

DIGGING INTO DEPTH

OF KNOWLEDGE

ROBERT KAPLINSKY

robert@robertkaplinsky.com

robertkaplinsky.com

[@robertkaplinsky](#)





COMMON CORE

STATE STANDARDS INITIATIVE

CCSS.MATH.CONTENT.4.MD.A.3


Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

meet the...
equal intensity, the...
of each grade: conceptual...
skills and fluency, and application.

What is the perimeter of a rectangle that measures 8 units by 4 units?

COMPONENTS OF RIGOR

- Procedural Skill and Fluency
- Conceptual Understanding



List the
dimensions of a
rectangle with
a perimeter
of 24 units.



COMPONENTS OF RIGOR

Procedural Skill and Fluency

Conceptual Understanding



Why?



COMPONENTS OF RIGOR

- Procedural Skill and Fluency
- Conceptual Understanding

List the
of a rectangle with a
perimeter of 24
units.



COMPONENTS OF RIGOR

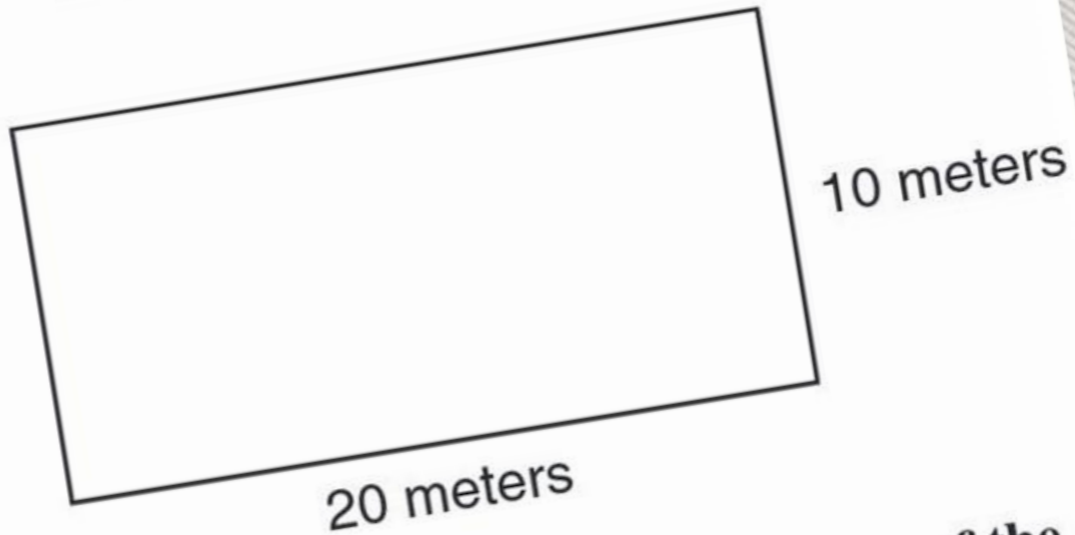
Procedural Skill and Fluency

Conceptual Understanding



71

A basketball court is shaped like a rectangle 20 meters long and 10 meters wide.



What is the perimeter in meters of the court?

