

DIGGING INTO DEPTH OF KNOWLEDGE

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COMMON CORE

STATE STANDARDS INITIATIVE

CCSS.MATH.CONTENT.4.MD.A.3

Apply the area and perimeter formulas for rectangles in real world and mathematical problems.


meet the same level of conceptual understanding and skills and fluency, and application.



What is the
perimeter of a
rectangle that
measures 8 units
by 4 units?

COMPONENTS OF RIGOR

- ☒ Procedural Skill and Fluency
- ☒ Conceptual Understanding



List the
dimensions of a
rectangle with
a perimeter
of 24 units.



COMPONENTS OF RIGOR

☒ Procedural Skill and
Fluency

☒ Conceptual
Understanding



why?



COMPONENTS OF RIGOR

- ☒ Procedural Skill and Fluency
- ☒ Conceptual Understanding

List the
of a rectangle with a
perimeter of 24
units.



COMPONENTS OF RIGOR

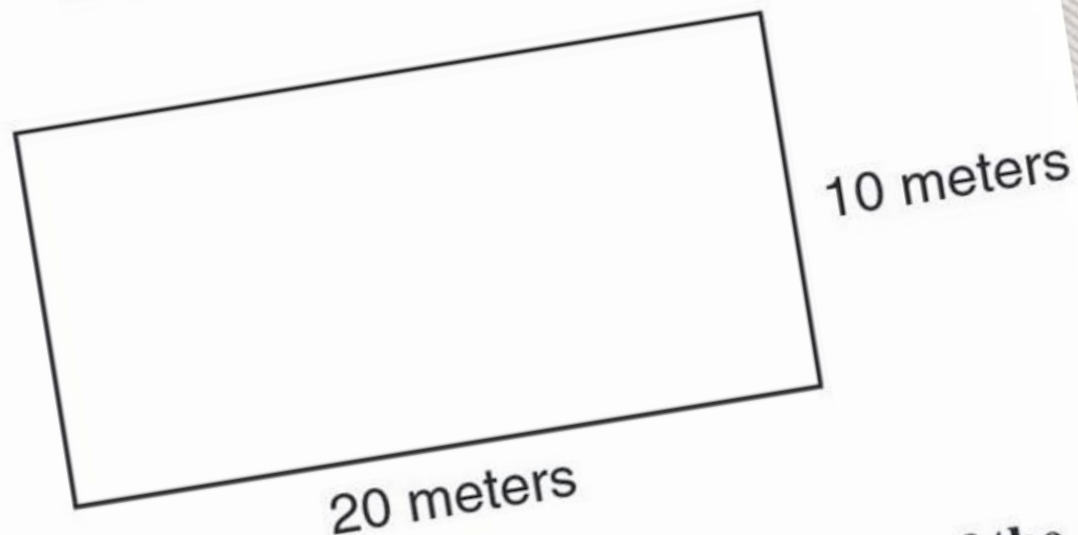
☒ Procedural Skill and
Fluency

☒ Conceptual
Understanding



71

A basketball court is shaped like a rectangle 20 meters long and 10 meters wide.



What is the perimeter in meters of the court?

