

**SCHOOL DISTRICT OF THE**

**MENOMONIE AREA**

**ROBERT KAPLINSKY**

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# GOALS

- ❑ **ENGAGING PROBLEM SOLVING**

- ❑ **REAL WORLD PROBLEM-BASED LEARNING**

- ❑ **HIGHER DEPTH OF KNOWLEDGE PROBLEMS**

- ❑ **BETTER IMPLEMENTATION**

- ❑ **IMPROVE QUESTION ASKING**

- ❑ **PRACTICE PREPARING FOR A LESSON**

- ❑ **DEAL WITH UNCOMFORTABLE SITUATIONS**



















**DOUBLE-DOUBLE**<sup>®</sup> *Double Meat & Double Cheese* **2<sup>65</sup>**

**CHEESEBURGER** **1<sup>75</sup>**

**HAMBURGER** **1<sup>50</sup>**

**FRENCH FRIES** **1<sup>05</sup>**

**SHAKES** *Chocolate  
Strawberry  
Vanilla* **1<sup>55</sup>**

<u>SM</u>	<u>MED</u>	<u>LG</u>	<u>X-LG</u>
<b>99</b>	<b>1<sup>10</sup></b>	<b>1<sup>29</sup></b>	<b>1<sup>49</sup></b>
<b>COKE</b> <i>Classic or Diet</i>			
<b>SEVEN-UP</b>			
<b>ROOT BEER</b>			
<b>DR PEPPER</b>			
<b>LEMONADE</b>			
<b>ICED TEA</b>			

**MILK** 70  
**COFFEE** 70



**OPEN 10:30 a.m. to 1:00 a.m.**  
.....**Fri. and Sat. until 1:30 a.m.**



2004-10-31

8:21 PM

YOUR GUEST NUMBER IS  
**98**

IN-N-OUT BURGER LAS VEGAS EASTERN  
2004-10-31

8:21 PM

165 1 5 98

---

Cashier: SAM

GUEST #: 98

---

Counter-Eat In

Db Db	2.65
98 Meat Pty XChz	88.20
Counter-Eat In	90.85
TAX 7.50%	6.81
Amount Due	97.66
CASH TENDER	\$97.66
Change	\$ .00

2004-10-31



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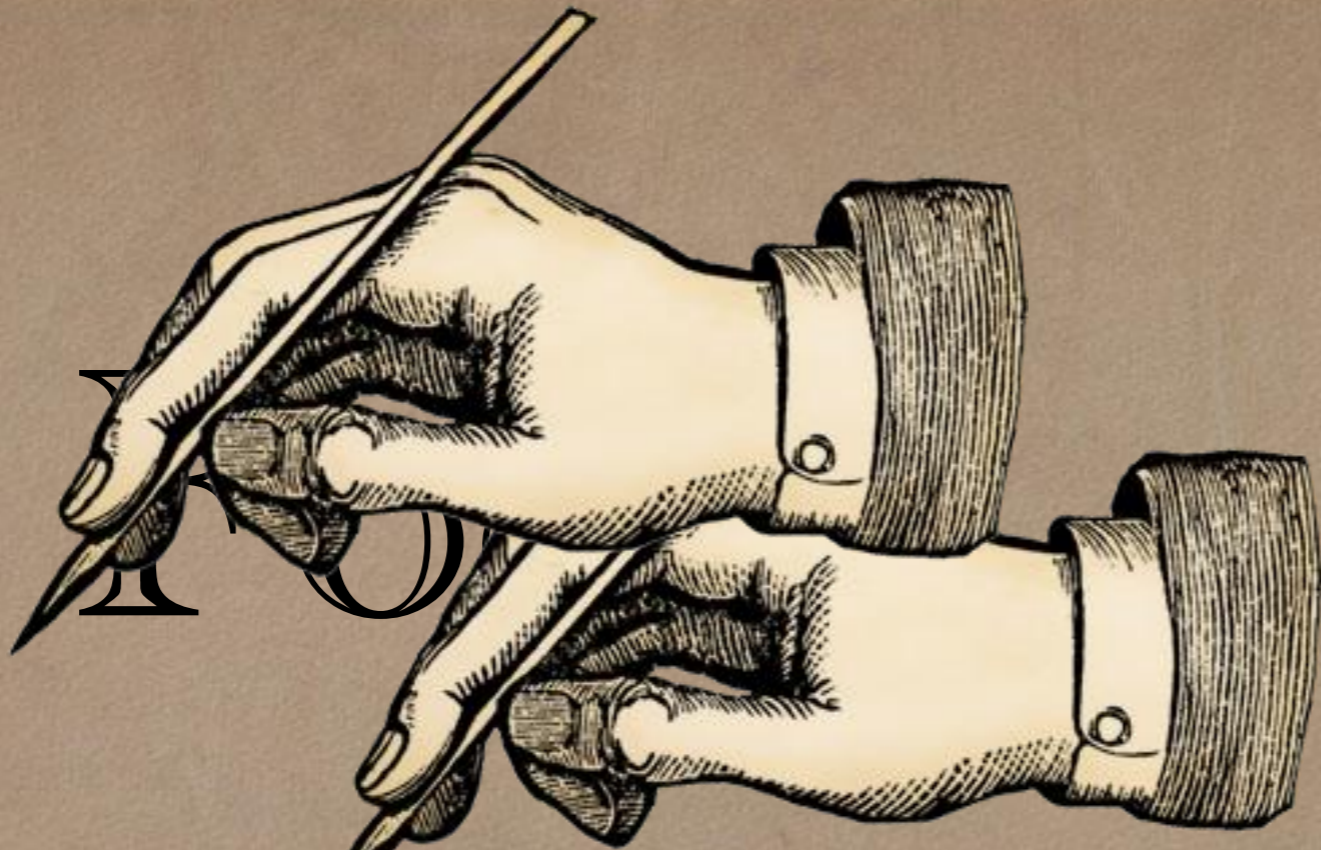
2004-10-31

8:21 PM



	Serving Size (g)	Calories
Hamburger w/Onion	243	390
Cheeseburger w/Onion	268	480
Double-Double w/Onion	330	670





•

Coherence

•

Rigor

•



Layers	Cost
1	\$1.75
2	\$2.65
3	\$3.55
4	\$4.45
.	.
.	.
20	\$18.85
.	.
.	.
100	\$90.85
.	.
.	.
N	$\$1.75 + (N-1)*\$0.90$

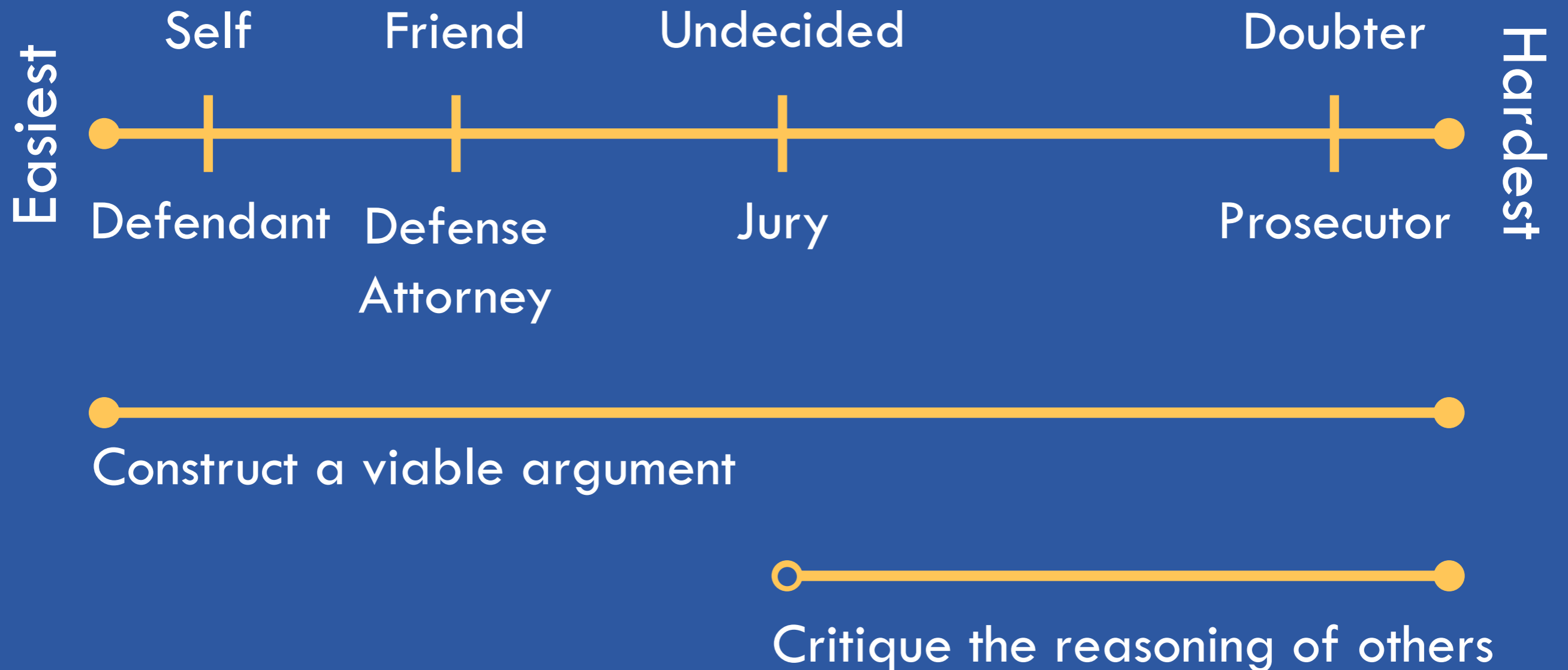


# MATH PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.



# LEVELS OF CONVINCING



Inspired by *Thinking Mathematically* by J. Mason, L. Burton, and K. Stacey



# MATH PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.



**bun + produce + meat + cheese + meat + cheese = \$2.65**

**bun + produce + meat + cheese = \$1.75**

**meat + cheese = \$0.90**



# MATH PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



# THE REALITY

- Students struggled to find a layer's cost.
- Common wrong answers included:
  - \$175.00 ( $\$1.75 \times 100$  cheeseburgers)
  - \$132.50 ( $\$2.65 \times 50$  Double-Doubles)
- Some classes were not ready for a 100x100.
- There were equations with more than N patties.
- Students were surprised to see many correct equations.

# STUDENT

# WORK

What problem are you trying to figure out?	
How much does a 100x100 burger cost? <span style="float: right;">Regular (one patty)</span>	
$\$132.50$ <span style="float: right;"><math>\\$175</math></span>	
What do you already know from the problem?	What do you need to know to solve the problem?
<ul style="list-style-type: none"><li>• there's 100 beef patties</li><li>• costs <math>2.50</math></li></ul>	<ul style="list-style-type: none"><li>• How much does a regular cheeseburger cost. <math>25.10</math></li></ul> $\begin{array}{r} 25.10 \\ \times 100 \\ \hline \end{array}$ $\begin{array}{r} 25.10 \\ \times 100 \\ \hline 2510 \end{array}$
What is your conclusion?	
<p>To get the answer, I first figured out what the price of a regular &amp; double-double cheeseburgers are. From there I subtracted the price of the produce &amp; buns, then multiplied by 100. That gave me the answer, which I once again had to add the price of the buns &amp; produce.</p> $22.8 + 0.1 - xOP_0 = 6$ $128.0 + xOP_0 = 6$	



What is your conclusion?

The only difference between a double double and a cheeseburger is one patty and one slice of cheese. So you subtract the prices of the two to find the price of only one patty & cheese. You then use that number (.90) & subtract it from the cost of one whole cheeseburger to find the price of all the extra stuff. Multiply by 100



What is your conclusion?