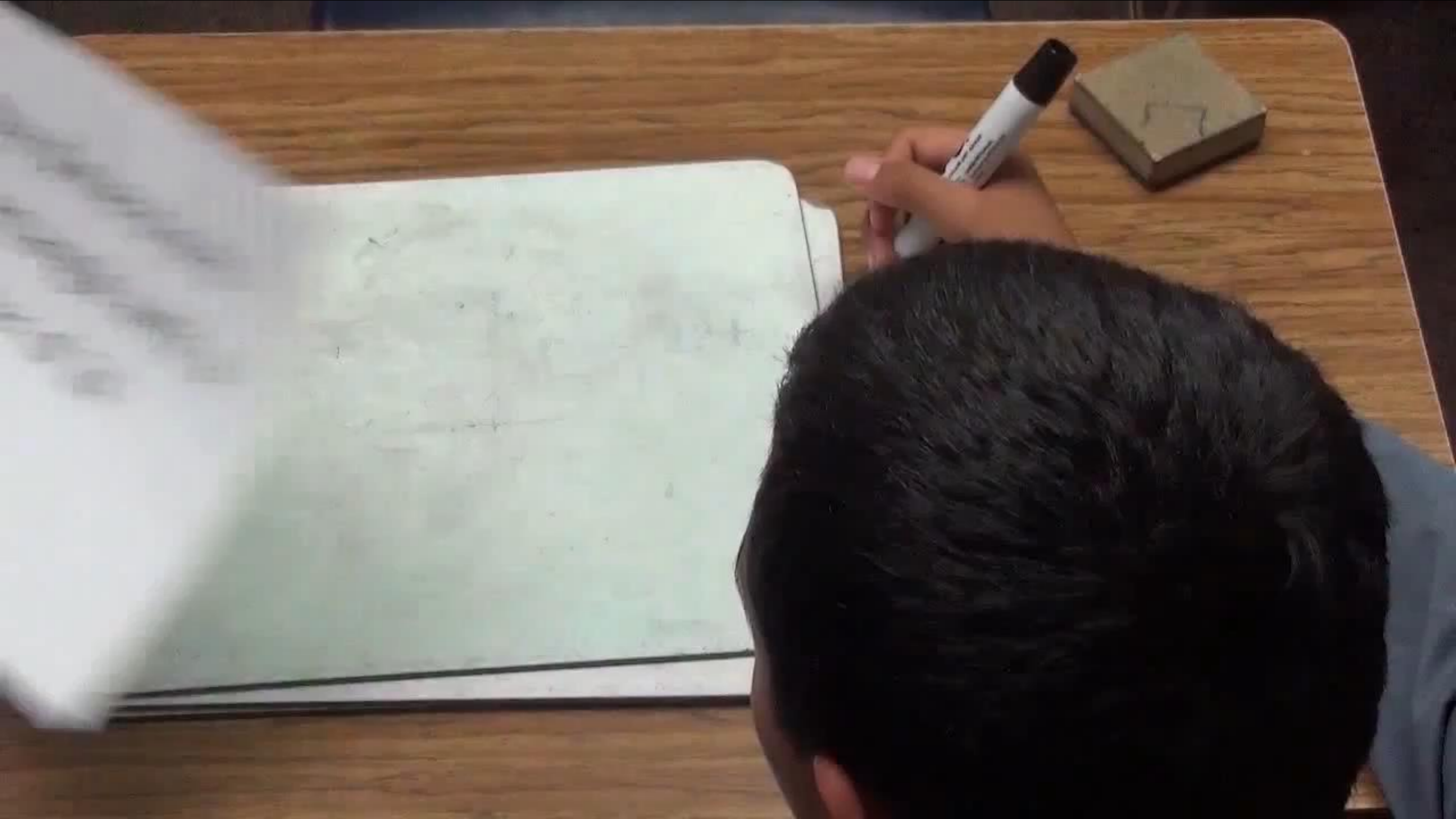
- A 30 meters
- B 50 meters
- C 60 meters
- D 200 meters

What is the perimeter
of a rectangle ~~with~~
that measures 8 units
by 4 units?




COMPONENTS OF RIGOR

- Procedural Skill and Fluency
- Conceptual Understanding



COMPONENTS OF RIGOR

- Procedural Skill and Fluency
- Conceptual Understanding



Of all the rectangles with a perimeter of 24 units, which one has the most area?

Of all the rectangles
with a perimeter of
24 units, which one
has the most area?

00:00:00:00

COMPONENTS OF RIGOR

Procedural Skill and Fluency

Conceptual Understanding

DEFINING THE PROBLEM

- Students appear to demonstrate “deep, authentic command of mathematical concepts” when given commonly used problems.
- However with more challenging problems, the same students seem to no longer demonstrate that command.

ADDRESSING THE PROBLEM

- First, we must have a clear understanding about why these problems are different from one another.
- Next, we need to practice using these problems so that we understand how students may react to them.
- Last, we need a source that can provide us with a variety of free problems.

