DIGGING INTO DEPTH OF KNOWLEDGE

ROBERT KAPLINSKY

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GOALS

- ☐ WHY DO WE NEED THEM?
- ☐ WHY ARE THEY DIFFERENT?
- ☐ HOW DO YOU IMPLEMENT THEM?
- ☐ HOW DO YOU CREATE YOUR OWN?
- ☐ WHERE DO YOU GET OTHERS?

Mathematics Clusters

(Clusters where the percent correct is shown in bold represent proficiency for that cluster.)

Student Name	ID Number	Perf. Level	Scaled Score	Rational numbers		Exponents, powers, and roots		Quantitative relationships and evaluating expressions		Multi-step problems, graphing, and functions		Measurement a geometry	
				Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Perc Corr
[redacted]	[redacted]	ADV	476	13	93%	8	100%	8	80%	14	93%	12	92
[redacted]	[redacted]	ADV	464	13	93%	7	88%	8	80%	15	100%	11	85
[redacted]	[redacted]	ADV	453	10	71%	8	100%	10	100%	14	93%	11	85
[redacted]	[redacted]	ADV	453	13	93%	8	100%	9	90%	12	80%	11	85
[redacted]	[redacted]	ADV	444	14	100%	7	88%	8	80%	13	87%	10	77
[redacted]	[redacted]	ADV	444	12	86%	8	100%	8	80%	15	100%	10	77
[redacted]	[redacted]	ADV	444	13	93%	8	100%	8	80%	14	93%	9	69
[redacted]	[redacted]	ADV	435	12	86%	6	75%	9	90%	14	93%	10	77
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[redacted]	[redacted]	ADV	435	13	93%	7	88%	9	90%	12	80%	10	77
[redacted]	[redacted]	ADV	427	13	93%	6	75%	9	90%	12	80%	10	77
[redacted]	[redacted]	ADV	427	13	93%	7	88%	6	60%	13	87%	11	85
[redacted]	[redacted]	ADV	427	14	100%	5	63%	7	70%	14	93%	10	77
[redacted]	[redacted]	ADV	421	13	93%	6	75%	6	60%	14	93%	10	77
[redacted]	[redacted]	ADV	421	11	79%	5	63%	9	90%	13	87%	11	85
[redacted]	[redacted]	ADV	414	12	86%	6	75%	8	80%	11	73%	11	85
[redacted]	[redacted]	ADV	414	12	86%	8	100%	8	80%	13	87%	8	62
[redacted]	[redacted]	PRO	408	11	79%	6	75%	9	90%	11	73%	10	77
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[redacted]	[redacted]	PRO	380	10	71%	7	88%	8	80%	11	73%	7	54
[redacted]	[redacted]	PRO	375	14	100%	5	63%	6	60%	10	67%	6	46
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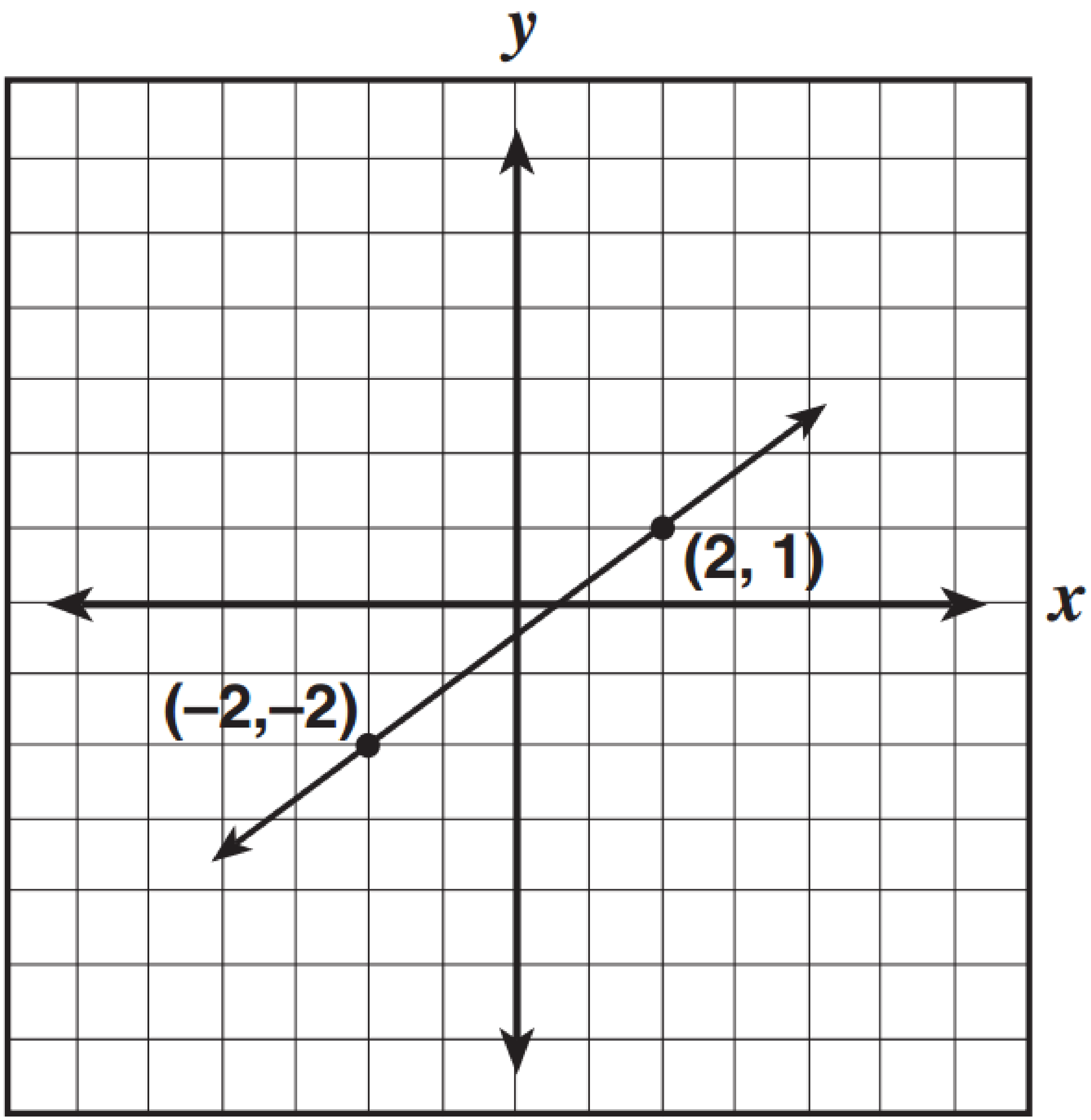
52 **What is the slope of this line?**