A 100x100 at In-n-Out cost \$90.85. To solve that, you start by subtracting the price of a cheeseburger from a double double. The answer (.90) is the price of a patty and cheese slice. You multiply (.90) by one less patty than what you want. (x-1), and you add the price of a cheeseburger (1.75). You end up with the eq.  $[y = .90(x-1) + 1.75]$ . For the 100x100, you plug in 100 to the (x) and you end up with \$90.85.

$$\left[ \begin{array}{l} y = .90(100-1) + 1.75 \\ y = 89.10 + 1.75 \\ y = 90.85 \end{array} \right]$$





## What is your conclusion?

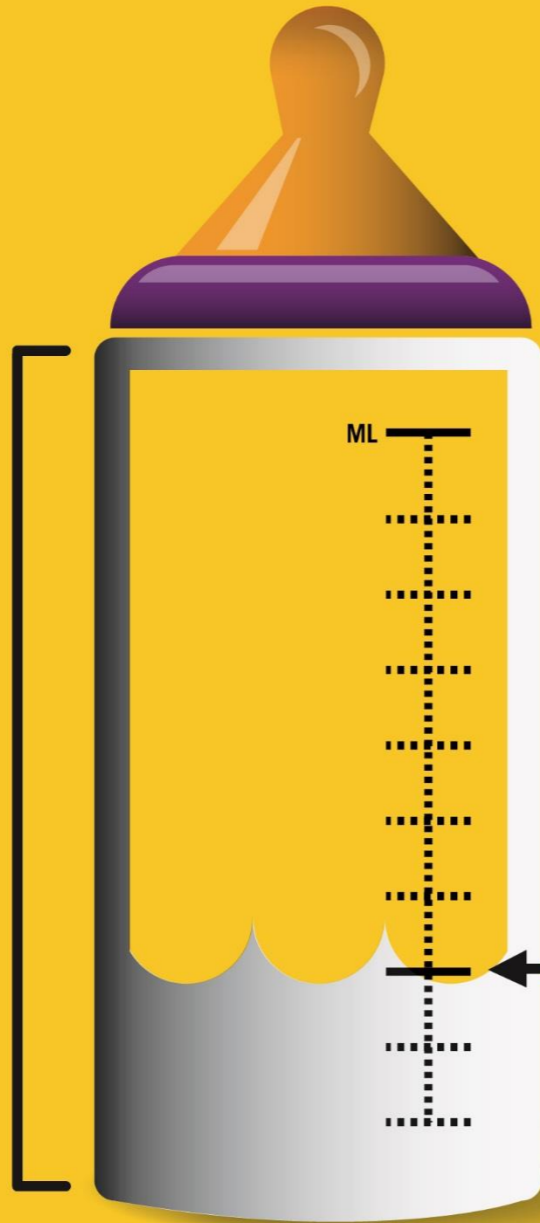
Figure the price difference from the Double-Double with a cheeseburger.  
Then find out the price for the produce and cheese-beef.

get total into \$90.85

CHOOSE CAR SEAT:  
BY AGE & SIZE



THE NUMBER  
OF PEOPLE  
**WHO  
THINK**  
THEY HAVE  
THEIR CHILD IN  
THE RIGHT  
SEAT.



THE ONES  
**WHO  
ACTUALLY  
DO.**

**KNOW FOR SURE**  
IF YOUR CHILD IS IN THE RIGHT CAR SEAT.



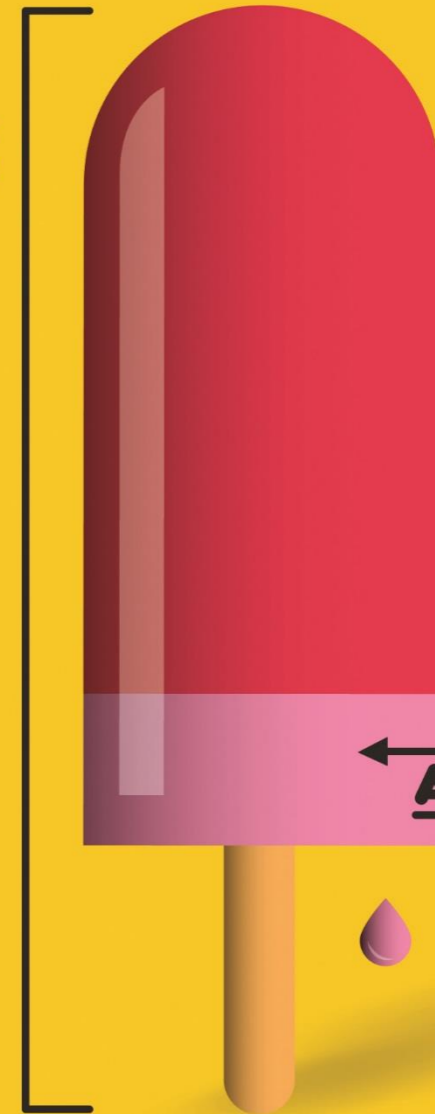
VISIT [SAFERCAR.GOV/THERIGHTSEAT](http://SAFERCAR.GOV/THERIGHTSEAT)



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There are 125  
sheep and 5 dogs  
in a flock. How old  
is the shepherd?





# 32 STUDENTS

- 75% of them gave me numerical responses
- 2 students calculated the answer to be 130 ( $125 + 5$ )
- 2 students calculated the answer to be 120 ( $125 - 5$ )
- 12 students calculated the answer to be 25 ( $125 \div 5$ )
- 0 students calculated the answer to be 625 ( $125 \times 5$ )
- 4 students stated that they guessed their answer (90, 5, 42, and 50)
- 4 students tried to divide 125 by 5 but could not correctly implement the procedure

# TAKEAWAYS

- Making sense of mathematics
- Intellectual autonomy
  - Intellectual autonomy is about being able to think for yourself and not being dependent on others for the direction and control of one's thinking.



# What Does the NHTSA Say?

## Key Statistics and Consumer Insights:

- Motor vehicle crashes are the leading cause of death for children age 1 through 12 years old.<sup>1</sup>

According to a NHTSA study, 3 out of 4 kids are not as secure in the car as they should be because their car seats are not being used correctly.

be reduced by about half if the correct child safety seats were always used.

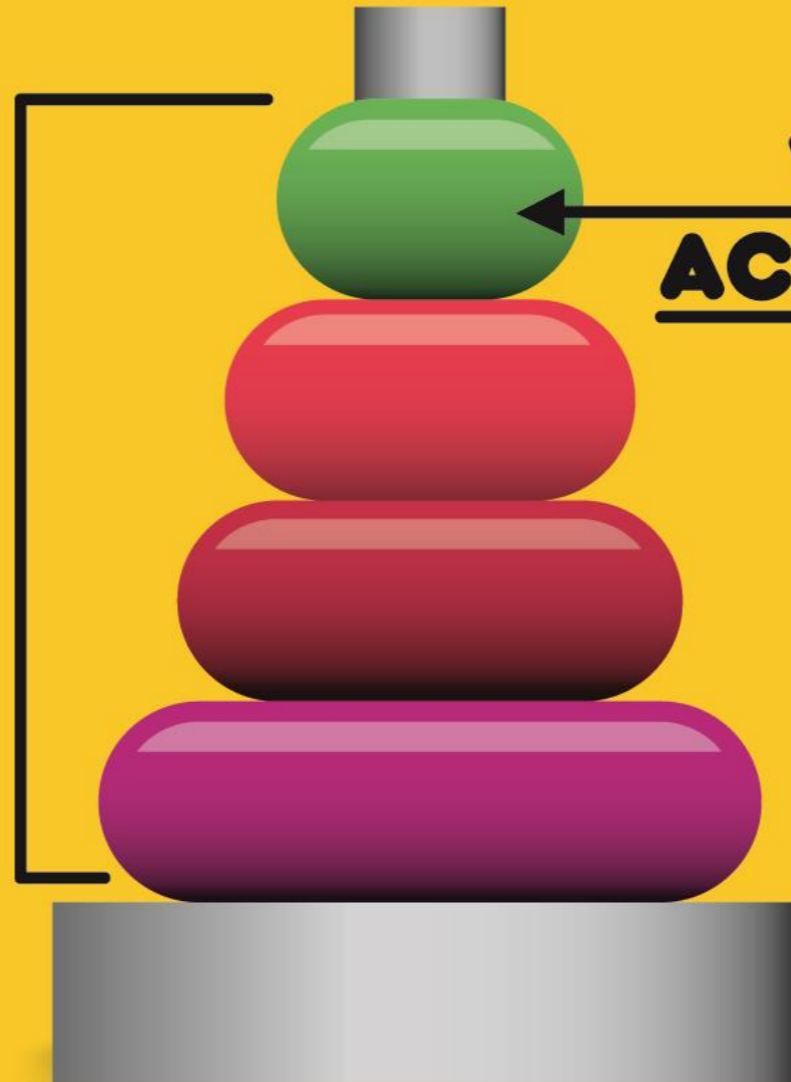
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<sup>1</sup> Source: Based on the latest mortality data currently available from the CDC's National Center for Health Statistics.



CHOOSE CAR SEAT:  
BY AGE & SIZE

THE NUMBER  
OF PEOPLE  
**WHO  
THINK**  
THEY HAVE  
THEIR CHILD  
IN THE RIGHT  
SEAT.



THE ONES  
**WHO  
ACTUALLY  
DO.**

- “because they have their child in the right seat”
- “because their car seats are not being used correctly”

**IF YOUR CHILD IS IN THE RIGHT CAR SEAT.**



VISIT [SAFERCAR.GOV/THERIGHTSEAT](http://SAFERCAR.GOV/THERIGHTSEAT)





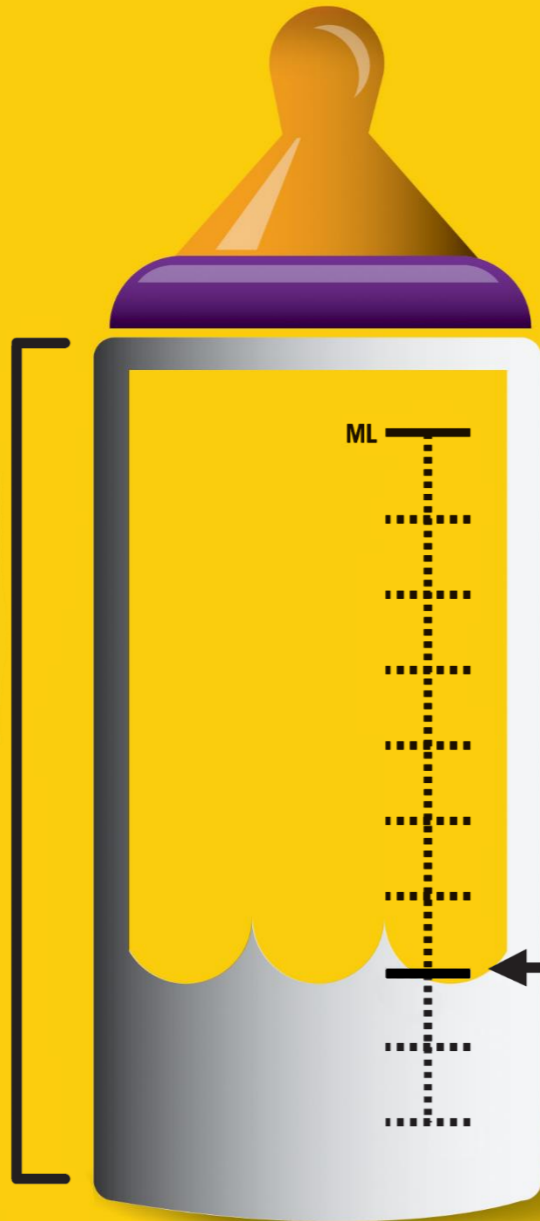
CHOOSE CAR SEAT:  
BY AGE & SIZE



THE NUMBER  
OF PEOPLE

**WHO  
THINK**

THEIR CAR  
SEATS ARE  
BEING USED  
CORRECTLY.



