- Classwork
  - Single problem for entire class
  - Extensions menu

QUESTION #1

Use the digits 1 to 9, at most one time each, to create an equation where x has the greatest possible value.

$$\square\square + x = \square\square$$

4 points

QUESTION #2

Solve for x.

$$3x + 7 = 19$$

1 point

QUESTION #3

Use the digits 1 to 9, at most one time each, to create two equations: one where x has a positive value and one where x has a negative value.

$$\square\square + x = \square\square$$

2 points

QUESTION #4

Use the digits 1 to 9, at most one time each, to make each equation true.

$$\square + a = \square$$

$$\square\, b = \square$$

$$c - \square = \square$$

$$a = \square,\ b = \square,$$

$$c = \square$$

SOLVING EQUATIONS EXTENSION MENU

You must earn <u>at least 12 points</u> by doing the problems of your choice. Circle the questions you have answered.

QUESTION #5

Use the digits 1 to 9, at most one time each, to create an equation where x has the greatest possible value.

$$\square x + \square = \square$$

6 points

IMPLEMENTATION

- Open Middle Worksheet
- Classwork
  - Single problem for entire class
  - Extensions menu
- Homework
- Assessments

GOALS

- [x] WHY DO WE NEED THEM?
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STEP ONE

- Find a One-Operation Problem
  - Addition
  - Subtraction
  - Multiplying
  - Dividing
  - Exponents (including square root)
  - Trigonometric functions

ADDING 2-DIGIT NUMBERS

Solve.

$$41 + 36 =$$

MULTIPLYING FRACTIONS

Solve.

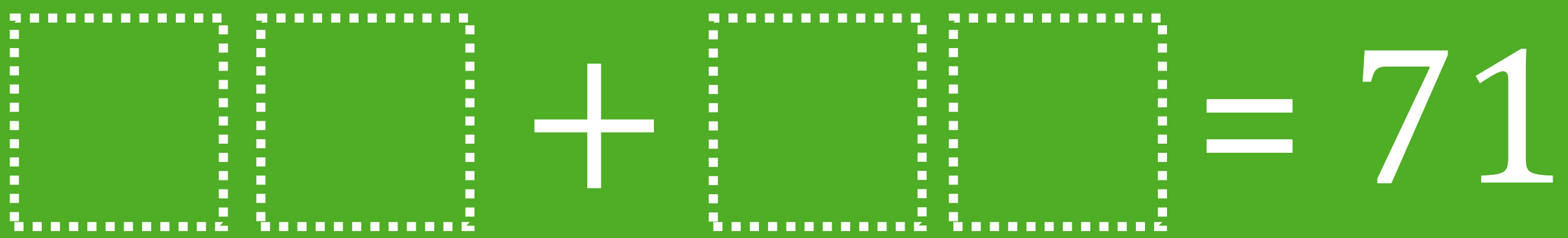
$$\frac{3}{7} \times \frac{2}{9} =$$

STEP TWO

- Go from DOK 1 to DOK 2
  - Strategically remove some information from the problem to prevent immediate calculation
  - Increase the quantity of solutions needed to increase the need to look for patterns

ADDING 2-DIGIT NUMBERS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of two-digit numbers that have a sum of 71.



MULTIPLYING FRACTIONS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of fractions that have a product of 2/3.

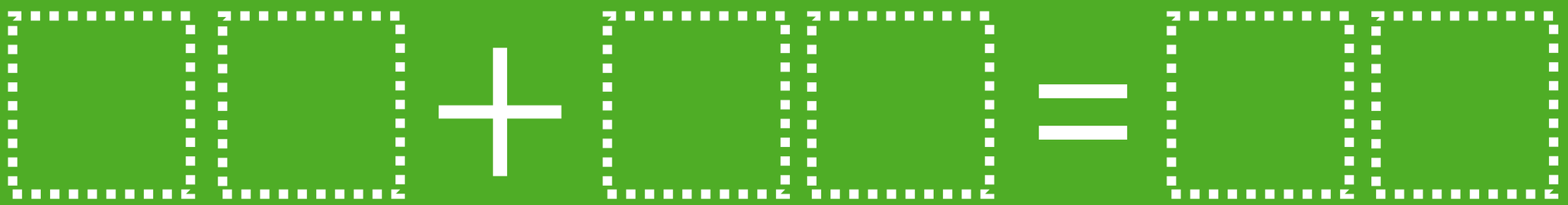
$$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{2}{3}$$