A $\frac{1}{2}$

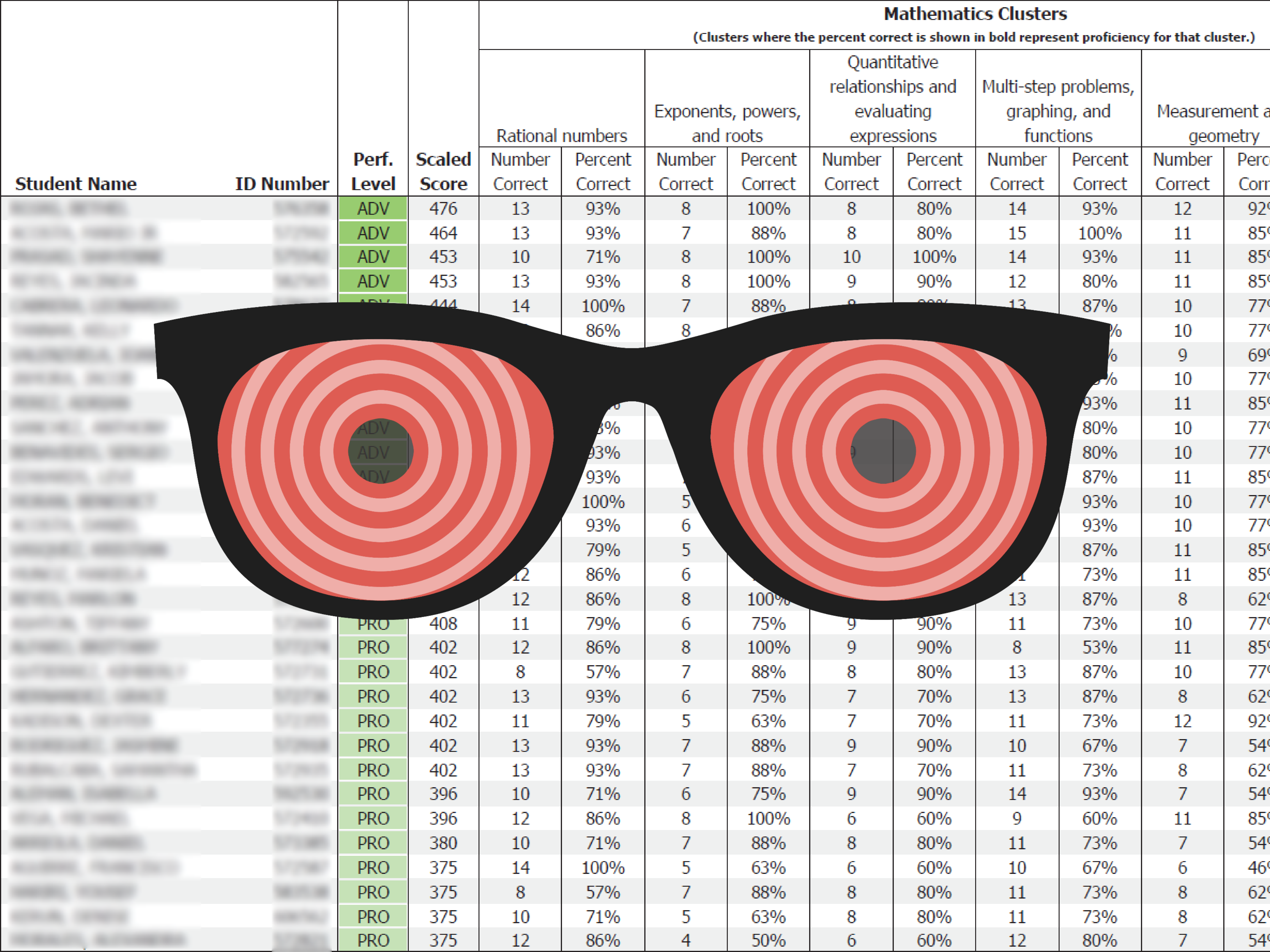
B $\frac{3}{4}$

C 1

D $\frac{4}{3}$

Source: California Released Test Questions (7th Grade Math)


X-RAY
VISION



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[redacted]	[illegible]	[illegible]	[illegible]	[illegible]	86%	8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10	77
[redacted]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9	69
[redacted]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10	77
[redacted]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	93%	11	85
[redacted]	[illegible]	ADV	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	80%	10	77
[redacted]	[illegible]	ADV	[illegible]	[illegible]	93%	[illegible]	[illegible]	9	[illegible]	[illegible]	80%	10	77
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PROBLEM ONE

Solve for x.