THE ONES  
**WHO  
ACTUALLY  
DO.**

**KNOW FOR SURE**  
IF YOUR CHILD IS IN THE RIGHT CAR SEAT.



VISIT [SAFERCAR.GOV/THERIGHTSEAT](http://SAFERCAR.GOV/THERIGHTSEAT)



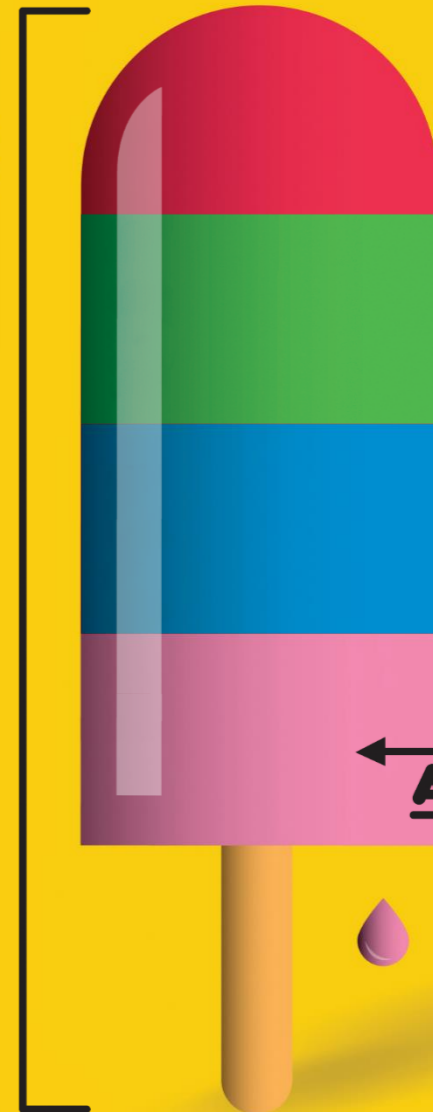
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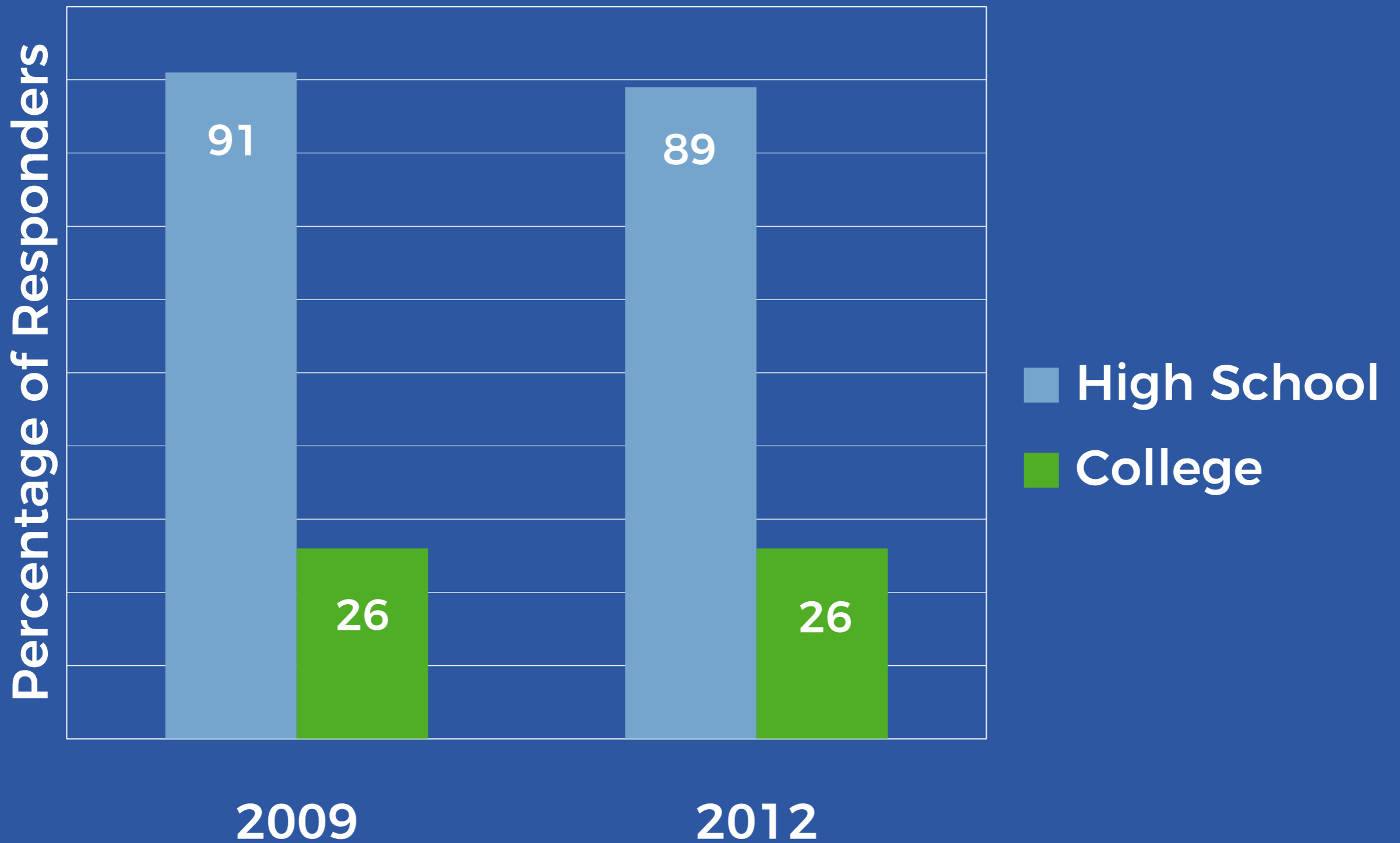
VISIT [SAFERCAR.GOV/THERIGHTSEAT](http://SAFERCAR.GOV/THERIGHTSEAT)



# PURPOSE OF K-12 ED?

- College readiness
  - ACT National Curriculum Survey
  - Surveyed 9,937 educators
  - What percent of students are “very well” or “well” prepared for college?





Source: [act.org/research/policymakers/pdf/NCS-PolicySummary2012.pdf](http://act.org/research/policymakers/pdf/NCS-PolicySummary2012.pdf)

# PURPOSE OF K-12 ED?

- College readiness
- Career readiness
  - Association of American Colleges and Universities survey
  - Surveyed over 300 employers with at least 25 employees and many new hires





■ More ■ Less ■ Same

Critical thinking and analytical reasoning skills

Analyzing and solving complex problems

Communicating effectively orally and in writing

Applying knowledge and skills to real-world setting

Working w/ numbers and understanding statistics

Source: [aacu.org/leap/documents/2013\\_EmployerSurvey.pdf](http://aacu.org/leap/documents/2013_EmployerSurvey.pdf)



















# SINKHOLE DIMENSIONS

- Slate Magazine
  - “A sinkhole, 65 feet across and 100 feet deep”





# How To Fix a Giant Sinkhole

The cement method vs. the graded-filter technique.



18

0

By Brian Palmer



A sinkhole in Guatamala

It's not clear whether cement is the best option, however. A 6,500-cubic-foot wad of concrete may serve to concentrate water runoff in other areas, leading to more sinkholes. Many engineers prefer the **graded-filter technique**, in which the hole is filled with a layer of boulders, then a layer of smaller rocks, and, finally, a layer of gravel. This fills the hole, more or less, while permitting water to drain through the area.



Reply

Reply All

Forward



## 2010 Guatemalan Sinkhole

Kaplinsky, Robert

To:



Wednesday, February 06, 2013 1:39 PM

Hi Brian,

I am using your "How to Fix a Giant Sinkhole" article for a math lesson on volume of a cylinder. I have one question for you. You mentioned.

"It's not clear whether cement is the best option, however. A 6,500-cubic-foot wad of concrete may serve to concentrate water runoff in other areas, leading to more sinkholes."

Can you please tell me where you got 6500 cubic feet from? Did you do  $65 \times 100$ ? We get something closer to 342,000 cubic feet.

Thanks,  
Robert

[Reply](#)

[Reply All](#)

[Forward](#)



# Re: 2010 Guatemalan Sinkhole

Brian Palmer

**To:** [Kaplinsky, Robert](#)

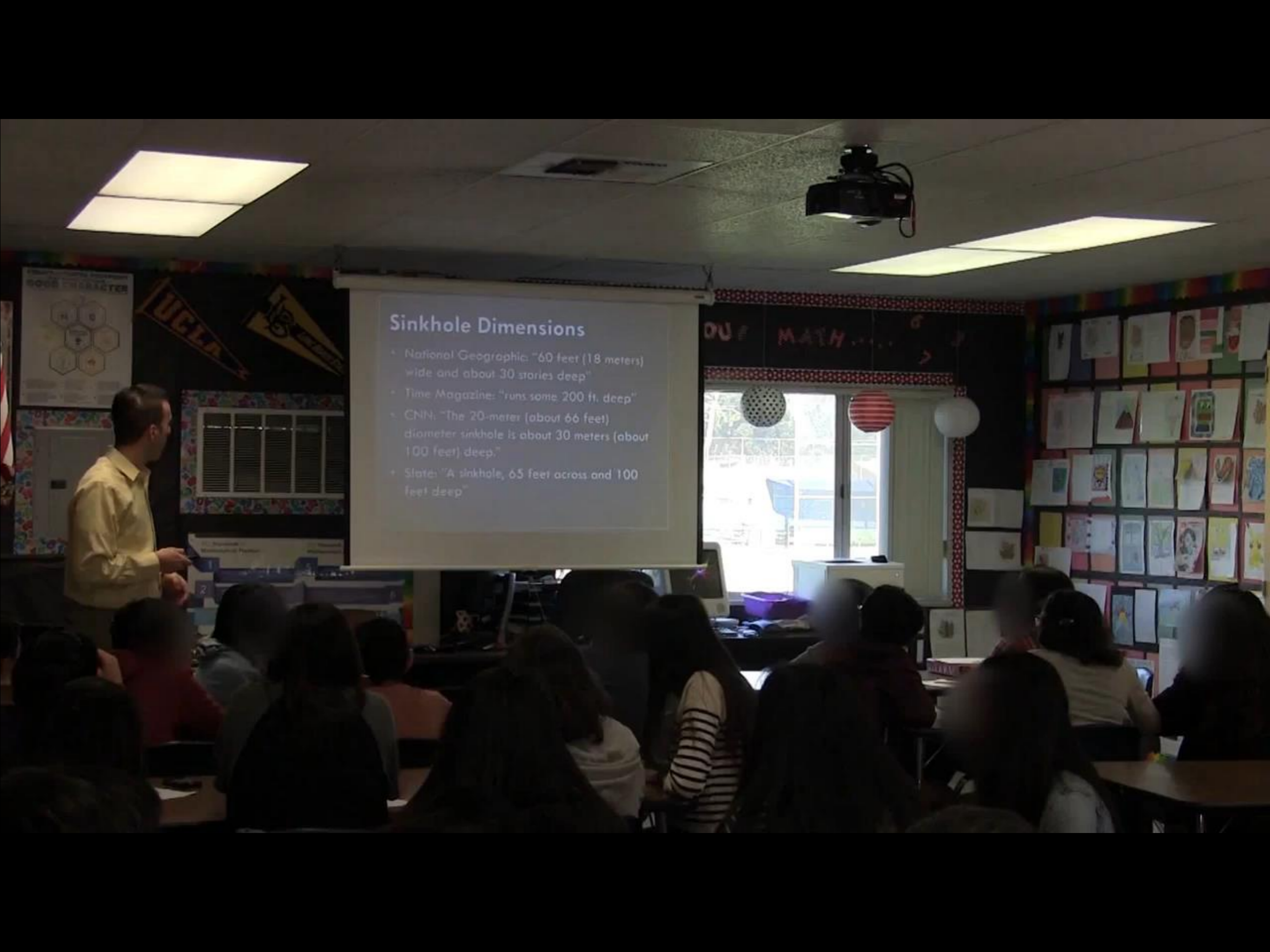
Wednesday, February 06, 2013 2:01 PM

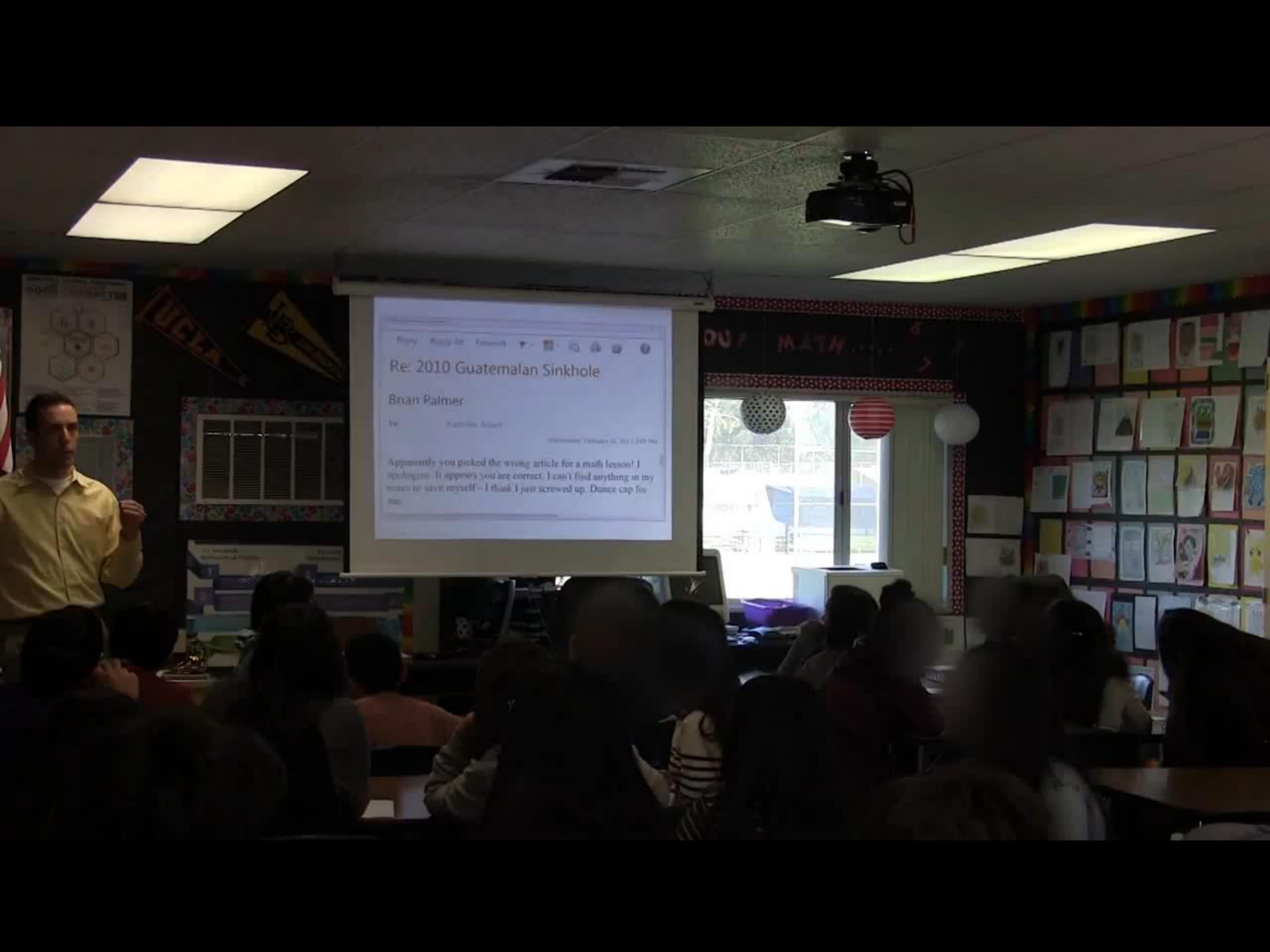
Apparently you picked the wrong article for a math lesson! I apologize. It appears you are correct. I can't find anything in my notes to save myself-- I think I just screwed up. Dunce cap for me.