STEP THREE

- Go from DOK 2 to DOK 3
  - Introduce the need to optimize the solution by making the greatest or least product / sum / difference / quotient / answer.
  - Another optimization option is make the answer closest to a specific value.

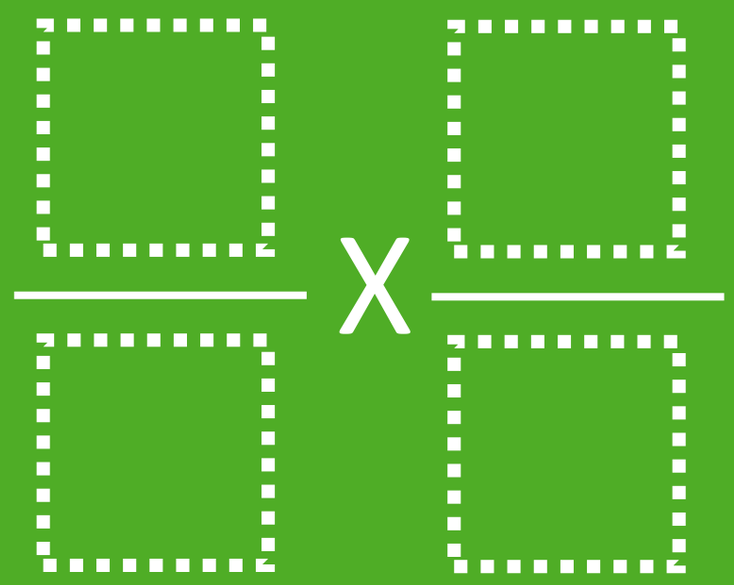
ADDING 2-DIGIT NUMBERS

Using the digits 1 to 9, at most one time each, fill in the boxes to make the smallest sum.



MULTIPLYING FRACTIONS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two fractions that have a product that is as close to 4/11 as possible.



3 Steps to Increase Math DOK Levels

Step 1: Find a One-Operation Problem

- Procedural problems with one operation are easiest to modify.
- Other problems may also be modified but may not be as easy.

Adding 2-Digit Numbers

Solve.

$41 + 36 = \_\_\_$

Multiplying Fractions

Solve.

$\frac{3}{7} \times \frac{2}{9} = \_\_\_$

Trigonometry

Solve.

$\sin \frac{\pi}{3} = \_\_\_$

Step 2: Go from DOK 1 to DOK 2

- Strategically remove some information from the problem to prevent immediate calculation
- Increase the quantity of solutions needed to increase the need to look for patterns

Adding 2-Digit Numbers

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of two-digit numbers that have a sum of 71.

$\square\square + \square\square = 71$

Multiplying Fractions

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of fractions that have a product of 2/3.

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{2}{3}$

Trigonometry

Using the digits 1 to 9, at most one time each, fill in the boxes to make two true number sentences.