$$21 + x = 70$$

PROBLEM TWO

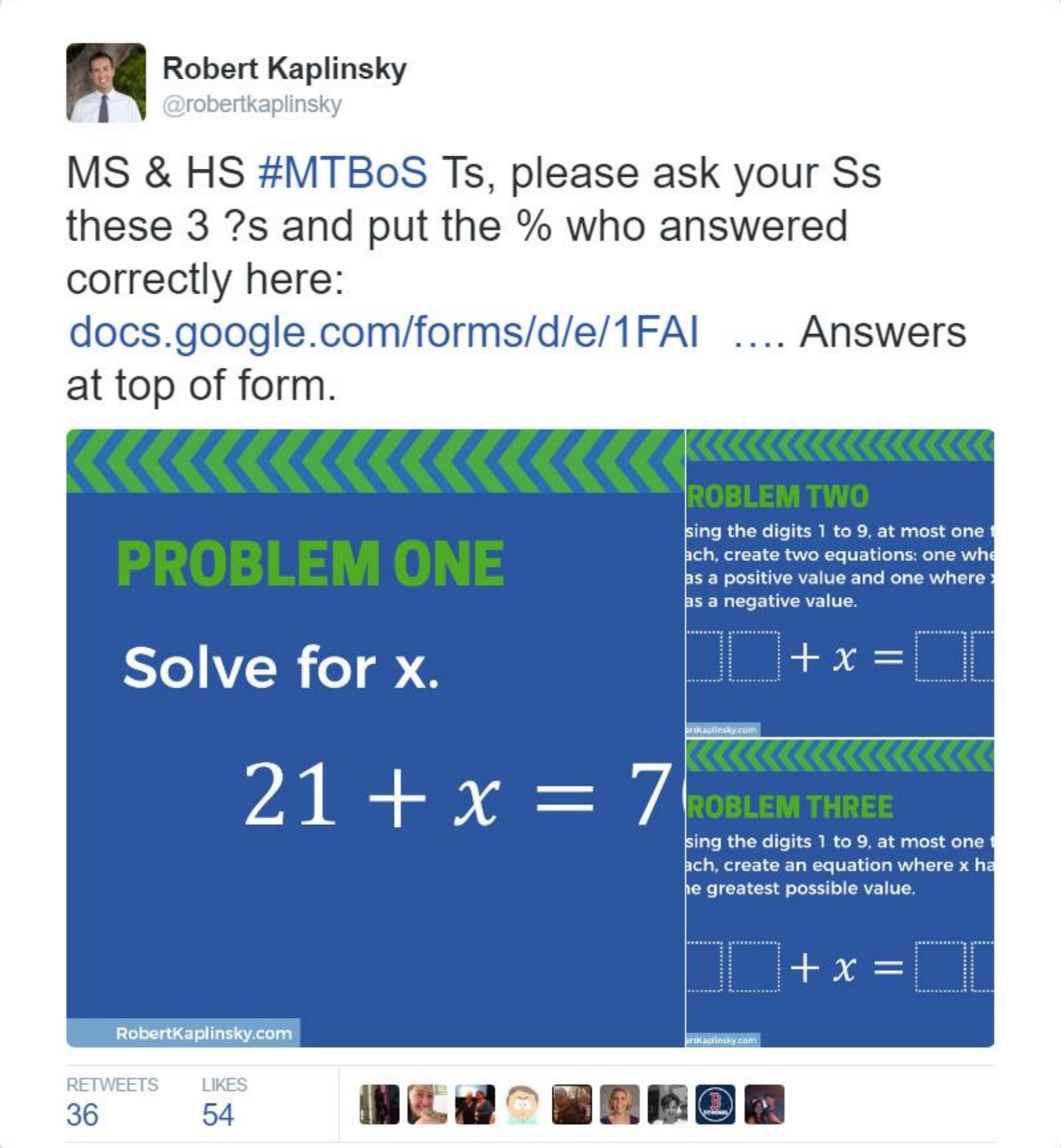
Using the digits 1 to 9, at most one time each, create two equations: one where x has a positive value and one where x has a negative value.

$$\square\,\square + x = \square\,\square$$

PROBLEM THREE

Using the digits 1 to 9, at most one time each, create an equation where x has the greatest possible value.

$$\square\square + x = \square\square$$



Robert Kaplinsky
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MS & HS #MTBoS Ts, please ask your Ss these 3 ?s and put the % who answered correctly here: docs.google.com/forms/d/e/1FAI …. Answers at top of form.

PROBLEM ONE

Solve for x.

$$21 + x = 7$$

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ROBLEM TWO

sing the digits 1 to 9, at most one t
ach, create two equations: one whe
as a positive value and one where x
as a negative value.

☐☐ + x = ☐☐

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ROBLEM THREE

sing the digits 1 to 9, at most one t
ach, create an equation where x ha
e greatest possible value.