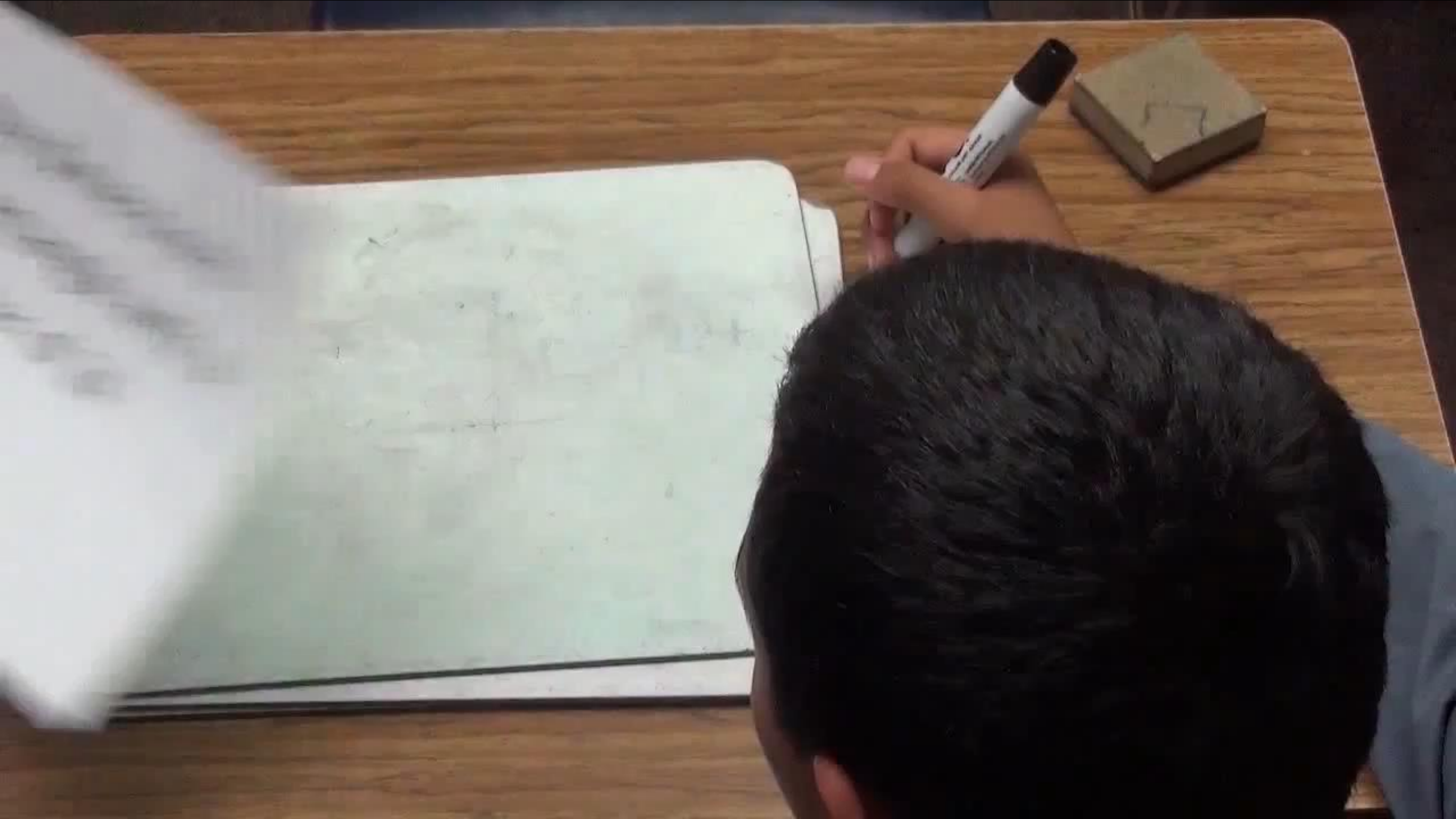
- A 30 meters
- B 50 meters
- C 60 meters
- D 200 meters

What is the perimeter
of a rectangle ~~with~~
that measures 8 units
by 4 units?




COMPONENTS OF RIGOR

- ☒ Procedural Skill and Fluency
- ☒ Conceptual Understanding



COMPONENTS OF RIGOR

- ☒ Procedural Skill and Fluency
- ☒ Conceptual Understanding



Of all the
rectangles with a
perimeter of 24
units, which one
has the most area?

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with a perimeter of
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00:00:00:00

COMPONENTS OF RIGOR

☒ Procedural Skill and
Fluency

☒ Conceptual
Understanding

DEFINING THE PROBLEM

- Students appear to demonstrate “deep, authentic command of mathematical concepts” when given commonly used problems.
- However with more challenging problems, the same students seem to no longer demonstrate that command.