## Sinkhole Dimensions

- National Geographic: "60 feet (18 meters) wide and about 30 stories deep"
- Time Magazine: "runs some 200 ft. deep"
- CNN: "The 20-meter (about 66 feet) diameter sinkhole is about 30 meters (about 100 feet) deep."
- Slate: "A sinkhole, 65 feet across and 100 feet deep"





Reply Reply All Forward

### Re: 2010 Guatemalan Sinkhole

Brian Palmer

To: [Kaprielian, Robert](#)

Wednesday, February 10, 2011 1:09 PM

Apparently you picked the wrong article for a math lesson! I apologize. It appears you are correct. I can't find anything in my notes to save myself—I think I just screwed up. Dunc cap for me.



# STUDENT REFLECTIONS

- “I didn’t say his answer was wrong since he is supposed to know more than an average 8th grader.”
- “Even though Brian was wrong, no one corrected him, because of fear of being wrong and lack of confidence in ourselves.”

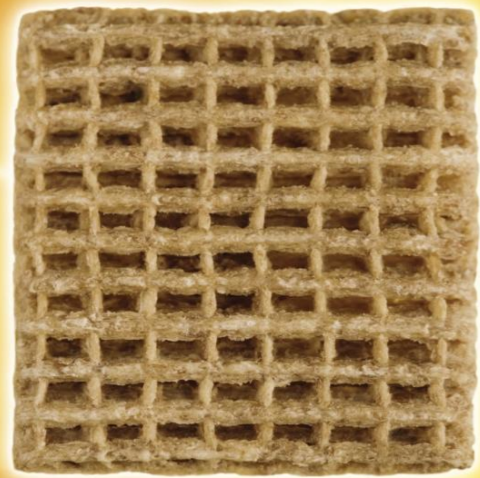
# STUDENT REFLECTIONS

- “I didn’t say anything when we were shown the ‘right’ answer because I thought that it must be right because he’s the author, but I knew in my mind he was actually wrong.”
- “I think that I should be the one who argues for my opinion, not just listening to others and accepting that my answer is wrong all the time.”









**OLD**  
**(Boring)**

**NEW**  
**Diamond**  
**Shreddies**

Cereal



**NEW**  
**(Exciting!)**







**SQUARE OR DIAMOND?**  
Vote for your Favourite at [DiamondShreddies.com](http://DiamondShreddies.com)

**Post**

# Diamond Shreddies Combo Pack



Square Diamond

**Made with 100% Whole Grain Wheat**

**620 g** Cereal  
SERVING SUGGESTION

COR 114

Limited Edition

**Sensible Solution**

- Very High Source of Fibre
- Good Source of 8 Essential Nutrients
- Low in Fat



**“Kraft Foods saw an immediate 18% increase in baseline sales of Shreddies within the first month alone, and for months thereafter.”**

Source: <http://www.visualtargeting.com/diamondshreddies.html>



# Complicated or Complex?





# Cookie Monster Cupcakes





**Nailed it**



## method

---

1. Using an electric mixer, whip the butter until it is pale. This will take at least 5 minutes on high.
2. Gradually add in the icing mixture and vanilla until well combined.
3. With the mixer running, add in food colouring until you get to the Cookie Monster colour. This may be a lot if you are using liquid food colouring or a little if using gel food colouring.
4. Add in the milk and mix until the frosting puffs up.
5. Fill a piping bag with a fluted nozzle and pipe on icing.
6. With the writing icing, place black spots on the marshmallows for pupils.
7. Place on each cupcake.
8. Cut cookies in half and place in 'mouth'.

# CUBES

A problem solving strategy

**C** - Circle the #s

**U** - underline the ques.

**B** - box the words



## Content Objective Example

- **SWBAT** apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. (MP4)

## Language Objective Example

- **SWBAT** explain correspondences between equations, verbal descriptions, tables, and graphs. (MP1)

- In early grades, this might be as simple as writing an addition equation to describe a situation. (MP4)
- In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. (MP4)
- By high school, a student might use geometry to solve a design problem or use a function to describe how one

# WHAT'S IT LOOK LIKE...

- when students have procedural skill but not conceptual understanding or the ability to apply mathematics?
- when students can work with numbers but cannot:
  - critically think
  - applying knowledge and skills to real-world settings
  - analyze and solve complex problems




How far apart are the exits on this freeway: Jct 90 and Jefferson Blvd?





Dixie Hwy	1
2nd St	1/2 mi
Junction Blvd	1/2 mi




$$\begin{array}{cccc} & 1 & & 1 \\ 1 & - & - & 1 - \\ & 2 & & 4 \end{array}$$

$$1\frac{1}{2} - 1\frac{1}{4}$$

Handwritten notes on a small piece of paper:

$1\frac{1}{2} - 1\frac{1}{4}$

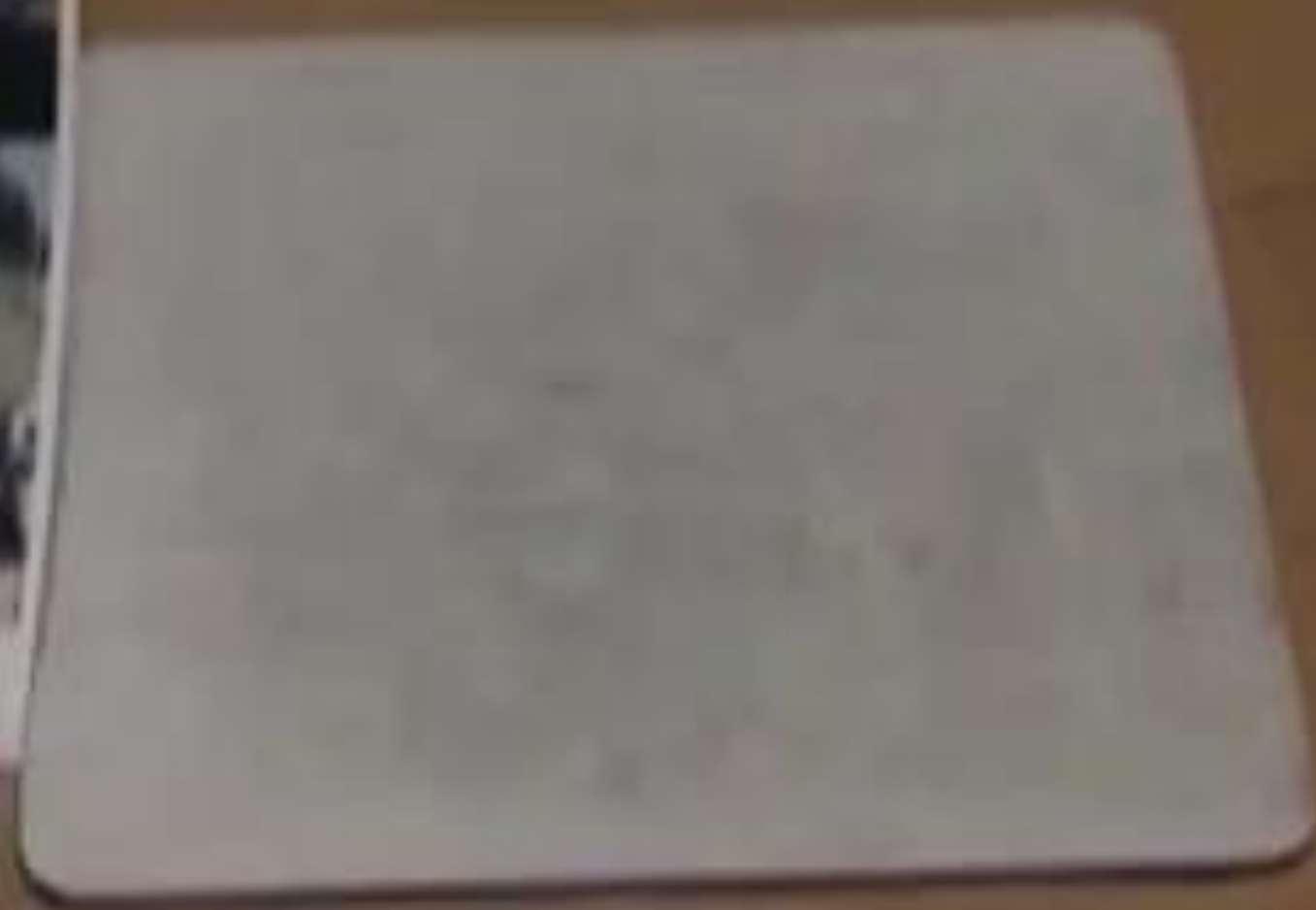
$\frac{3}{2} - \frac{5}{4}$

$\frac{6}{4} - \frac{5}{4}$

$\frac{1}{4}$





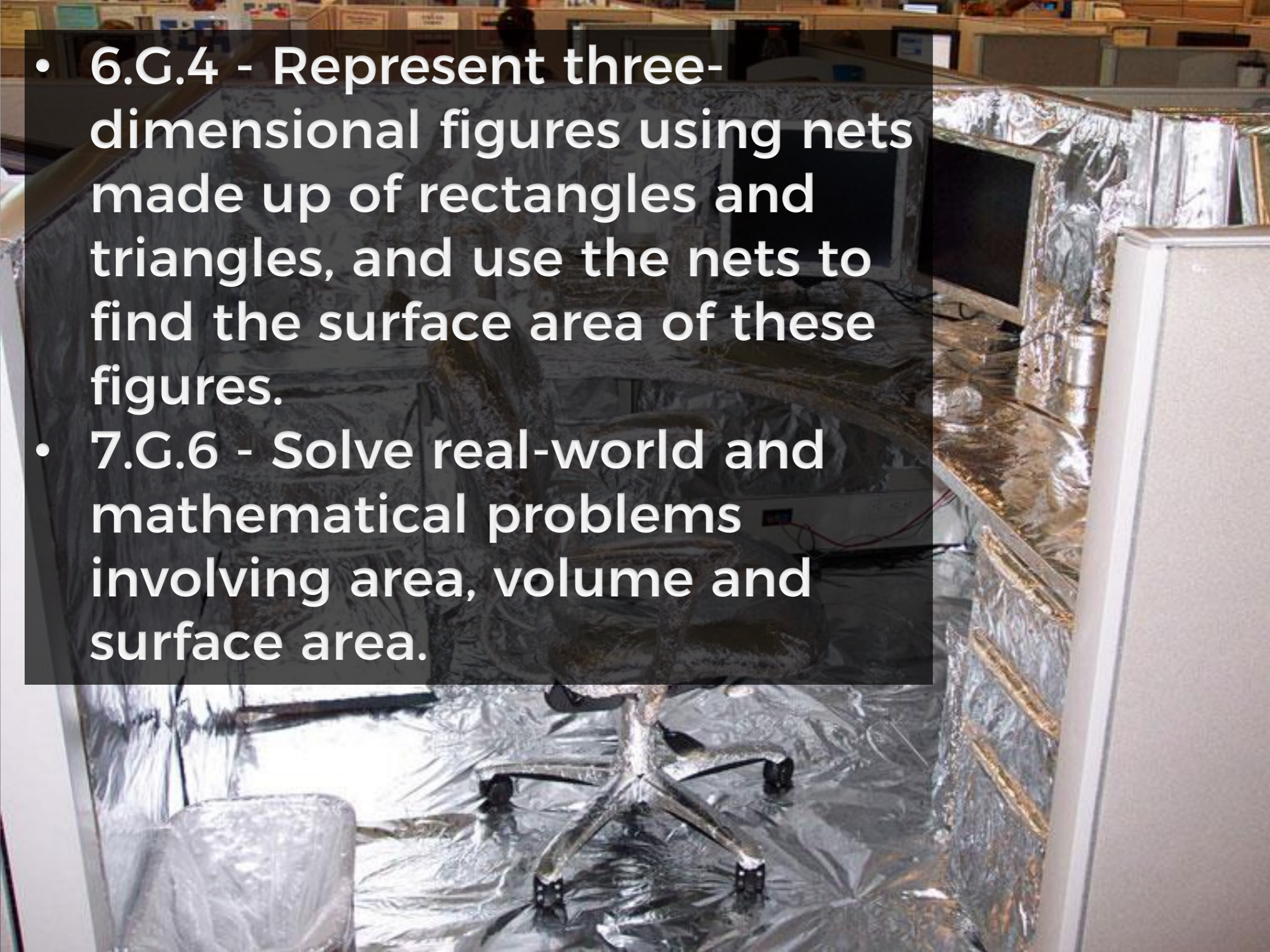


# THE FOUR C's

- Communication
- Curiosity

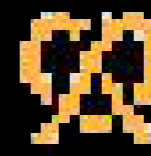


- **6.G.4 - Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures.**
- **7.G.6 - Solve real-world and mathematical problems involving area, volume and surface area.**



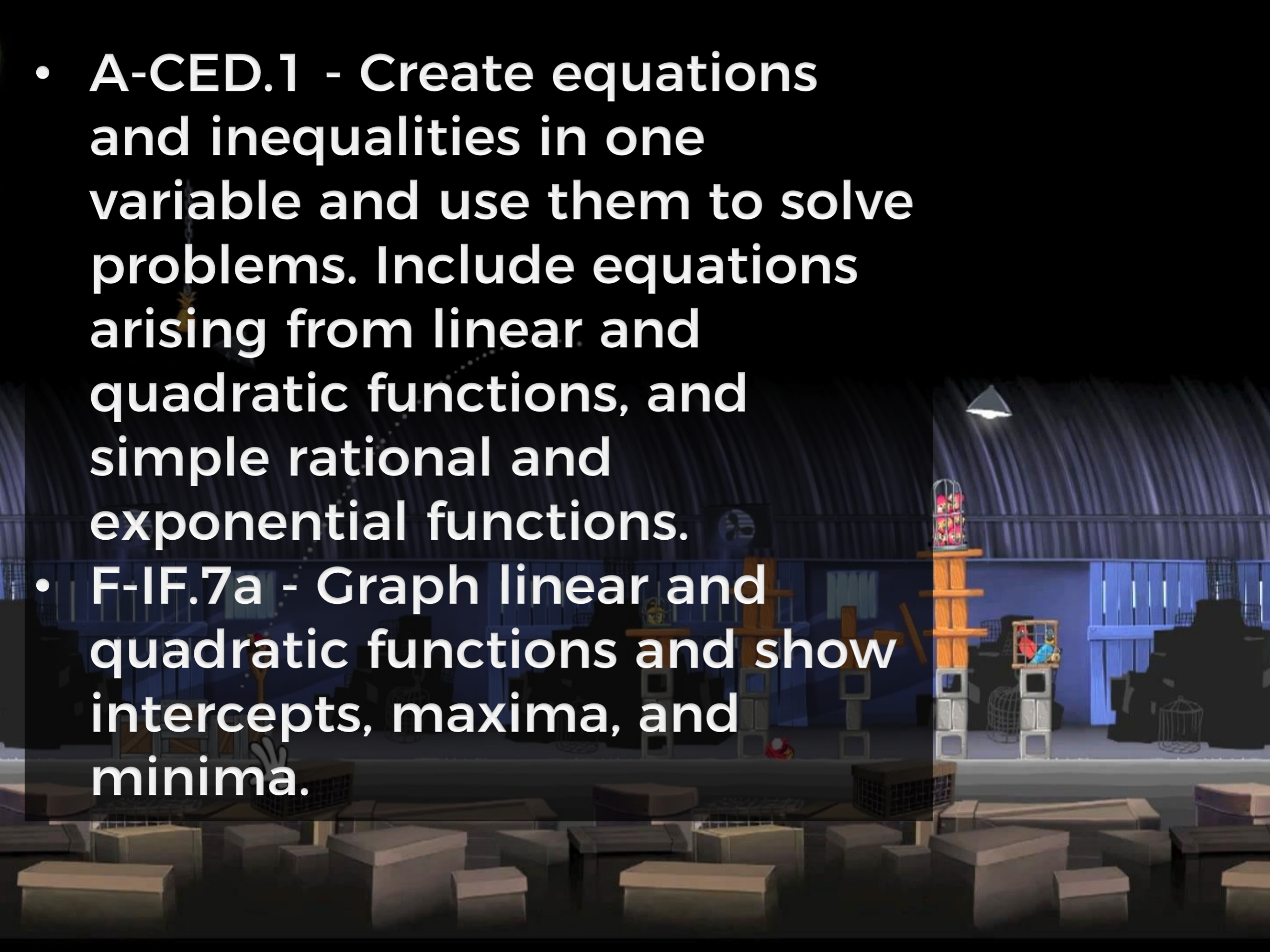


- 8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- G-CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure.





- **A-CED.1 - Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**
- **F-IF.7a - Graph linear and quadratic functions and show intercepts, maxima, and minima.**





- 6.RP.2 - Understand the concept of a unit rate





# THE FOUR C's

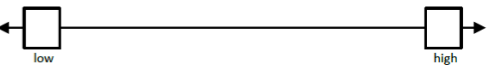
- Communication
- Curiosity
- Critical Thinking

# PROBLEM

# SOLVING

# FRAMEWORK

Inspired by Geoff Krall's  
resources at  
[emergentmath.com](http://emergentmath.com)

What problem are you trying to figure out?	What estimates do you have?  Place your estimate on the number line.
What info do you already know about the problem?	What info do you need about the problem?
What is your conclusion? How did you reach that conclusion?	



# THE FOUR C's

- Communication
- Curiosity
- Critical Thinking
- Content Knowledge

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IMPROVE QUESTION ASKING

PRACTICE PREPARING FOR A LESSON

DEAL WITH UNCOMFORTABLE SITUATIONS



# QUESTIONING SCENARIOS

- The activity begins with teachers in groups of three taking the roles of teacher, student, or observer.
- The individuals playing the role of teacher and student each receive a slip of paper describing their scenario.
- The individual playing the role of observer waits to record all of the teacher's questions to the student.
- Once the activity begins, the teacher will talk to the student in the context of the scenario they read about on the slips of paper.

What did you get for the area of a square with a side length of 4 units?

16

Great. Do you have any questions?

No



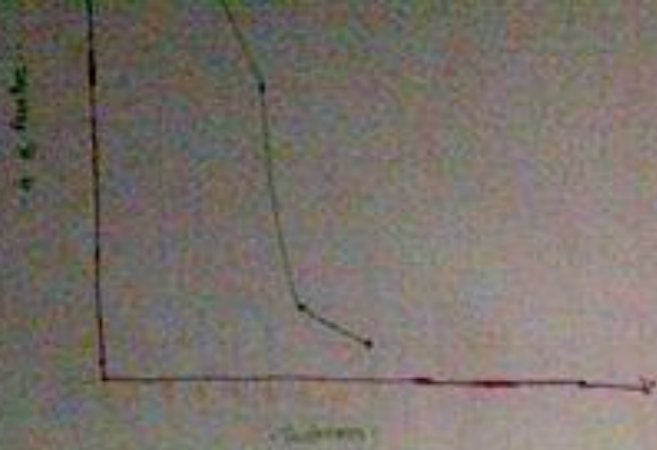
What did you get for the area of a square with a side length of 4 units?

16

Great. How did you get your answer?

Each side is 4 so I added 4 together 4 times and got 16.

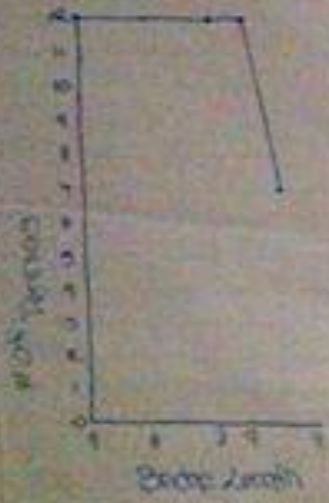




Table

Length	4	6	8	9	11
# of bases	12	16	12	16	17

Graph



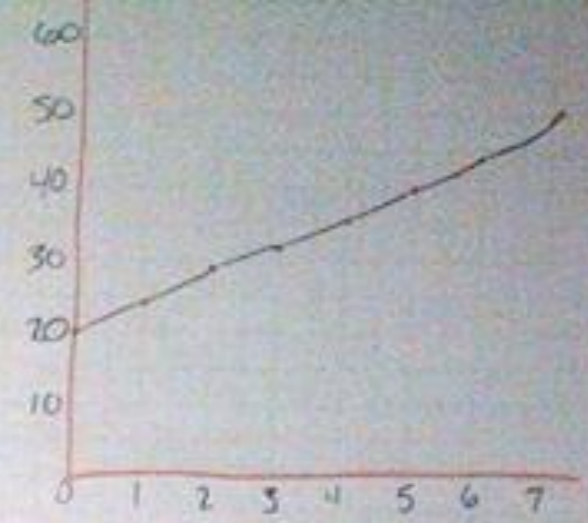
Concave Up  
Jada Dixon  
Diphenia Cahya

WHY?

1	24
2	28
3	32
4	36
5	40

How do you know?

$$y = 20 + 4x$$

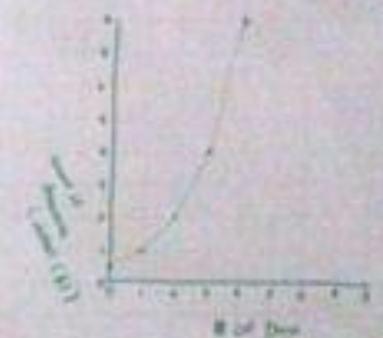


Johnnie King

Convince me.

# Exponential

Explain that please.



$B = \frac{1}{2}(2)^x$

Draw a picture.

By: Ashli, Anel

Day	0	1	2	3	4
Amount	1	2	4	8	16



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# COMMON CORE

## STATE STANDARDS INITIATIVE

CCSS.MATH.CONTENT.4.MD.A.3

Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

meet the

equal intensity, the

of each grade: conceptual

skills and fluency, and application.




What is the  
perimeter of a  
rectangle that  
measures 8 units  
by 4 units?





# COMPONENTS OF RIGOR

- Procedural Skill and Fluency
- Conceptual Understanding



List the  
dimensions of a  
rectangle with  
a perimeter  
of 24 units.





# COMPONENTS OF RIGOR

Procedural Skill and Fluency

Conceptual Understanding





Why?





# COMPONENTS OF RIGOR

- Procedural Skill and Fluency
- Conceptual Understanding

List the  
of a rectangle with a  
perimeter of 24  
units.