☐☐ + x = ☐☐

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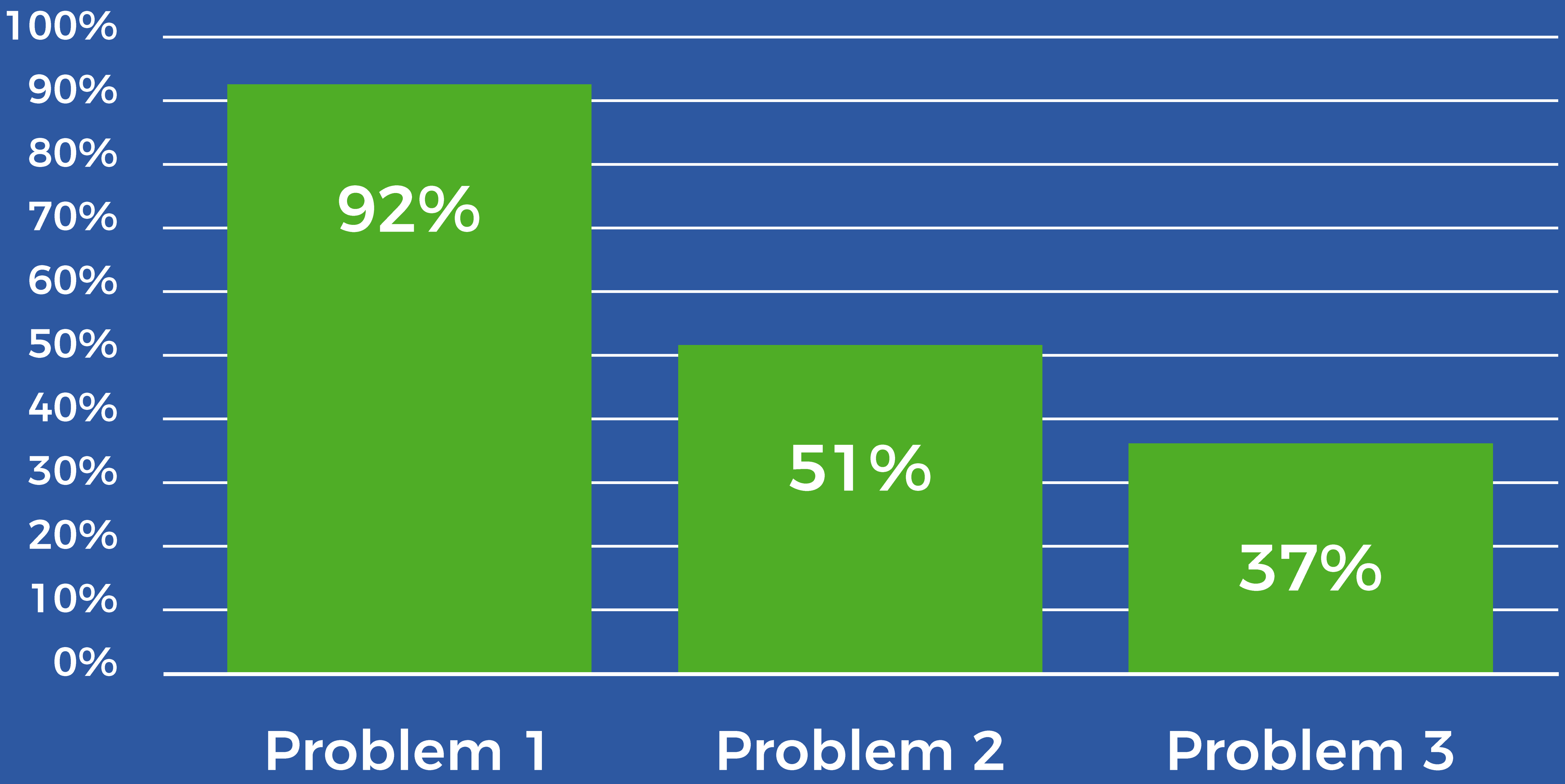
PROBLEM RESULTS



Depth of Knowledge Matrix - Secondary Math

Topic	Dividing Fractions	Solving One-Step Equations	Exponents	Solving Equations with Variables on Both Sides
CCSS Standard(s)	- 6.NS.1	- 7.EE.4a	- 8.EE.1	- 8.EE.8 - A-REI.3
DOK 1 Example	Evaluate. $\frac{4}{9} \div \frac{2}{5}$	Solve for x. $21 + x = 70$	Evaluate. 3^4	Solve for x. $3x + 2 = -2x + 4$
DOK 2 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two different pairs of fractions that have a quotient of 2/3. $\frac{\square}{\square} \div \frac{\square}{\square} = \frac{2}{3}$	Use the digits 1 to 9, at most one time each, to create two equations: one where x has a positive value and one where x has a negative value. $\square\square + x = \square\square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two true number sentences. $\square^{\square} = 64$	Use the digits 1 to 9, at most <u>two</u> times each, to fill in the boxes to make an equation with no solutions. $\square x + \square = \square x + \square$
DOK 3 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two fractions that have a quotient that is as close to 4/11 as possible. $\frac{\square}{\square} \div \frac{\square}{\square}$	Use the digits 1 to 9, at most one time each, to create an equation where x has the greatest possible value. $\square\square + x = \square\square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a result that has the greatest value possible. $\square^{\square} = \square\square\square$	Use the digits 1 to 9, at most one time each, to fill in the boxes so that the solution is closest to zero. $\square x + \square = \square x + \square$

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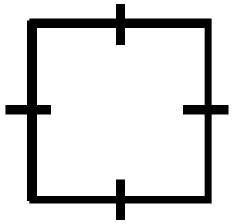
Depth of Knowledge Matrix - Secondary Math

Topic	Geometric Proofs	Complex Numbers	Trigonometric Functions	Definite Integral
CCSS Standard(s)	• G-CO.11	• N-CN.2	• F-TF.3	• N/A
DOK 1 Example	Add one geometric marking to demonstrate the quadrilateral is a square. 	Multiply the binomials. $(3+4i)(2+3i)$	Evaluate. $\sin\frac{\pi}{3}$	Solve. $\int_{2}^{6} x^{3}\,dx$
DOK 2 Example	Use exactly 5 geometric markings to show that a quadrilateral is a square.	Use the integers -9 to 9, at most one time each, to fill in the boxes twice: once to make a positive real number product and once to make a negative real number product. $(\square+\square i)(\square+\square i)$	Use the digits 1 to 9, at most one time each, to fill in the boxes and make two true number sentences. $\sin\frac{\square\pi}{\square}=1$	Use the digits 1 to 9, at most one time each, to fill in the boxes and make a positive and a negative solution. $\int_{\square}^{\square} x^{\square}\,dx$
DOK 3 Example	What is the least number of geometric markings needed to demonstrate that a quadrilateral is a square?	Use the integers -9 to 9, at most one time each, to fill in the boxes and make a real number product with the greatest value. $(\square+\square i)(\square+\square i)$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two true number sentences. $\sin\frac{\square\pi}{\square}=\frac{\sqrt{\square}}{\square}$	Use the digits 1 to 9, at most one time each, to fill in the boxes and make a solution that is as close to 100 as possible. $\int_{\square}^{\square} x^{\square}\,dx$