$\sin \frac{\square\pi}{\square} = 0$

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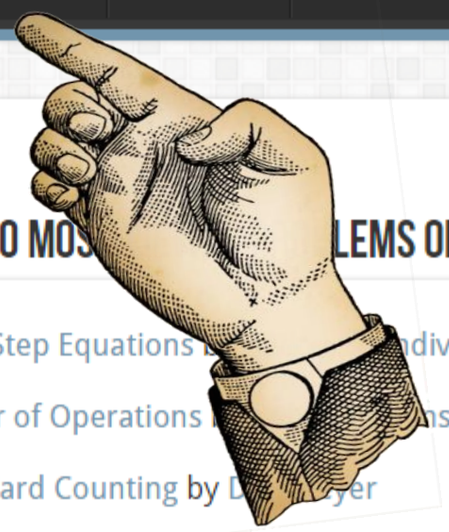
Open Middle

Challenging math problems worth solving



THE TOP 10 MOS[illegible]LEMS OF 2016

1. Two-Step Equations [illegible]ndivil, Daniel Luevanos, and Robert Kaplinsky
2. Order of Operations [illegible]nsky with answer from Michael Fenton and his students
3. Dot Card Counting by D[illegible]yer
4. Rational and Irrational Numbers by Bryan Anderson
5. One Solution, No Solutions, Infinite Solutions by Bryan Anderson
6. Multiplying a Two-Digit Number by a Single-Digit Number by Robert Kaplinsky
7. Exponents and Order of Operations by Zack Miller
8. Converting Between Fractions and Decimals by Robert Kaplinsky
9. Interpretting Percentages by Robert Kaplinsky
10. Two-Step Equations 3 by Erick Lee

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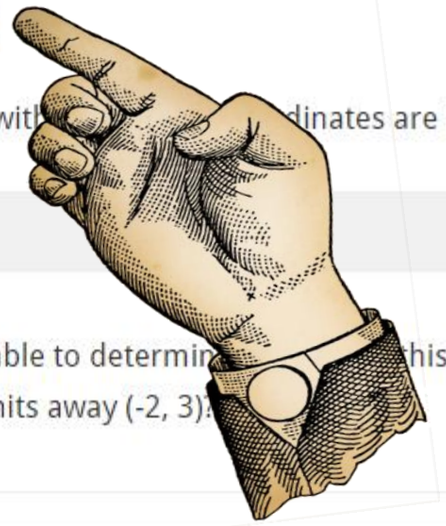
  - Counting & Cardinality (2)
  - Number & Operations in Base Ten (1)
  - Operations & Algebraic Thinking (3)

EQUIDISTANT POINTS



Directions: How many points with ... dinates are 5 units away from (-2, 3)?

Hint

Which methods are available to determin... this problem? What shape is defined by *all* of the points that are 5 units away (-2, 3)?

Answer

12 points: (-5, 7), (-7, 3), (-5, -1), (-2, -2), (3, 3), (1, -1), (-2, 8), (1, 7), (2,6), (-6, -6), (-6, 0), and (2, 0)

Source: Dylan Kane

Tags 8.G.8 • DOK 2: SKILL / CONCEPT • DYLAN KANE • G-GPE.1

BROWSE BY COMMON CORE STATE STANDARDS

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Hey @openmiddle fans, we want to hear from you. Why do you use our problems with your students? Share your success stories or lessons learned.

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GOALS

- [x] WHY DO WE NEED THEM?
- [x] WHY ARE THEY DIFFERENT?
- [x] HOW DO YOU IMPLEMENT THEM?
- [x] HOW DO YOU CREATE YOUR OWN?
- [x] WHERE DO YOU GET OTHERS?

Home

How I Can Help You

Real World Problems

My workshops help teachers implement problem-based lessons by helping them experience them from both student and teacher perspective, leading to increase students' success with performance tasks and the Common Core State Standards.

Depth of Knowledge

Problems at higher depth of knowledge levels have the potential to challenge your most talented student yet remain accessible to everyone. I can help teachers develop best practices for implementing them so that students persevere longer towards finding the solution.

Lessons

View all | Kinder | 1st | 2nd | 3rd | 4th | 5th | 6th | 7th | 8th | Alg 1 | Geo | Alg 2

How Much Money Were Those Pennies?

How Can We #SaveNelly?

How Many Chip Bags Will There Be?

How Can We Make Stronger Passwords?

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First Name

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DIGGING INTO DEPTH OF KNOWLEDGE

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