ADDRESSING THE PROBLEM

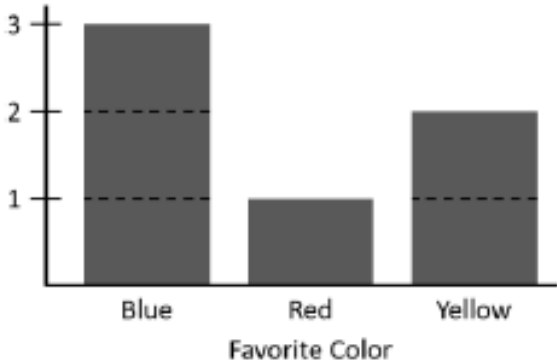
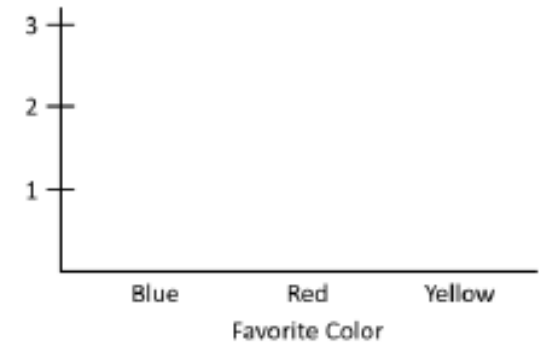
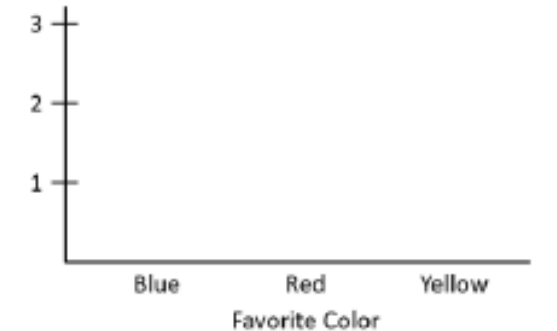
- First, we must have a clear understanding about why these problems are different from one another.
- Next, we need to practice using these problems so that we understand how students may react to them.
- Last, we need a source that can provide us with a variety of free problems.

ADDING DECIMALS

Use the numbers 1 through 9, exactly one time each, to fill in the boxes and make three decimals whose sum is as close to 1 as possible.

$$\begin{array}{r} 0.\square\square\square \\ 0.\square\square\square \\ + 0.\square\square\square \\ \hline \end{array}$$

Depth of Knowledge Matrix - Elementary Math

Topic	Adding 1-Digit Numbers (< 5)	Equality	Interpreting Data	Money
CCSS Stand.	<ul style="list-style-type: none"> K.OA.5 	<ul style="list-style-type: none"> 1.OA.7 	<ul style="list-style-type: none"> 1.MD.4 	<ul style="list-style-type: none"> 2.MD.8
DOK 1 Example	<p>Solve.</p> $3 + 1 =$	<p>Determine whether the number sentence is true or false.</p> $4 + 1 = 5 - 2$	<p>How many people were surveyed?</p> 	<p>If you have 1 quarter, 4 dimes, 2 nickels, and 3 pennies, how many cents do you have?</p>
DOK 2 Example	<p>Use the digits 1 to 5, at most one time each, to fill in the boxes to create two true number sentences.</p> $\square + \square = \square$	<p>Use the digits 1 to 9, at most one time each, to fill in the boxes to create two true number sentences.</p> $\square + \square = \square - \square$	<p>Make a graph that shows a possible result of 7 students' favorite color.</p> 	<p>Make 72¢ in two different ways with either quarters, dimes, nickels, or pennies.</p>
DOK 3 Example	<p>Use the digits 1 to 5, at most one time each, to fill in the boxes to create a true number sentence with the greatest possible sum.</p> $\square + \square = \square$	<p>Use the digits 1 to 9, at most one time each, to fill in the boxes to create a true number sentence with the greatest possible value.</p> $\square + \square = \square - \square$	<p>Make a graph that shows a possible result of 7 students' favorite color with red being the most popular color.</p> 	<p>Make 72¢ using exactly 9 coins that are either quarters, dimes, nickels, or pennies.</p>

Depth of Knowledge Matrix - Elementary Math

Topic	Subtracting 3-Digit Numbers	Operations with Time	Comparing Fractions	Multiplying Decimals
CCSS Stand.	• 3.NBT.2	• 3.MD.1	• 4.NF.2	• 5.NBT.7
DOK 1 Example	Solve. $821 - 357 =$	What time will it be 14 minutes after 1:27 pm?	Place a < or > between the two fractions to make a true number sentence. $\frac{4}{7} \quad \frac{3}{5}$	Solve. $3.4 \times 2.5 =$
DOK 2 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two different pairs of three-digit numbers that form a true number sentence. $\square\square\square - 291 = \square\square\square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a time that is 4:37 pm. $\square\square$ minutes after $\square:\square\square$ pm	Use the digits 1 to 9, at most one time each, to fill in the boxes to create two different fractions: one that is less than one half and one that is more than one half. $\frac{\square}{\square} < \frac{1}{2}$ and $\frac{\square}{\square} > \frac{1}{2}$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a true number sentence. $\square.\square \times 3.2 = \square.\square$
DOK 3 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a difference that is as close to 329 as possible. $\square\square\square - \square\square\square =$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make the latest possible time. $\square\square$ minutes after $\square:\square\square$ pm	Use the digits 1 to 9, at most one time each, to fill in the boxes to create a fraction that is as close to $\frac{5}{11}$ as possible. $\frac{\square}{\square}$	Use the digits 1 to 9, at most one time each, so that the product is as close to 50 as possible. $\square.\square \times \square.\square =$