# COMPONENTS OF RIGOR

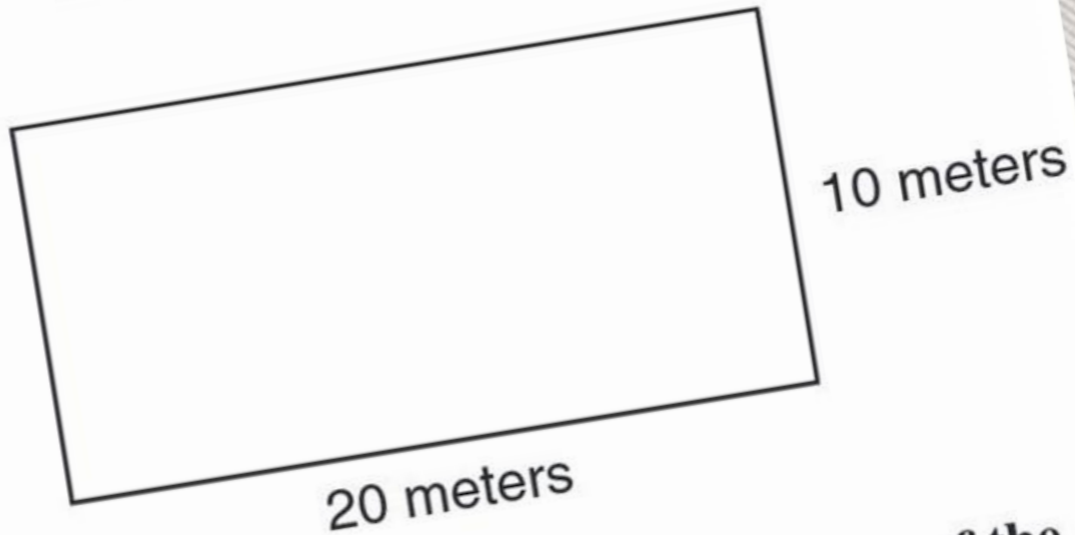
Procedural Skill and Fluency

Conceptual Understanding



71

A basketball court is shaped like a rectangle 20 meters long and 10 meters wide.



What is the perimeter in meters of the court?

- A 30 meters
- B 50 meters
- C 60 meters
- D 200 meters



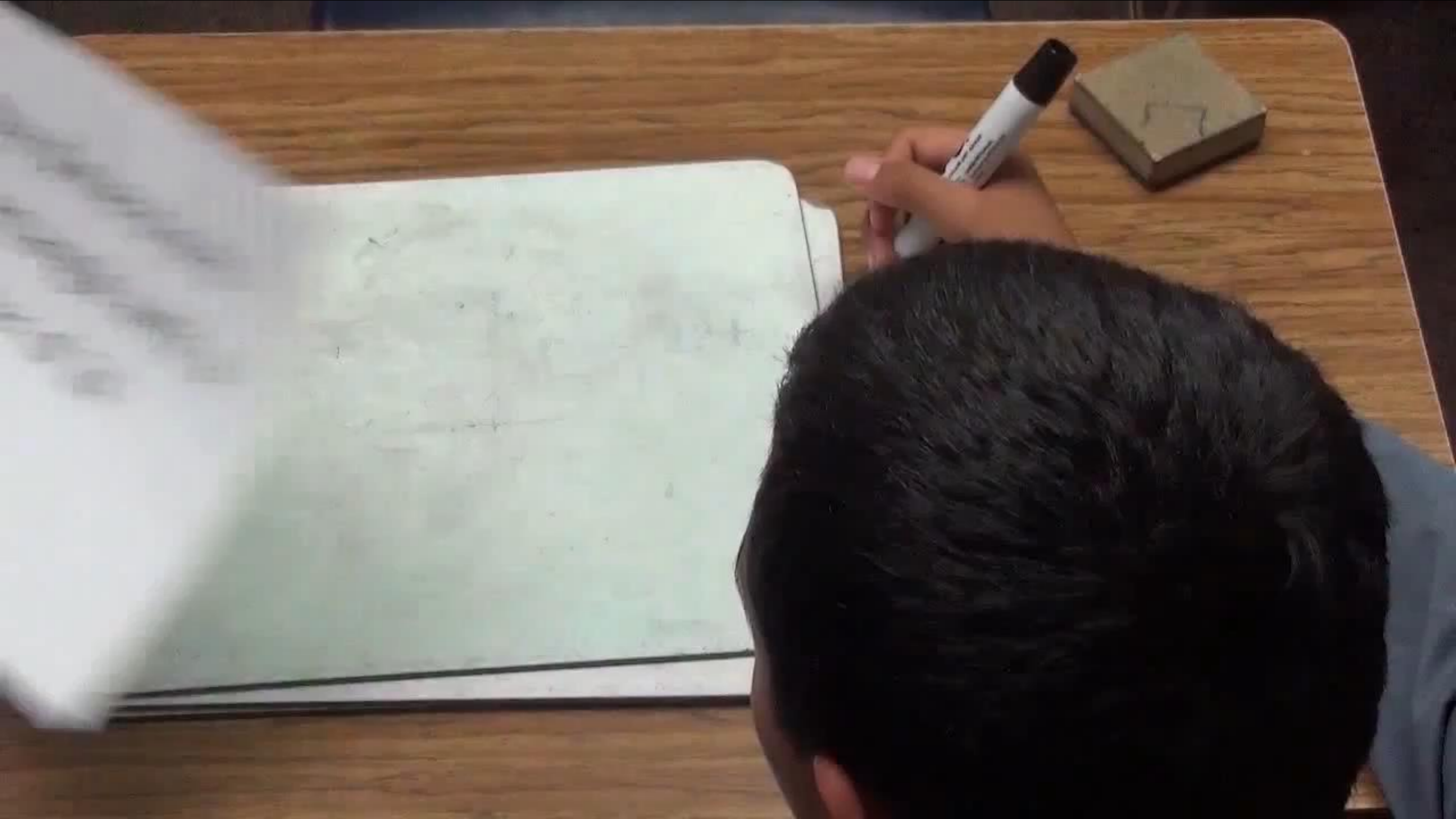
What is the perimeter  
of a rectangle ~~with~~  
that measures 8 units  
by 4 units?



# COMPONENTS OF RIGOR

- Procedural Skill and Fluency
- Conceptual Understanding






# COMPONENTS OF RIGOR

- Procedural Skill and Fluency
- Conceptual Understanding





Of all the  
rectangles with a  
perimeter of 24  
units, which one  
has the most area?

Of all the rectangles  
with a perimeter of  
24 units, which one  
has the most area?

00:00:00:00



# COMPONENTS OF RIGOR

Procedural Skill and Fluency

Conceptual Understanding

# DEFINING THE PROBLEM


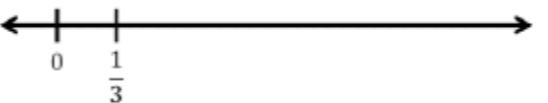
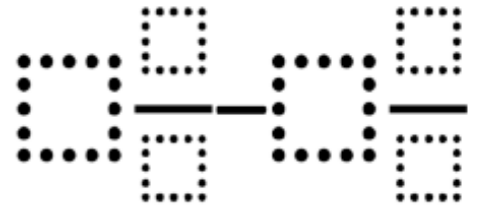
- Students appear to demonstrate “deep, authentic command of mathematical concepts” when given commonly used problems.
- However with more challenging problems, the same students seem to no longer demonstrate that command.



# ADDRESSING THE PROBLEM

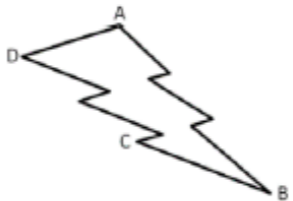
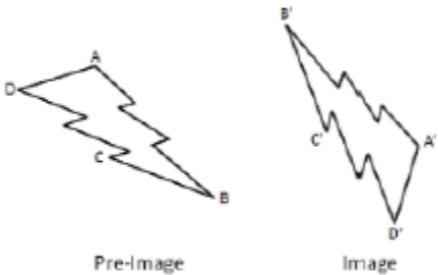
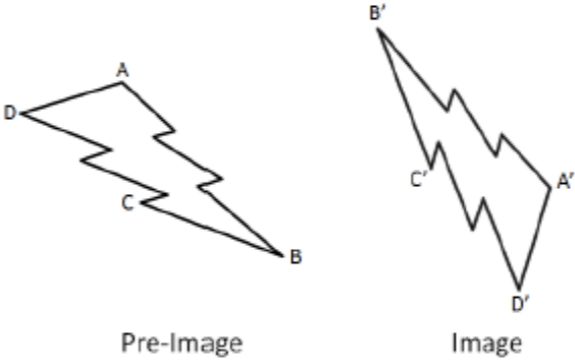
- First, we must have a clear understanding about why these problems are different from one another.
- Next, we need to practice using these problems so that we understand how students may react to them.
- Last, we need a source that can provide us with a variety of free problems.

# Depth of Knowledge Matrix - Elementary & Secondary Math

Topic	Adding Whole Numbers	Money	Fractions on a Number Line	Area and Perimeter	Subtracting Mixed Numbers
CCSS Standard(s)	<ul style="list-style-type: none"> <li>1.NBT.4</li> <li>2.NBT.5</li> </ul>	<ul style="list-style-type: none"> <li>2.MD.8</li> </ul>	<ul style="list-style-type: none"> <li>3.NF.2</li> </ul>	<ul style="list-style-type: none"> <li>3.MD.8</li> <li>4.MD.3</li> </ul>	<ul style="list-style-type: none"> <li>5.NF.1</li> </ul>
DOK 1 Example	Find the sum.  $44 + 27 =$	If you have 2 dimes and 3 pennies, how many cents do you have?	Which point is located at $\frac{7}{12}$ below? 	Find the perimeter of a rectangle that measures 4 units by 8 units.	Find the difference.  $5\frac{1}{2} - 4\frac{2}{3} =$
DOK 2 Example	Fill in the boxes below using the whole numbers 1 through 9, no more than one time each, so that you make a true equation.  $\square\square + 53 = \square\square$	Make 47¢ in three different ways with either quarters, dimes, nickels, or pennies.	Label the point where $\frac{3}{4}$ belongs on the number line below. Be as precise as possible.  	List the measurements of three different rectangles that each has a perimeter of 20 units.	Create three different mixed numbers that will make the equation true by using the whole numbers 1 through 9, no more than one time each. You may reuse the same whole numbers for each of the three mixed numbers.  $5\frac{4}{5} - \square\frac{\square}{\square} = 3\frac{1}{20}$
DOK 3 Example	Make the largest sum by filling in the boxes below using the whole numbers 1 through 9, no more than one time each.  $\square\square + \square\square =$	Make 47¢ using exactly 6 coins with either quarters, dimes, nickels, or pennies.	Create 5 fractions using the whole numbers 0 through 9, exactly one time each as numerators and denominators, and place them all on a number line.	What is the greatest area you can make with a rectangle that has a perimeter of 24 units?	Make the smallest difference by filling in the boxes below using the whole numbers 1 through 9, no more than one time each.  



# Depth of Knowledge Matrix - Elementary & Secondary Math

Topic	Surface Area and Volume	Probability	Transformations	Factoring Quadratics	Quadratics in Vertex Form
CCSS Standard(s)	<ul style="list-style-type: none"> <li>6.G.4</li> <li>7.G.6</li> </ul>	<ul style="list-style-type: none"> <li>7.SP.5</li> <li>7.SP.7</li> </ul>	<ul style="list-style-type: none"> <li>8.G.1</li> <li>G-CO.5</li> </ul>	<ul style="list-style-type: none"> <li>A-SSE.3a</li> </ul>	<ul style="list-style-type: none"> <li>F-IF.7a</li> </ul>
DOK 1 Example	Find the surface area of a rectangular prism that measures 3 units by 4 units by 5 units.	What is the probability of rolling a sum of 5 using two 6-sided dice?	Rotate the image below 90° counterclockwise about point D and reflect it across a horizontal line. 	Find the factors: $2x^2 + 7x + 3$	Find the roots and maximum of the quadratic equation below. $y = -3(x - 4)^2 - 3$
DOK 2 Example	List the measurements of three different rectangular prisms that each have a surface area of 20 square units.	What value(s) have a 1/12 probability of being rolled as the sum of two 6-sided dice?	List three sequences of transformations that take pre-image ABCD to image A'B'C'D'. 	Find three different integers to put in the blank that will make the quadratic expression factorable. $x^2 + \_\_x + 4$	Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values.
DOK 3 Example	What is the greatest volume you can make with a rectangular prism that has a surface area of 20 square units?	Fill in the blanks to complete this sentence using the whole numbers 1 through 9, no more than one time each.  Rolling a sum of $\_\_$ on two $\_\_$ -sided dice is the same probability as rolling a sum of $\_\_$ on two $\_\_$ -sided dice.	What is the fewest number of transformations needed to take pre-image ABCD to image A'B'C'D'? 	Fill the blank by finding the largest and smallest integers that will make the quadratic expression factorable. $2x^2 + 3x + \_\_$	Create a quadratic equation with the largest maximum value using the whole numbers 1 through 9, no more than one time each. $y = -\square(x - \square)^2 + \square$