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GOALS

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IMPLEMENTATION

- Open Middle Worksheet

First attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

IMPLEMENTATION

- Open Middle Worksheet
- Classwork
    - Single problem for entire class
    - Extensions menu

QUESTION #1

Use the digits 1 to 9, at most one time each, to create an equation where x has the greatest possible value.

$$\square\square + x = \square\square$$

4 points

QUESTION #2

Solve for x.

$$3x + 7 = 19$$

1 point

QUESTION #3

Use the digits 1 to 9, at most one time each, to create two equations: one where x has a positive value and one where x has a negative value.

$$\square\square + x = \square\square$$

2 points

QUESTION #4

Use the digits 1 to 9, at most one time each, to make each equation true.

$$\square + a = \square$$

$$\square b = \square$$

$$c - \square = \square$$

$$a = \square,\ b = \square,$$

$$c = \square$$

SOLVING EQUATIONS EXTENSION MENU

You must earn <u>at least 12 points</u> by doing the problems of your choice. Circle the questions you have answered.

QUESTION #5

Use the digits 1 to 9, at most one time each, to create an equation where x has the greatest possible value.

$$\square x + \square = \square$$

4 points

IMPLEMENTATION

- Open Middle Worksheet
- Classwork
  - Single problem for entire class
  - Extensions menu
- Homework
- Assessments

GOALS

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STEP ONE

- Find a One-Operation Problem
  - Addition
  - Subtraction
  - Multiplying
  - Dividing
  - Exponents (including square root)
  - Trigonometric functions

ADDING 2-DIGIT NUMBERS

Solve.

$$41 + 36 =$$

MULTIPLYING FRACTIONS

Solve.

$$\frac{3}{7} \times \frac{2}{9} =$$

STEP TWO

- Go from DOK 1 to DOK 2
  - Strategically remove some information from the problem to prevent immediate calculation
  - Increase the quantity of solutions needed to increase the need to look for patterns

ADDING 2-DIGIT NUMBERS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of two-digit numbers that have a sum of 71.

$$\square\,\square + \square\,\square = 71$$

MULTIPLYING FRACTIONS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of fractions that have a product of 2/3.

$$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{2}{3}$$

STEP THREE

- Go from DOK 2 to DOK 3
  - Introduce the need to optimize the solution by making the greatest or least product / sum / difference / quotient / answer.
  - Another optimization option is make the answer closest to a specific value.

ADDING 2-DIGIT NUMBERS

Using the digits 1 to 9, at most one time each, fill in the boxes to make the smallest sum.

☐☐ + ☐☐ = ☐☐

MULTIPLYING FRACTIONS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two fractions that have a product that is as close to 4/11 as possible.

$$\frac{\square}{\square} \times \frac{\square}{\square}$$

3 Steps to Increase Math DOK Levels

Step 1: Find a One-Operation Problem

- Procedural problems with one operation are easiest to modify.
- Other problems may also be modified but may not be as easy.

Adding 2-Digit Numbers
Solve.

$41 + 36 = \_\_\_$

Multiplying Fractions
Solve.

$\frac{3}{7} \times \frac{2}{9} = \_\_\_$

Trigonometry
Solve.

$\sin\frac{\pi}{3} = \_\_\_$

Step 2: Go from DOK 1 to DOK 2

- Strategically remove some information from the problem to prevent immediate calculation
- Increase the quantity of solutions needed to increase the need to look for patterns

Adding 2-Digit Numbers
Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of two-digit numbers that have a sum of 71.

$\square\square + \square\square = 71$

Multiplying Fractions
Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of fractions that have a product of 2/3.

$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{2}{3}$

Trigonometry
Using the digits 1 to 9, at most one time each, fill in the boxes to make two true number sentences.

$\sin\frac{\square\pi}{\square} = 0$

Step 3: Go from DOK 2 to DOK 3

Problem Drives Inquiry

Teacher Drives Inquiry	Yes	No
Yes	Using the digits 1 to 9, at most one time each, fill in the boxes to make the smallest sum. ☐☐ + ☐☐ = ☐☐	37 + 27
No	Using the digits 1 to 9, at most one time each, fill in the boxes to make the smallest sum. ☐☐ + ☐☐ = ☐☐	37 + 27

WHAT TEACHER MOVES?

- What conversations would you want to happen when using the Adding 2-Digit Number DOK 3 problem?
  - How will you ensure they happen?
- Where might students get stuck?
  - What might you say or do if they do get stuck?