Complex or Complicated?

DOK

VERB

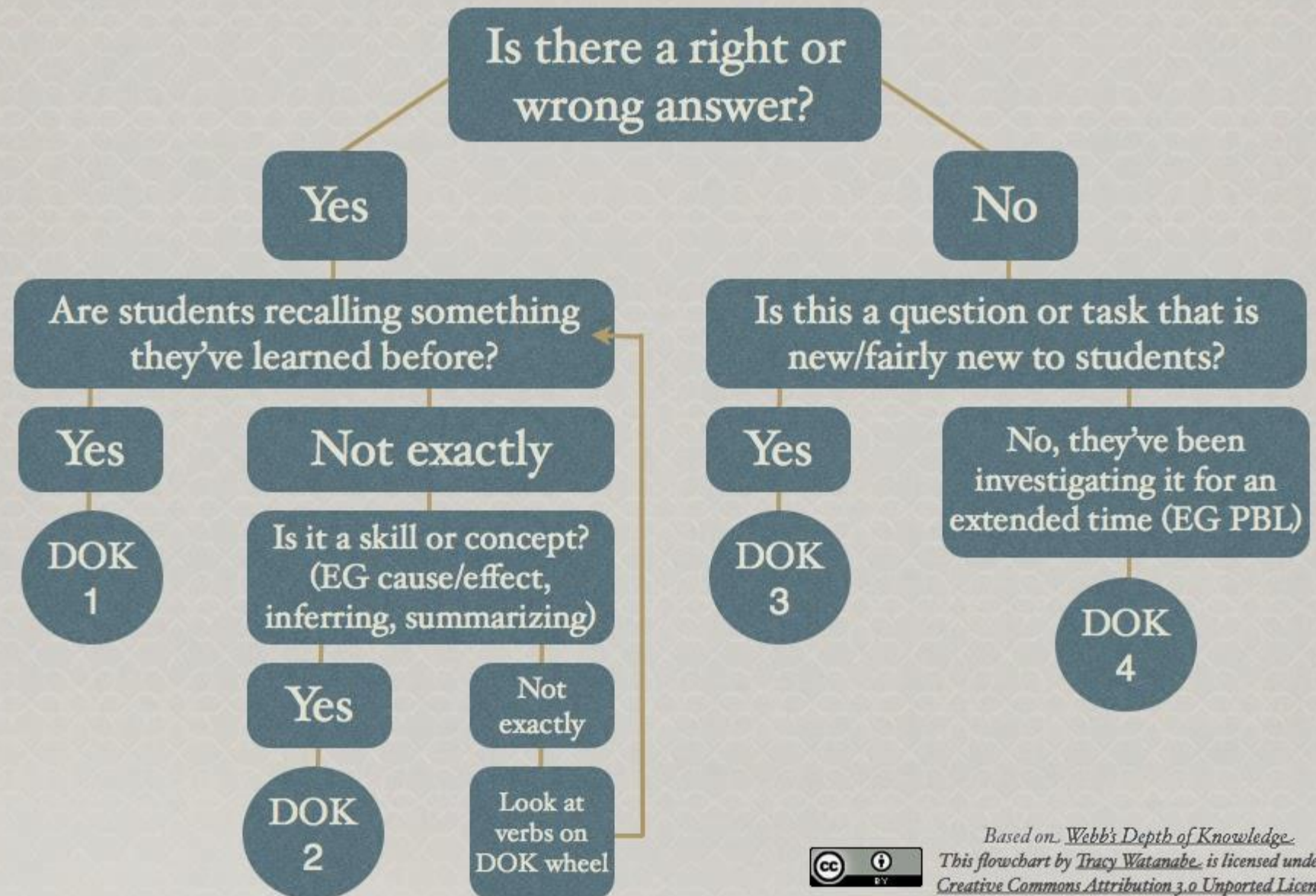
WHEEL



Source: Unknown

DOK FLOWCHART

Depth of Knowledge (DOK) Flowchart for Questions



Source:

Tracy Watanabe
@tracywatanabe



Based on: *Webb's Depth of Knowledge*.
This flowchart by Tracy Watanabe is licensed under a
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DOK

POSTERS

Source: Penny Lund

isntitelementary.blogspot.com/

RobertKaplinsky.com

DOK 1

Routine Thinking

- Can you recall ____?
- Can you identify ____?
- How would you describe ____?
- What might you include on a list about ____?
- Can you select ____?
- How can you find the meaning of ____?

arrange	calculate	memorize
measure	name	recognize
recall	repeat	identify
illustrate	match	label
state	list	state

DOK 2

Conceptual Thinking

- Can you explain how ____ affected ____?
- How would you apply what you learned to develop ____?
- How would you summarize ____?
- What do you notice about ____?
- How would you estimate ____?
- How could you organize ____?

compare	classify	categorize
measure	graph	distinguish
predict	modify	construct
organize	infer	summarize
interpret	make observations	

DOK 3

Strategic Reasoning

- How is ____ related to ____?
- What conclusions can be drawn?
- Can you elaborate on ____?
- How would you test ____?
- What evidence supports ____?
- What would happen if ____?
- Why is that the best answer?

assess	compare	construct
apprise	revise	hypothesize
critique	investigate	
draw conclusions		
develop a logical argument		

DOK 4

Extended Reasoning

- Write a research paper.
- What information can you gather to support your idea about ____?
- Write a thesis, drawing conclusions from multiple sources.
- Apply information from one text to another to develop an persuasive argument.

design	connect	prove
analyze	critique	synthesize
create	apply concepts	

Created by Penny Lund 2013

DOK LEVEL DIFFERENCES

Level 1: Recall & Reproduction

- Often a trivial application of facts.
- Generally requires little effort beyond remembering a formula.
- Usually only one answer.

Level 2: Skills & Concepts

- Usually requires more than one step to solve.
- Often multiple answers.

Level 3: Strategic Thinking

- Usually requires critical thinking about how to approach a problem.
- May be multiple answers or a single optimal answer.
- Often challenging enough to make your head hurt.

Level 4: Extended Thinking

- These are generally represented by performance tasks or problem-based lessons.

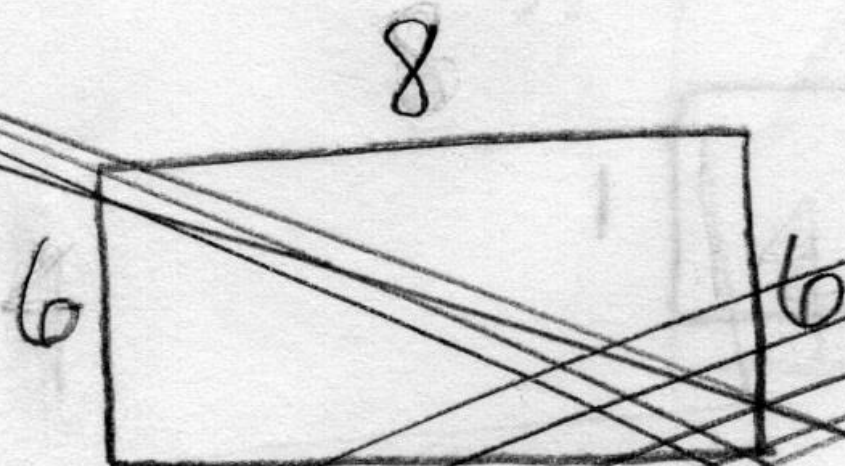
First attempt:

Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

First attempt:

Points: ____/2 attempt ____/2 explanation



area:

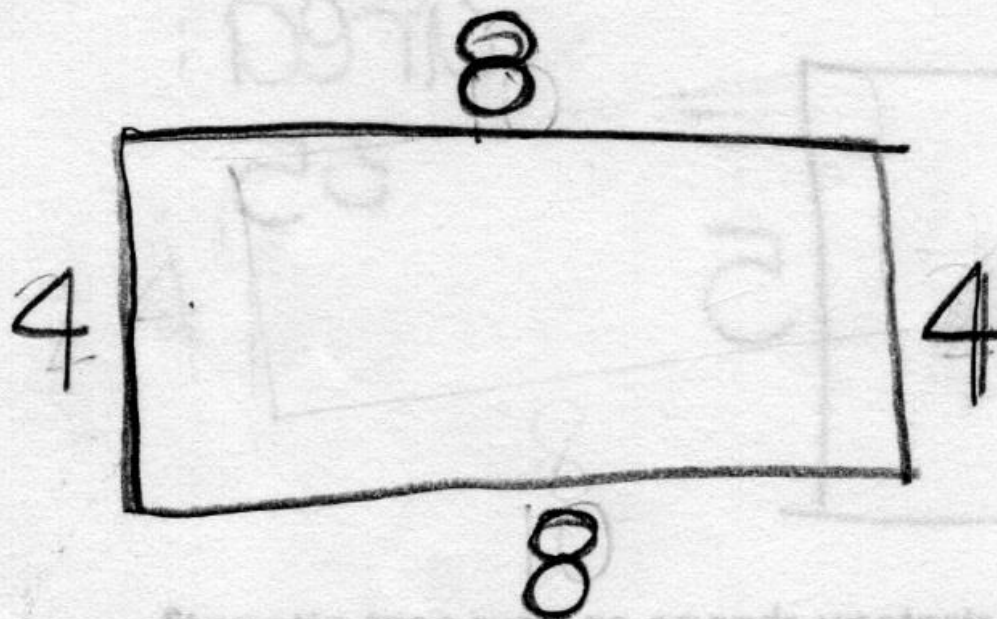
48

What did you learn from this attempt? How will your strategy change on your next attempt?

~~This attempt doesn't equal 24.~~

Second attempt:

Points: ____/2 attempt ____/2 explanation



area:

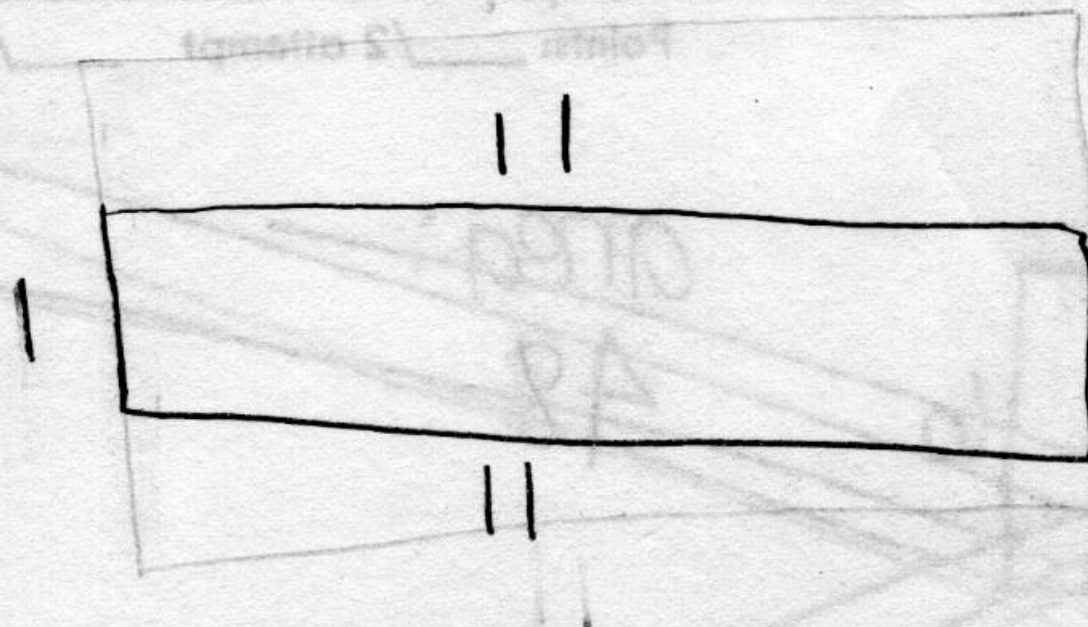
32

What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter was 24, and the area was 32
but I think there's a blazer #

Fourth attempt:

Points: ____/2 attempt ____/2 explanation

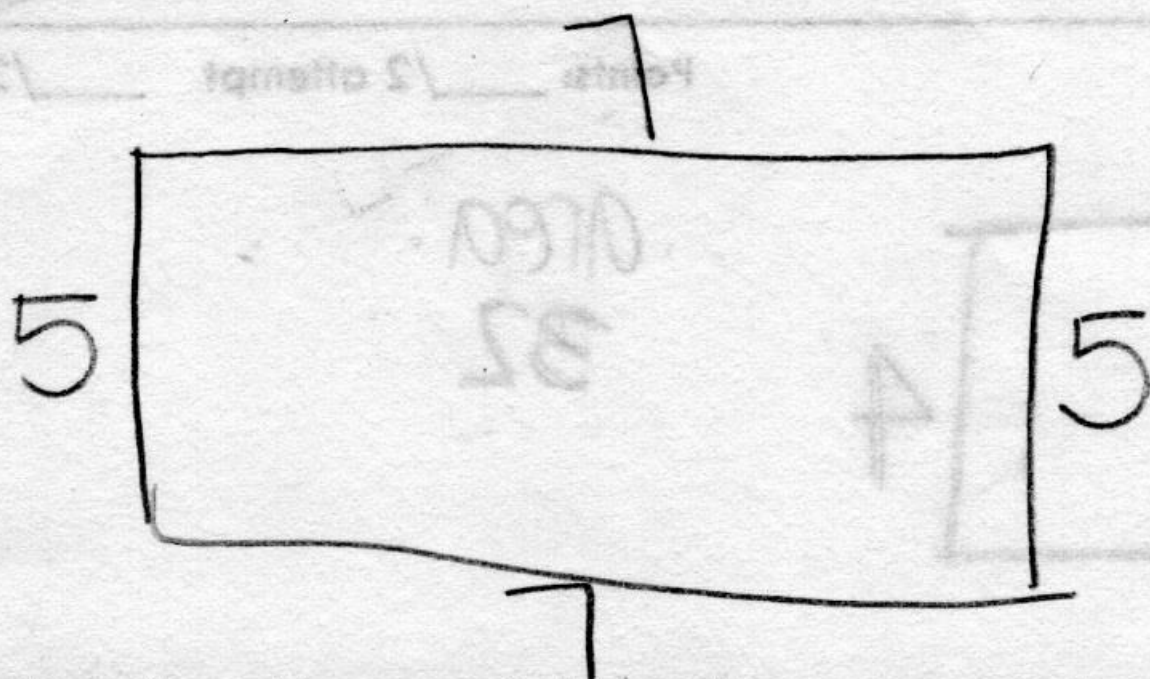


What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter is 24, but the area is 11 and attempt #2 the area is 32
Strategy: Use #'s with more than one row.

Fifth attempt:

Points: ____/2 attempt ____/2 explanation



area:
35


What did you learn from this attempt? How will your strategy change on your next attempt?

Open Middle

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- 
1. Two-Step Equations 1 by Robert Kaplinsky, Daniel Luevanos, and Robert Kaplinsky
 2. Order of Operations 1 by Robert Kaplinsky with answer from Michael Fenton and his students
 3. Dot Card Counting by Daniel Luevanos
 4. Rational and Irrational Numbers by Bryan Anderson
 5. One Solution, No Solutions, Infinite Solutions by Bryan Anderson
 6. Multiplying a Two-Digit Number by a Single-Digit Number by Robert Kaplinsky
 7. Exponents and Order of Operations by Zack Miller
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 9. Interpreting Percentages by Robert Kaplinsky
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- Kindergarten (6)
 - Counting & Cardinality (2)
- Number & Operations in Base Ten (1)
- Operations & Algebraic Thinking (3)

Home > High School: Geometry > Expressing Geometric Properties with Equations > Equidistant Points

EQUIDISTANT POINTS

Directions: How many points with integer coordinates are 5 units away from $(-2, 3)$?

Hint

Which methods are available to determine the answer to this problem? What shape is defined by *all* of the points that are 5 units away $(-2, 3)$?

Answer

12 points: $(-5, 7)$, $(-7, 3)$, $(-5, -1)$, $(-2, -2)$, $(3, 3)$, $(1, -1)$, $(-2, 8)$, $(1, 7)$, $(2, 6)$, $(-6, -6)$, $(-6, 0)$, and $(2, 0)$

Source: [Dylan Kane](#)



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