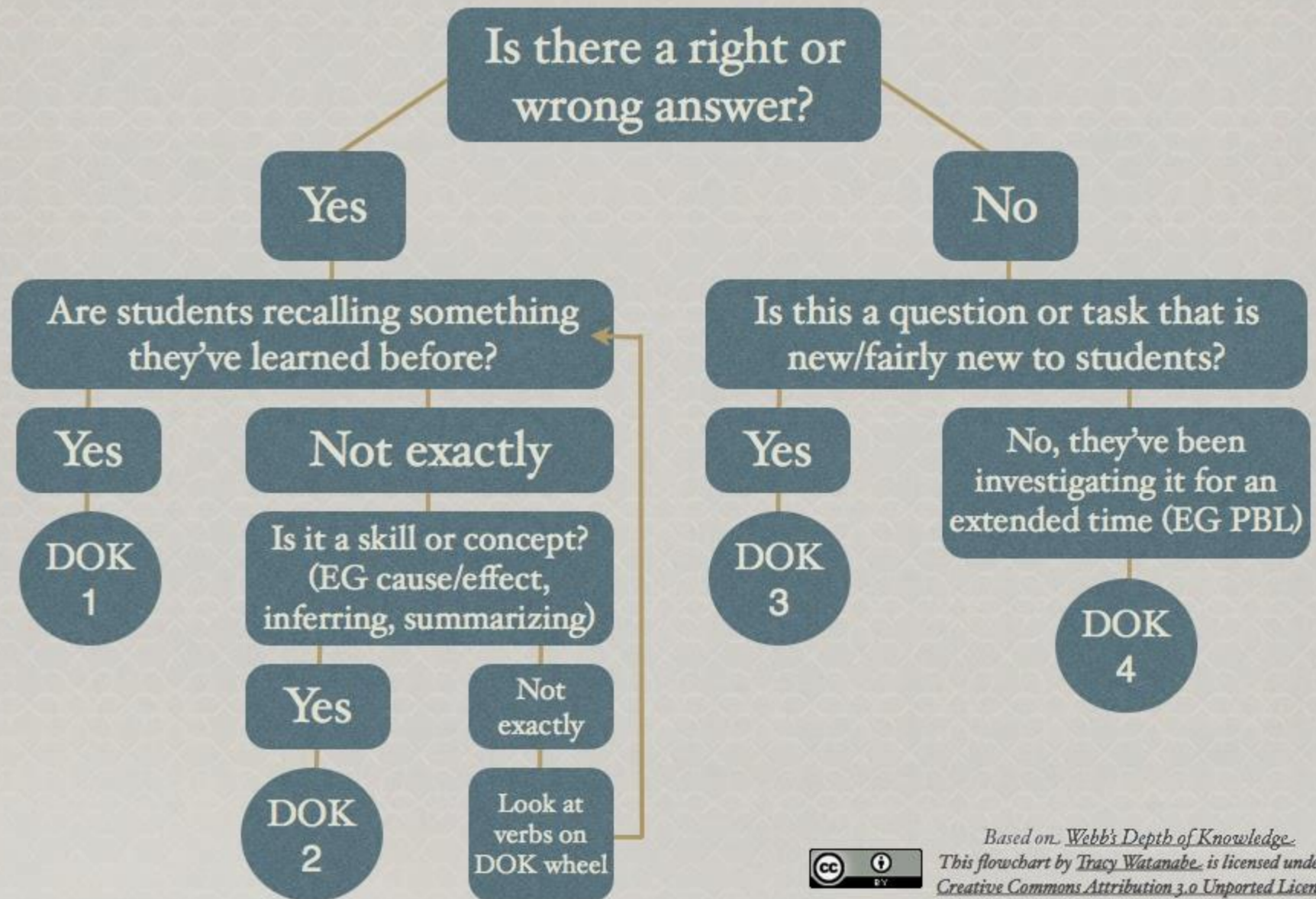
# Complex or Complicated?





# DOK FLOWCHART

## Depth of Knowledge (DOK) Flowchart for Questions



Source:

Tracy Watanabe

@tracywatanabe



Based on *Webb's Depth of Knowledge*.  
This flowchart by Tracy Watanabe is licensed under a  
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# DOK

# POSTERS

## DOK 1

### Routine Thinking

- Can you recall \_\_\_?
- Can you identify \_\_\_?
- How would you describe \_\_\_?
- What might you include on a list about \_\_\_?
- Can you select \_\_\_?
- How can you find the meaning of \_\_\_?

arrange    calculate    memorize  
measure    name    recognize  
recall    repeat    identify  
illustrate    match    label  
state    list    state

## DOK 2

### Conceptual Thinking

- Can you explain how \_\_\_ affected \_\_\_?
- How would you apply what you learned to develop \_\_\_?
- How would you summarize \_\_\_?
- What do you notice about \_\_\_?
- How would you estimate \_\_\_?
- How could you organize \_\_\_?

compare    classify    categorize  
measure    graph    distinguish  
predict    modify    construct  
organize    infer    summarize  
interpret    make observations

## DOK 3

### Strategic Reasoning

- How is \_\_\_ related to \_\_\_?
- What conclusions can be drawn?
- Can you elaborate on \_\_\_?
- How would you test \_\_\_?
- What evidence supports \_\_\_?
- What would happen if \_\_\_?
- Why is that the best answer?

assess    compare    construct  
apprise    revise    hypothesize  
critique    investigate  
draw conclusions  
develop a logical argument

## DOK 4

### Extended Reasoning

- Write a research paper.
- What information can you gather to support your idea about \_\_\_?
- Write a thesis, drawing conclusions from multiple sources.
- Apply information from one text to another to develop an persuasive argument.

design    connect    prove  
analyze    critique    synthesize  
create    apply concepts

Source: Penny Lund

[isntitelementary.blogspot.com/](http://isntitelementary.blogspot.com/)

Created by Penny Lund 2013

RobertKaplinsky.com

# DOK LEVEL DIFFERENCES

## Level 1: Recall & Reproduction

- Often a trivial application of facts.
- Generally requires little effort beyond remembering a formula.
- Usually only one answer.

## Level 2: Skills & Concepts

- Usually requires more than one step to solve.
- Often multiple answers.

## Level 3: Strategic Thinking

- Usually requires critical thinking about how to approach a problem.
- May be multiple answers or a single optimal answer.
- Often challenging enough to make your head hurt.

## Level 4: Extended Thinking

- These are generally represented by performance tasks or problem-based lessons.



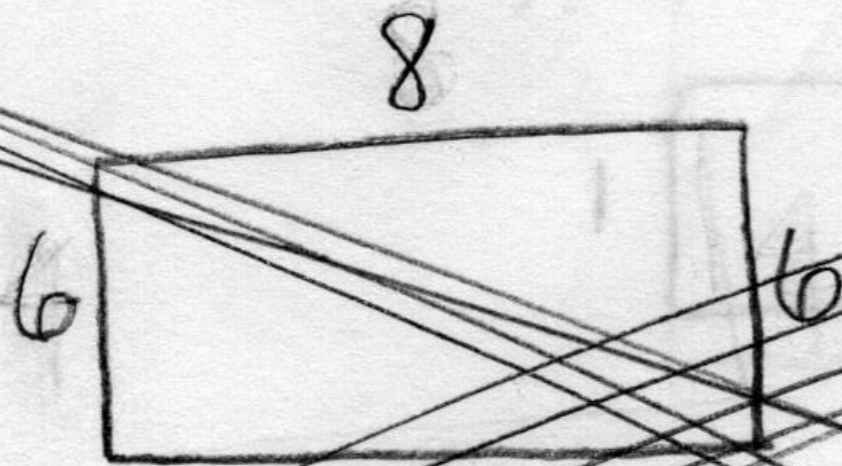
# ADDING DECIMALS

Use the numbers 1 through 9, exactly one time each, to fill in the boxes and make three decimals whose sum is as close to 1 as possible.

$$\begin{array}{r} 0.\square\square\square \\ 0.\square\square\square \\ + 0.\square\square\square \\ \hline \end{array}$$

First attempt:

Points: \_\_\_/2 attempt \_\_\_/2 explanation



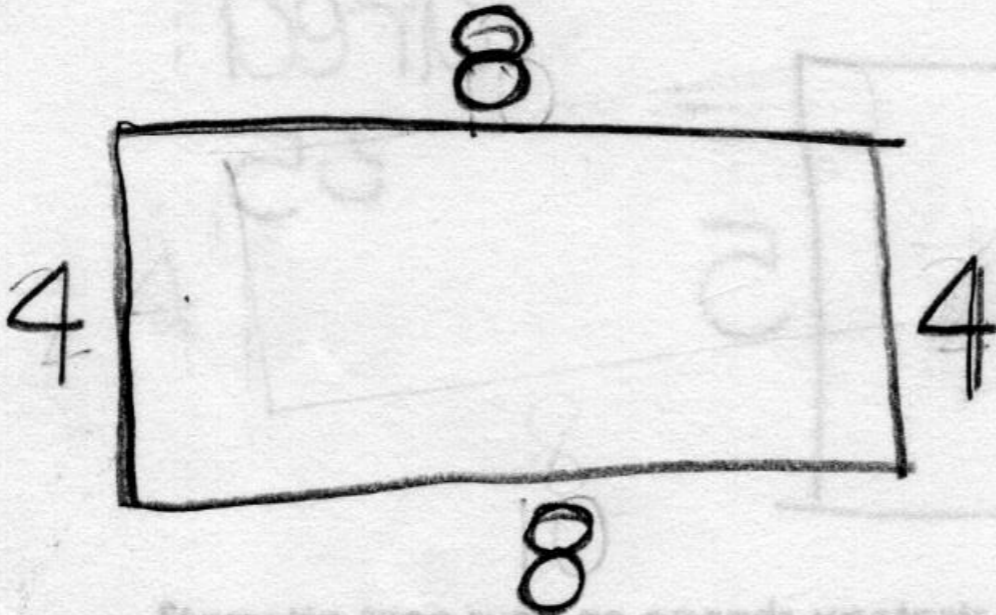
area:  
48

What did you learn from this attempt? How will your strategy change on your next attempt?

~~This attempt doesn't equal 24.~~

Second attempt:

Points: \_\_\_/2 attempt \_\_\_/2 explanation



area:  
32

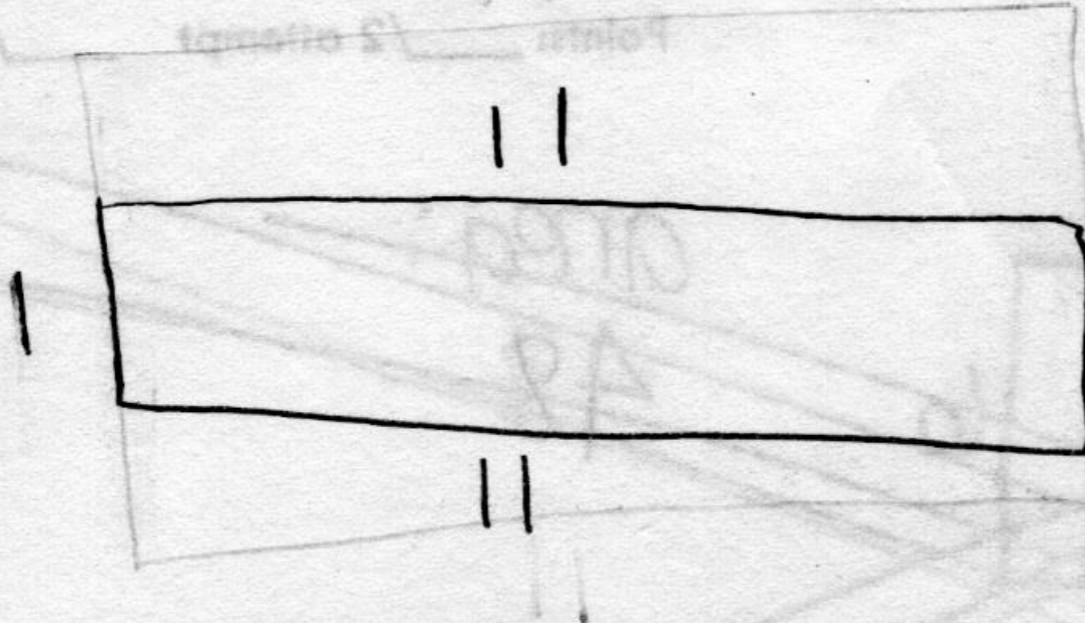
What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter was 24, and the area was 32  
but I think there's a blader #



Fourth attempt:

Points: \_\_\_/2 attempt \_\_\_/2 explanation

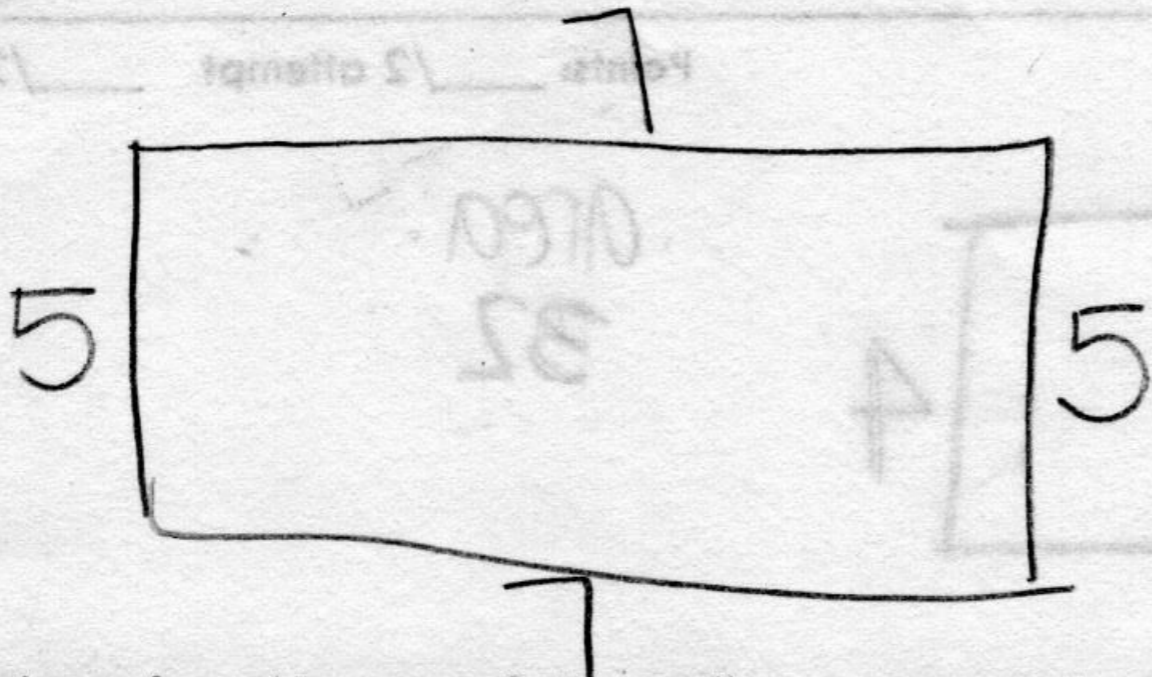


What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter is 24, but the area is 11 and attempt #2 the area is 32  
Strategy: Use #'s with more than one row.

Fifth attempt:

Points: \_\_\_/2 attempt \_\_\_/2 explanation



area:  
35

What did you learn from this attempt? How will your strategy change on your next attempt?

# DOK FAQ

- When will students ever use this?
- What DOK level should I start students with?
- How do teachers fit these problems in?
- How do I help prevent students from giving up after trying the problem once or twice?
- Where can I find other DOK 2 and DOK 3 problems or submit ones I've made?



# Open Middle

Challenging math problems worth solving

- Home
- Kinder ▾
- Grade 1 ▾
- Grade 2 ▾
- Grade 3 ▾
- Grade 4 ▾
- Grade 5 ▾
- Grade 6 ▾
- Grade 7 ▾
- Grade 8 ▾
- High School ▾
- About
- Submit



## THE TOP 10 MOST POPULAR PROBLEMS OF 2016

1. Two-Step Equations with One Variable by Daniel Luevanos, and Robert Kaplinsky
2. Order of Operations with answer from Michael Fenton and his students
3. Dot Card Counting by Dan Meyer
4. Rational and Irrational Numbers by Bryan Anderson
5. One Solution, No Solutions, Infinite Solutions by Bryan Anderson
6. Multiplying a Two-Digit Number by a Single-Digit Number by Robert Kaplinsky
7. Exponents and Order of Operations by Zack Miller
8. Converting Between Fractions and Decimals by Robert Kaplinsky
9. Interpreting Percentages by Robert Kaplinsky
10. Two-Step Equations 3 by Erick Lee

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**Brian Marks**

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- Kindergarten (6)
- Counting & Cardinality (2)
- Number & Operations in Base Ten (1)
- Operations & Algebraic Thinking (3)

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Home > High School: Geometry > Expressing Geometric Properties with Equations > Equidistant Points

## EQUIDISTANT POINTS

Directions: How many points with integer coordinates are 5 units away from  $(-2, 3)$ ?

Hint

Which methods are available to determine the answer to this problem? What shape is defined by *all* of the points that are 5 units away  $(-2, 3)$ ?

Answer

12 points:  $(-5, 7)$ ,  $(-7, 3)$ ,  $(-5, -1)$ ,  $(-2, -2)$ ,  $(3, 3)$ ,  $(1, -1)$ ,  $(-2, 8)$ ,  $(1, 7)$ ,  $(2, 6)$ ,  $(-6, -6)$ ,  $(-6, 0)$ , and  $(2, 0)$

Source: [Dylan Kane](#)

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-  Number & Operations in Base Ten (1)
-  Operations & Algebraic Thinking (3)



# GOALS

- ENGAGING PROBLEM SOLVING
- REAL WORLD PROBLEM-BASED LEARNING
- HIGHER DEPTH OF KNOWLEDGE PROBLEMS
- BETTER IMPLEMENTATION
- IMPROVE QUESTION ASKING
- PRACTICE PREPARING FOR A LESSON
- DEAL WITH UNCOMFORTABLE SITUATIONS

# PBL RESOURCES

- Problem-based lesson search engine:  
[robertkaplinsky.com/prbl-search-engine](http://robertkaplinsky.com/prbl-search-engine)
- My lessons (Elementary, Middle, and High School)  
[robertkaplinsky.com/lessons](http://robertkaplinsky.com/lessons)
- Dan Meyer (Middle and High School)  
[threeacts.mrmeyer.com](http://threeacts.mrmeyer.com)
- Andrew Stadel (Elementary and Middle School)  
[www.estimated180.com/lessons.html](http://www.estimated180.com/lessons.html)
- Graham Fletcher (Elementary and Middle School)  
[gfletchy.com/3-act-lessons](http://gfletchy.com/3-act-lessons)





## Home



How Big Is The World's Largest Deliverable Pizza?  
(Area of Rectangles)

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Type and hit enter ...



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### How I Can Help You



#### Real World Problems

My workshops help teachers implement problem-based lessons by helping them experience them from both student and teacher perspective, leading to increase students' success with performance tasks and the Common Core State Standards.



#### Depth of Knowledge

Problems at higher depth of knowledge levels have the potential to challenge your most talented student yet remain accessible to everyone. I can help teachers develop best practices for implementing them so that students persevere longer towards finding the solution.



## Lessons

- [View all](#)
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- [2nd](#)
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- [7th](#)
- [8th](#)
- [Alg 1](#)
- [Geo](#)
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### How Much Money Were Those Pennies?



### How Can We #SaveNelly?



### How Many Chip Bags Will There Be?



### How Can We Make Stronger Passwords?

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# Robert Kaplinsky's Problem-Based Lessons

File Edit View Insert Format Data Tools Add-ons Help All changes saved in Drive

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*fx*

	A	B	C	D	E
1	Lesson	Concept / Skill	Standard 1	Standard 2	Standard 3
2	<a href="#">How Much Money Were Those Pennies?</a>	Money, Multiplying Decimals, Proportions	4.MD.2	5.NBT.5	5.NF.5
3	<a href="#">How Can We #SaveNelly?</a>	Dividing Decimals	6.NS.3		
4	<a href="#">How Many Chip Bags Will There Be?</a>	Ratio and Proportions, Population Sampling	6.RP.3	6.RP.3c	7.SP.8
5	<a href="#">How Can We Make Stronger Passwords?</a>	Permutations, Combinations, Probability, Exponents, Exponential Growth	7.SP.8	8.EE.1	9.SP.8
6	<a href="#">How Many Hot Dogs And Buns Should He Buy?</a>	Least Common Multiple (LCM)	6.NS.4		
7	<a href="#">What Does 2000 Calories Look Like?</a>	Unit Rates, Ratios, Solving Equations, and Solving Inequalities	6.EE.3	6.EE.4	6.EE.5
8	<a href="#">How Much Money Are The Coins Worth?</a>	Decimal Operations and Coin Counting	2.MD.8	5.NBT.7	6.NS.3
9	<a href="#">How Many Times Will A Case of Paper Jam?</a>	Interpreting Percentages	6.RP.3c	7.RP.3	
10	<a href="#">How Many Soda Combinations Are There On A Coke Freestyle?</a>	Counting, Composing, and Decomposing Numbers	K.CC.5	K.CC.6	K.CC.7
11	<a href="#">What Should The Freeway Sign Show?</a>	Fractions on Number Lines, Converting Units, Decimal and Fraction Operations	3.NF.1	3.NF.2	3.NF.3
12	<a href="#">How Fast Was The Fastest Motorcycle Speeding Ticket Ever?</a>	Converting Units and Unit Rates	5.MD.1	6.RP.3d	7.SP.8
13	<a href="#">How Much Did Patrick Peterson Lose By Not Cashing His Check?</a>	Compound and/or Simple Interest	7.RP.3	N-RN.2	A-SSE.1
14	<a href="#">How Many Biscuits Can You Make?</a>	Dividing Fractions and Mixed Numbers	5.NF.7	5.NF.7a	5.NF.7b
15	<a href="#">How Much Bigger Should They Make Zoolander's School?</a>	Scale and Proportions	5.NF.5A	7.RP.2	7.SP.8
16	<a href="#">Where Is The Freeway Sign Located?</a>	Identifying Fractions on a Number Line	3.NF.1	3.NF.2	3.NF.3
17	<a href="#">How Far Apart Are Exits On A Ring Road?</a>	Arc length measures	G-C.5		
18	<a href="#">How Much Is One Third Of A Cup Of Butter?</a>	Identifying Fractions on a Number Line	3.NF.1	3.NF.2	3.NF.3
19	<a href="#">How Do Skytypers Write Messages?</a>	Transformations (Rotations, Reflections, Dilations, and Translations)	8.G.1	8.G.2	8.G.3
20	<a href="#">How Big Is The Bermuda Triangle?</a>	Coordinate Geometry: Area of Triangle	G-GPE.7		
21	<a href="#">What Fraction Of Children Are In The Right Car Seat?</a>	Representing and Comparing Fractions	3.NF.1	3.NF.2	3.NF.3
22	<a href="#">How Much Did The Temperature Drop?</a>	Absolute Value	6.NS.7c	7.NS.1c	
23	<a href="#">How Much Shorter Are Staggered Pipe Stacks?</a>	Circles, Pythagorean Theorem, trigonometric ratios, and linear functions	8.G.7	A-CED.1	A-SSE.1
24	<a href="#">How Do You Write A Check To Pay For Something?</a>	Expanded Form	2.NBT.3	4.NBT.2	5.NF.5
25	<a href="#">How Can We Correct The Scarecrow?</a>	Pythagorean Theorem	8.G.6	G-SRT.4	
26	<a href="#">How Much Does A 100x100 In-N-Out Cheeseburger Cost?</a>	Building and Interpreting Linear Functions	8.F.1	8.F.3	8.F.4
27	<a href="#">How Can We Water All Of The Grass?</a>	Circles, Pythagorean Theorem, trigonometric ratios	7.G.4	8.G.7	8.G.8
28	<a href="#">How Much Money IS That?!</a>	Volume of rectangular prism	5.MD.3	5.MD.4	5.MD.5
29	<a href="#">How Much Money Should Dr. Evil Demand?</a>	Exponential Growth	N-RN.2	A-SSE.1	A-SSE.2
30	<a href="#">How Tall Is Mini-Me?</a>	Scale and Dividing Decimals	5.NF.5	5.NF.5a	5.NF.5b
31	<a href="#">How Did They Make Ms. Pac-Man?</a>	Transformations (Rotations, Reflections, and Translations)	8.G.1	8.G.2	8.G.3
32	<a href="#">Which Ticket Option Is The Best Deal?</a>	Unit Rates and Ratios	6.RP.2	6.RP.3	6.RP.4
33	<a href="#">How Far Apart Are The Freeway Exits?</a>	Fractions on a Number Line and Subtracting Fractions	3.NF.2	3.NF.2b	4.NF.3
34	<a href="#">Do We Have Enough Paint?</a>	Area	3.MD.5	3.MD.6	3.MD.7

**SCHOOL DISTRICT OF THE**

**MENOMONIE AREA**

**ROBERT KAPLINSKY**

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