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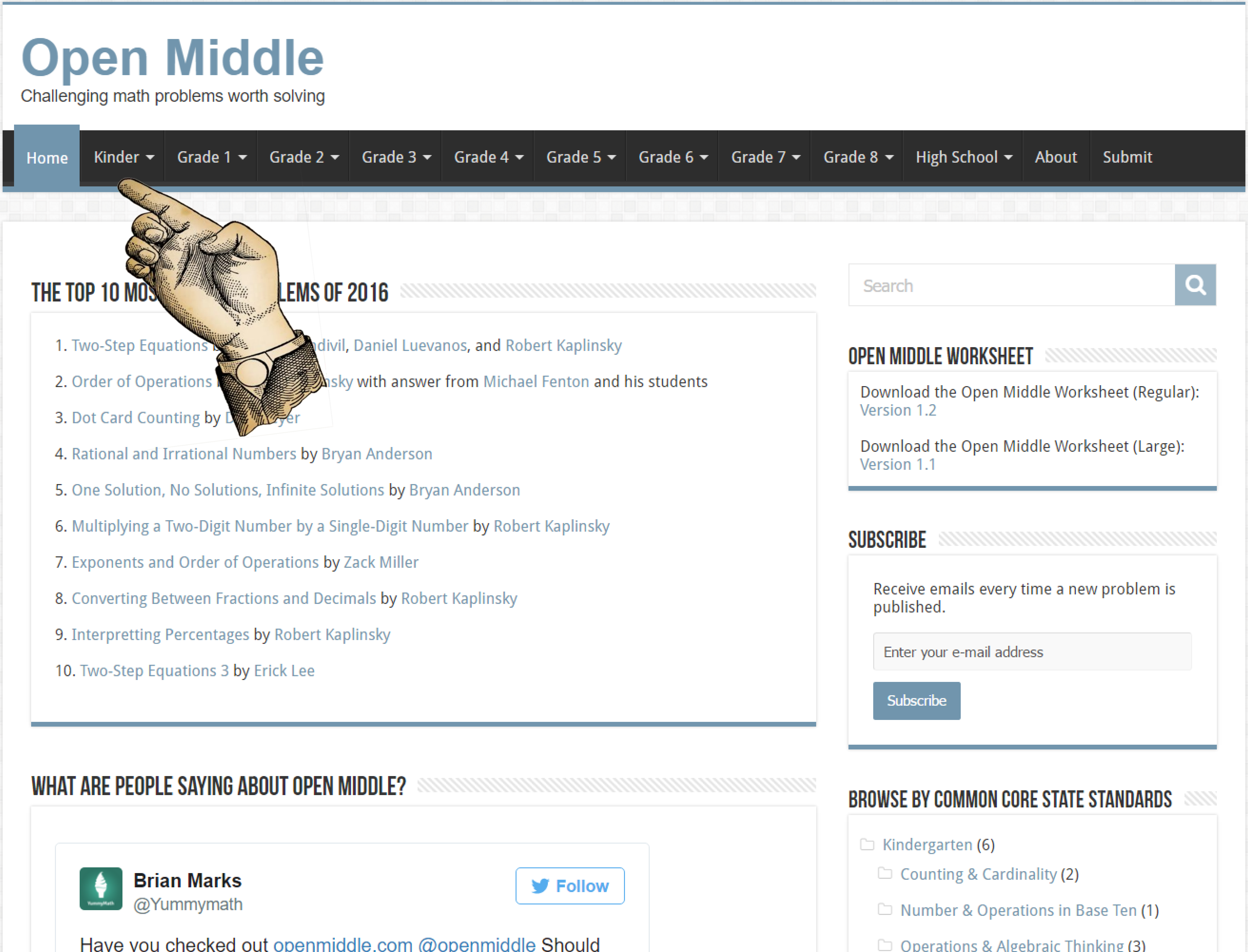

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Challenging math problems worth solving

THE TOP 10 MOS LEMS OF 2016

1. Two-Step Equations ndivil, Daniel Luevanos, and Robert Kaplinsky
2. Order of Operations nsky with answer from Michael Fenton and his students
3. Dot Card Counting by D yer
4. Rational and Irrational Numbers by Bryan Anderson
5. One Solution, No Solutions, Infinite Solutions by Bryan Anderson
6. Multiplying a Two-Digit Number by a Single-Digit Number by Robert Kaplinsky
7. Exponents and Order of Operations by Zack Miller
8. Converting Between Fractions and Decimals by Robert Kaplinsky
9. Interpretting Percentages by Robert Kaplinsky
10. Two-Step Equations 3 by Erick Lee

WHAT ARE PEOPLE SAYING ABOUT OPEN MIDDLE?