ADDING DECIMALS

Use the numbers 1 through 9, exactly one time each, to fill in the boxes and make three decimals whose sum is as close to 1 as possible.

$$\begin{array}{r} 0.\square\square\square \\ 0.\square\square\square \\ + 0.\square\square\square \\ \hline \end{array}$$

Depth of Knowledge Matrix - Elementary Math

| Topic | Adding 1-Digit Numbers (< 5) | Equality | Interpreting Data | Money |
|------------------|--|--|---|---|
| CCSS Stand. | <ul style="list-style-type: none"> K.OA.5 | <ul style="list-style-type: none"> 1.OA.7 | <ul style="list-style-type: none"> 1.MD.4 | <ul style="list-style-type: none"> 2.MD.8 |
| DOK 1 Example | Solve. $3 + 1 =$ | Determine whether the number sentence is true or false. $4 + 1 = 5 - 2$ | How many people were surveyed? <p>A bar graph with the y-axis labeled from 1 to 3 and the x-axis labeled 'Favorite Color' with categories Blue, Red, and Yellow. The bars show 3 for Blue, 1 for Red, and 2 for Yellow.</p> | If you have 1 quarter, 4 dimes, 2 nickels, and 3 pennies, how many cents do you have? |
| DOK 2 Example | Use the digits 1 to 5, at most one time each, to fill in the boxes to create two true number sentences. $\square + \square = \square$ | Use the digits 1 to 9, at most one time each, to fill in the boxes to create two true number sentences. $\square + \square = \square - \square$ | Make a graph that shows a possible result of 7 students' favorite color. <p>A blank bar graph with the y-axis labeled from 1 to 3 and the x-axis labeled 'Favorite Color' with categories Blue, Red, and Yellow.</p> | Make 72¢ in two different ways with either quarters, dimes, nickels, or pennies. |
| DOK 3 Example | Use the digits 1 to 5, at most one time each, to fill in the boxes to create a true number sentence with the greatest possible sum. $\square + \square = \square$ | Use the digits 1 to 9, at most one time each, to fill in the boxes to create a true number sentence with the greatest possible value. $\square + \square = \square - \square$ | Make a graph that shows a possible result of 7 students' favorite color with red being the most popular color. <p>A blank bar graph with the y-axis labeled from 1 to 3 and the x-axis labeled 'Favorite Color' with categories Blue, Red, and Yellow.</p> | Make 72¢ using exactly 9 coins that are either quarters, dimes, nickels, or pennies. |

Depth of Knowledge Matrix - Elementary Math

| Topic | Subtracting 3-Digit Numbers | Operations with Time | Comparing Fractions | Multiplying Decimals |
|---------------|--|---|---|--|
| CCSS Stand. | <ul style="list-style-type: none"> 3.NBT.2 | <ul style="list-style-type: none"> 3.MD.1 | <ul style="list-style-type: none"> 4.NF.2 | <ul style="list-style-type: none"> 5.NBT.7 |
| DOK 1 Example | Solve. $821 - 357 =$ | What time will it be 14 minutes after 1:27 pm? | Place a < or > between the two fractions to make a true number sentence. $\frac{4}{7}$ $\frac{3}{5}$ | Solve. $3.4 \times 2.5 =$ |
| DOK 2 Example | Use the digits 1 to 9, at most one time each, to fill in the boxes to make two different pairs of three-digit numbers that form a true number sentence. $\square\square\square - 291 = \square\square\square$ | Use the digits 1 to 9, at most one time each, to fill in the boxes to make a time that is 4:37 pm. $\square\square$ minutes after $\square:\square\square$ pm | Use the digits 1 to 9, at most one time each, to fill in the boxes to create two different fractions: one that is less than one half and one that is more than one half. $\frac{\square}{\square} < \frac{1}{2}$ and $\frac{\square}{\square} > \frac{1}{2}$ | Use the digits 1 to 9, at most one time each, to fill in the boxes to make a true number sentence. $\square.\square \times 3.2 = \square.\square$ |
| DOK 3 Example | Use the digits 1 to 9, at most one time each, to fill in the boxes to make a difference that is as close to 329 as possible. $\square\square\square - \square\square\square =$ | Use the digits 1 to 9, at most one time each, to fill in the boxes to make the latest possible time. $\square\square$ minutes after $\square:\square\square$ pm | Use the digits 1 to 9, at most one time each, to fill in the boxes to create a fraction that is as close to 5/11 as possible. $\frac{\square}{\square}$ | Use the digits 1 to 9, at most one time each, so that the product is as close to 50 as possible. $\square.\square \times \square.\square =$ |



Complex or Complicated?



Cookie Monster Cupcakes



Nailed it

method

1. Using an electric mixer, whip the butter until it is pale. This will take at least 5 minutes on high.
2. Gradually add in the icing mixture and vanilla until well combined.
3. With the mixer running, add in food colouring until you get to the Cookie Monster colour. This may be a lot if you are using liquid food colouring or a little if using gel food colouring.
4. Add in the milk and mix until the frosting puffs up.
5. Fill a piping bag with a fluted nozzle and pipe on icing.
6. With the writing icing, place black spots on the marshmallows for pupils.
7. Place on each cupcake.
8. Cut cookies in half and place in 'mouth'.

DOK

VERB

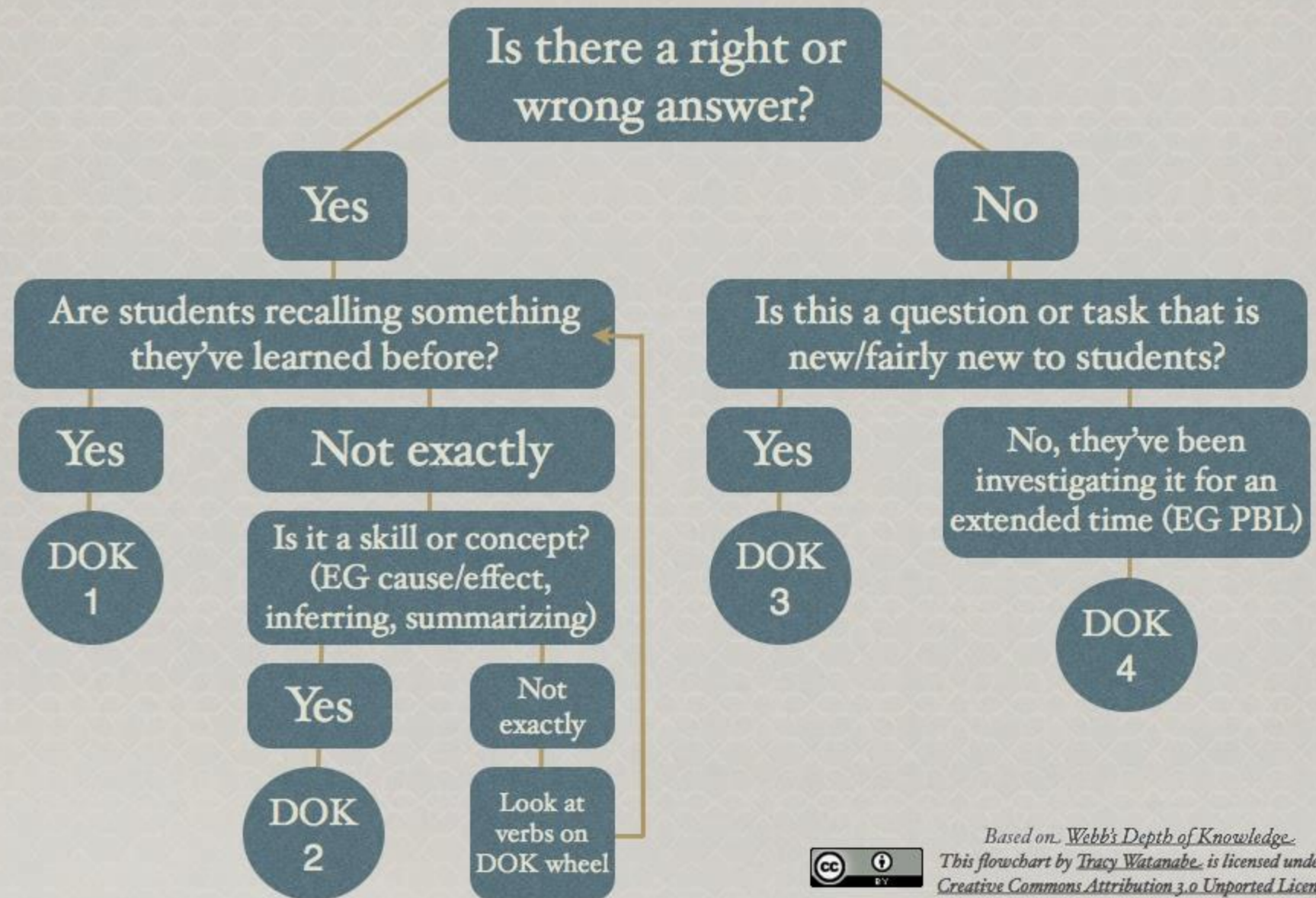
WHEEL



Source: Unknown

DOK FLOWCHART

Depth of Knowledge (DOK) Flowchart for Questions



Source:

Tracy Watanabe

@tracywatanabe



Based on *Webb's Depth of Knowledge*.
This flowchart by Tracy Watanabe is licensed under a
Creative Commons Attribution 3.0 Unported License.