Brian Marks
@Yummymath

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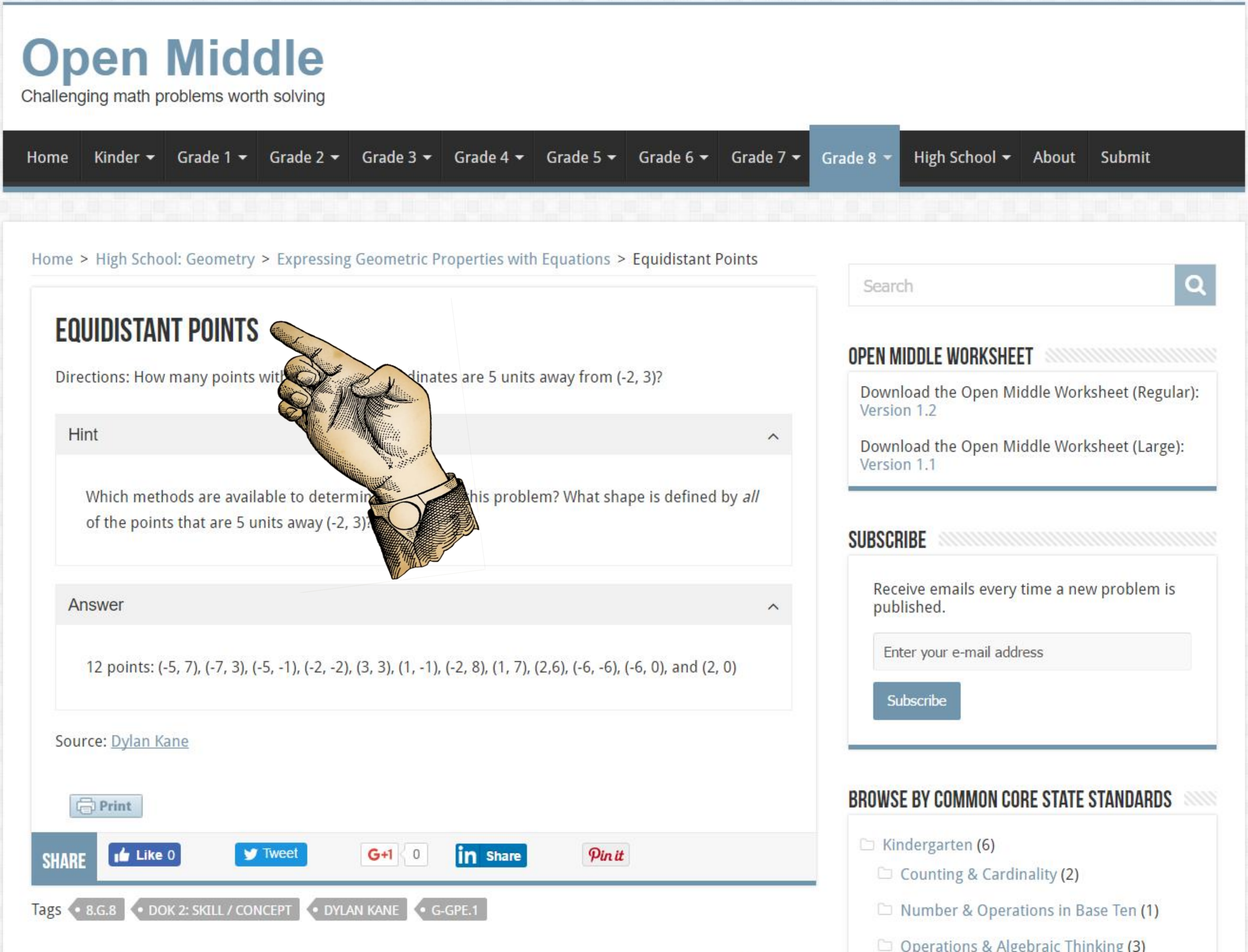
  - Number & Operations in Base Ten (1)
  - Operations & Algebraic Thinking (3)



Home > High School: Geometry > Expressing Geometric Properties with Equations > Equidistant Points

EQUIDISTANT POINTS

Directions: How many points wit dinates are 5 units away from (-2, 3)?

Hint

Which methods are available to determin his problem? What shape is defined by *all* of the points that are 5 units away (-2, 3)?

Answer

12 points: (-5, 7), (-7, 3), (-5, -1), (-2, -2), (3, 3), (1, -1), (-2, 8), (1, 7), (2,6), (-6, -6), (-6, 0), and (2, 0)

Source: Dylan Kane

Tags 8.G.8 DOK 2: SKILL / CONCEPT DYLAN KANE G-GPE.1

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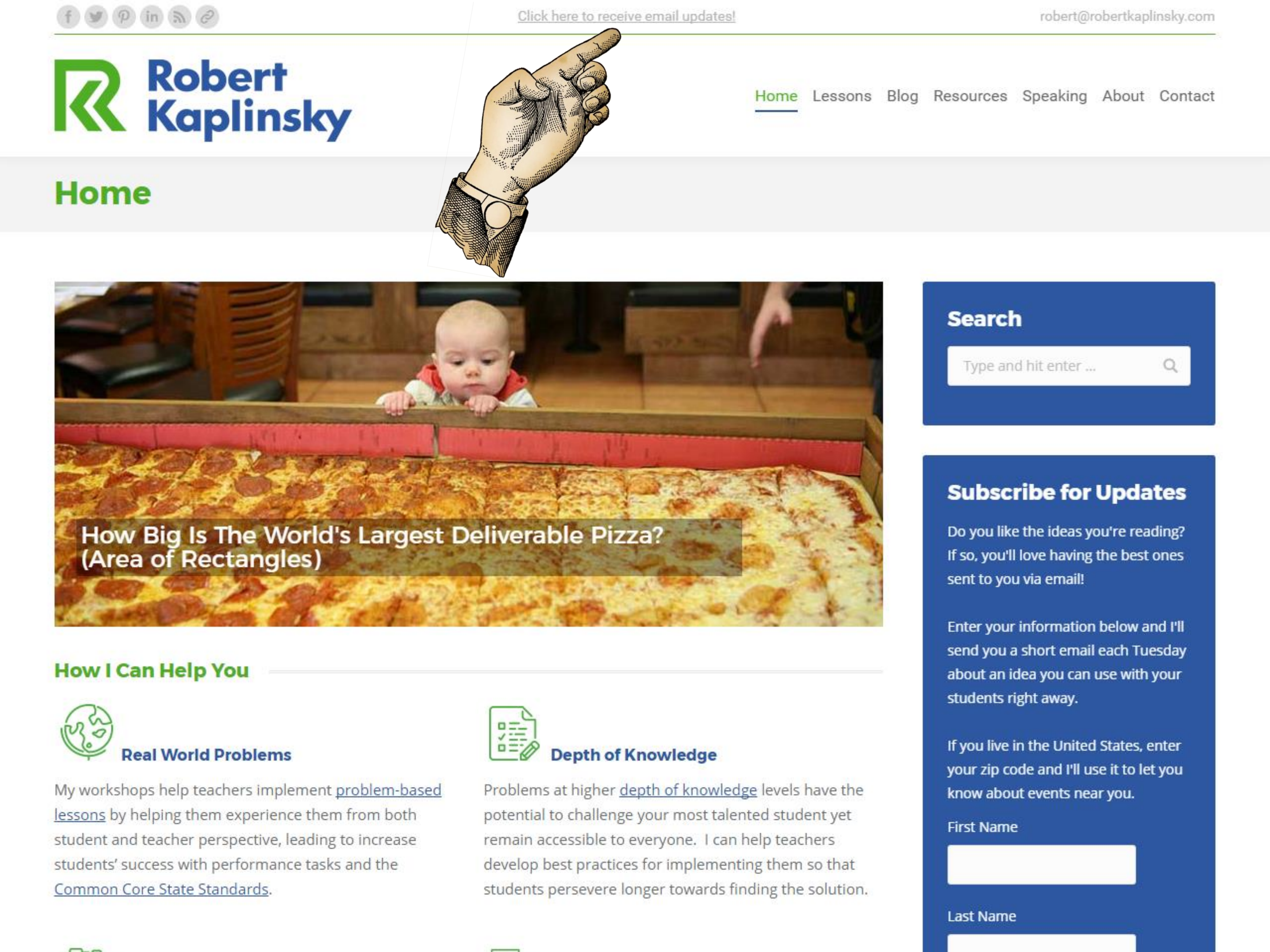

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Real World Problems

My workshops help teachers implement problem-based lessons by helping them experience them from both student and teacher perspective, leading to increase students' success with performance tasks and the Common Core State Standards.

Depth of Knowledge

Problems at higher depth of knowledge levels have the potential to challenge your most talented student yet remain accessible to everyone. I can help teachers develop best practices for implementing them so that students persevere longer towards finding the solution.

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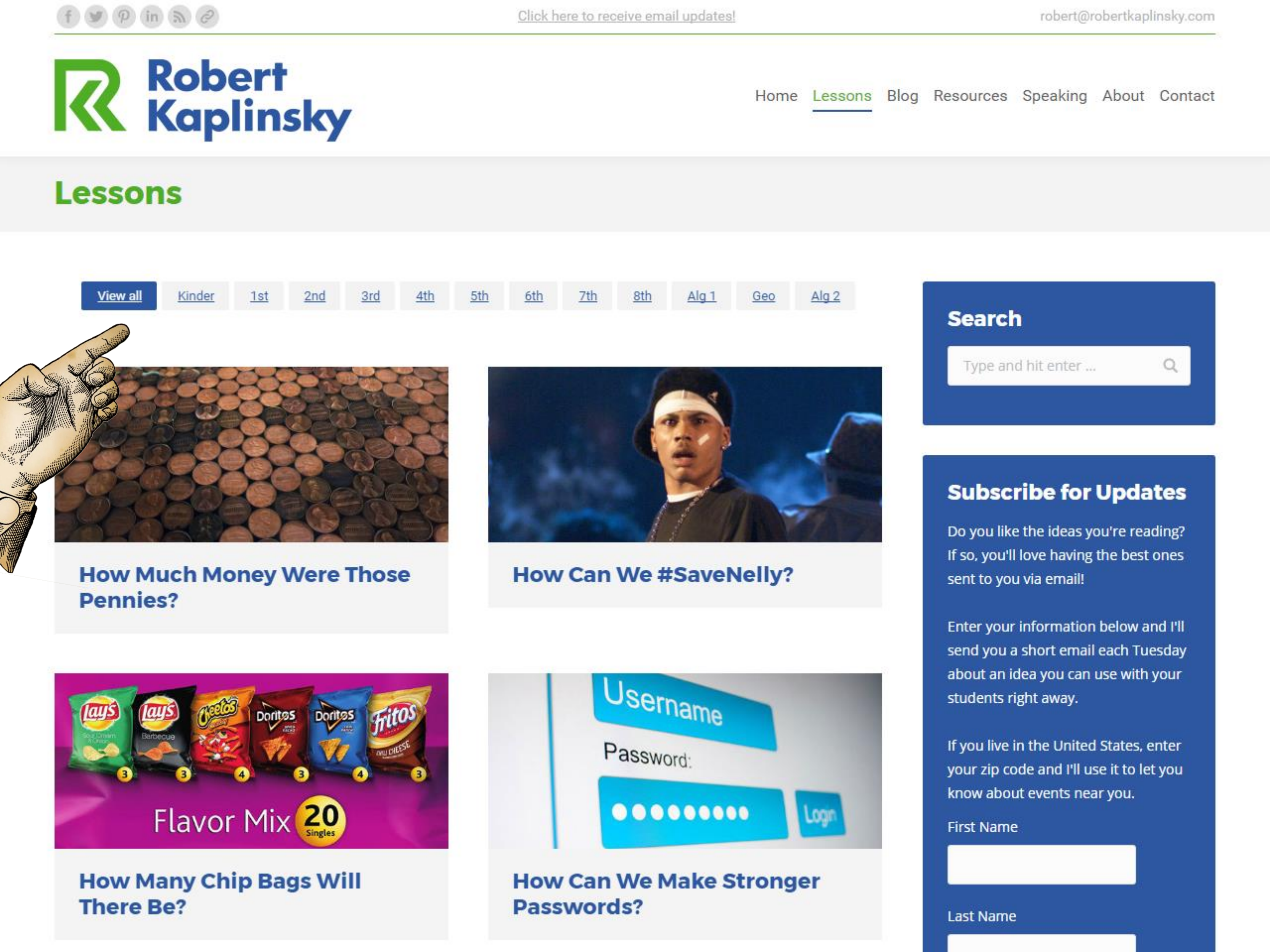

Lessons

How Much Money Were Those Pennies?

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CALL TO ACTION

Action	Do Now	Start Planning	Yes & No	Don’t Do
Incorporate higher DOK problems on assessments		X		
Replace all DOK 1 problems with higher DOK problems				X
Share these resources with colleague to make them aware.	X			
Find problems I can integrate on Open Middle.	X			
Use the 3 steps process to strengthen existing problems.			X	

DIGGING INTO DEPTH OF KNOWLEDGE

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