DOK

POSTERS

DOK 1

Routine Thinking

- Can you recall ___?
- Can you identify ___?
- How would you describe ___?
- What might you include on a list about ___?
- Can you select ___?
- How can you find the meaning of ___?

arrange calculate memorize
measure name recognize
recall repeat identify
illustrate match label
state list state

DOK 2

Conceptual Thinking

- Can you explain how ___ affected ___?
- How would you apply what you learned to develop ___?
- How would you summarize ___?
- What do you notice about ___?
- How would you estimate ___?
- How could you organize ___?

compare classify categorize
measure graph distinguish
predict modify construct
organize infer summarize
interpret make observations

DOK 3

Strategic Reasoning

- How is ___ related to ___?
- What conclusions can be drawn?
- Can you elaborate on ___?
- How would you test ___?
- What evidence supports ___?
- What would happen if ___?
- Why is that the best answer?

assess compare construct
apprise revise hypothesize
critique investigate
draw conclusions
develop a logical argument

DOK 4

Extended Reasoning

- Write a research paper.
- What information can you gather to support your idea about ___?
- Write a thesis, drawing conclusions from multiple sources.
- Apply information from one text to another to develop an persuasive argument.

design connect prove
analyze critique synthesize
create apply concepts

Source: Penny Lund

isntitelementary.blogspot.com/

Created by Penny Lund 2013

RobertKaplinsky.com

DOK LEVEL DIFFERENCES

Level 1: Recall & Reproduction

- Often a trivial application of facts.
- Generally requires little effort beyond remembering a formula.
- Usually only one answer.

Level 2: Skills & Concepts

- Usually requires more than one step to solve.
- Often multiple answers.

Level 3: Strategic Thinking

- Usually requires critical thinking about how to approach a problem.
- May be multiple answers or a single optimal answer.
- Often challenging enough to make your head hurt.

Level 4: Extended Thinking

- These are generally represented by performance tasks or problem-based lessons.

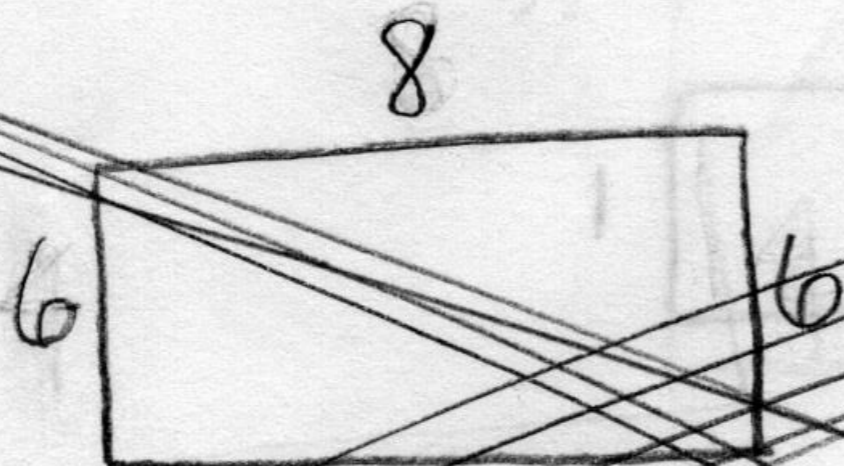
First attempt:

Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

First attempt:

Points: ___/2 attempt ___/2 explanation



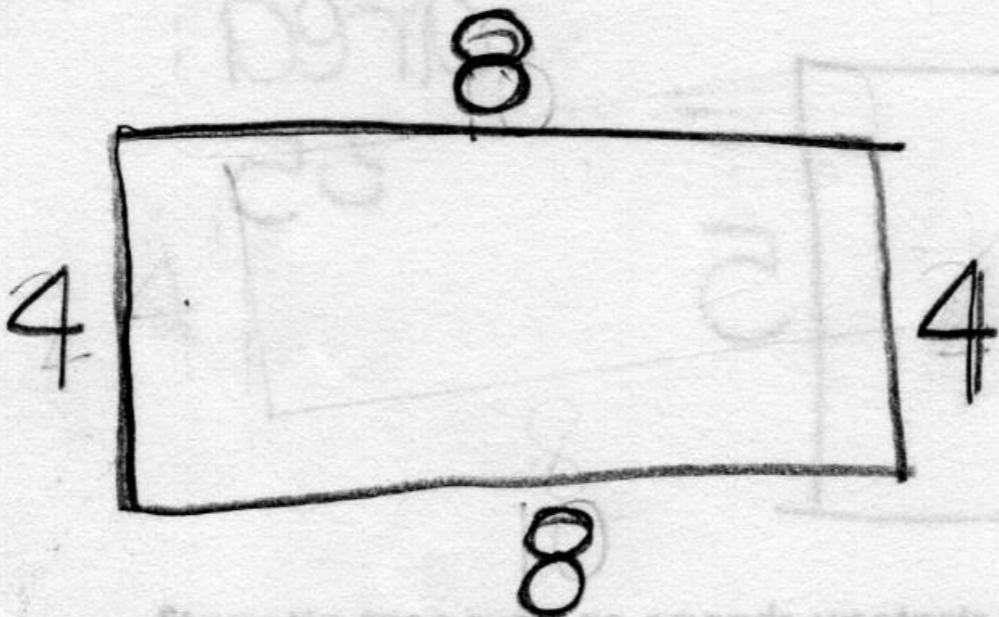
area:
48

What did you learn from this attempt? How will your strategy change on your next attempt?

~~This attempt doesn't equal 24.~~

Second attempt:

Points: ___/2 attempt ___/2 explanation



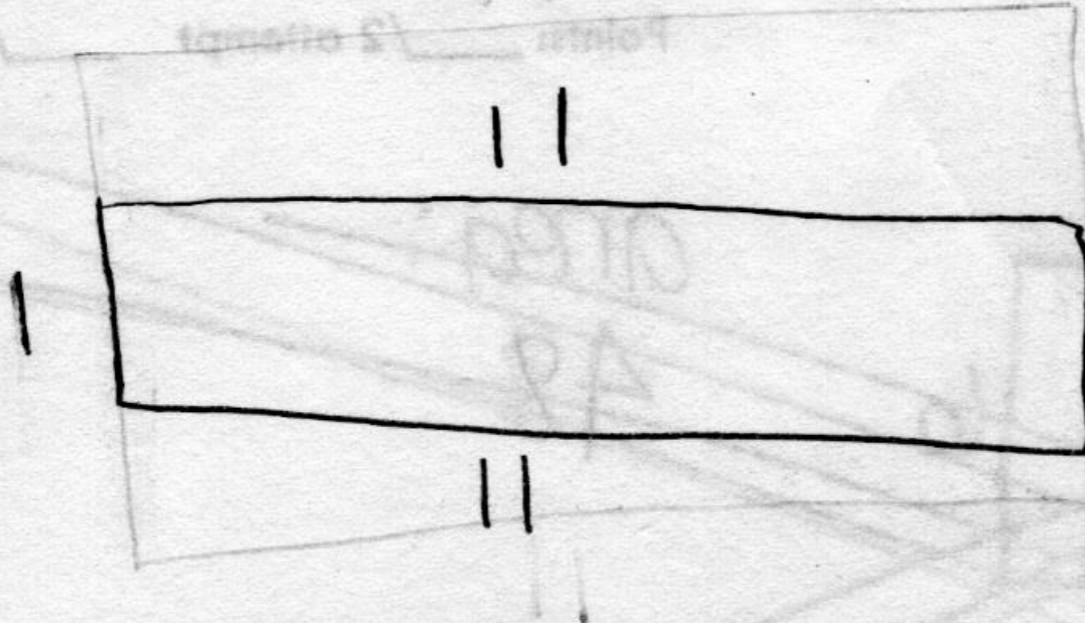
area:
32

What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter was 24, and the area was 32
but I think there's a blader #

Fourth attempt:

Points: ___/2 attempt ___/2 explanation

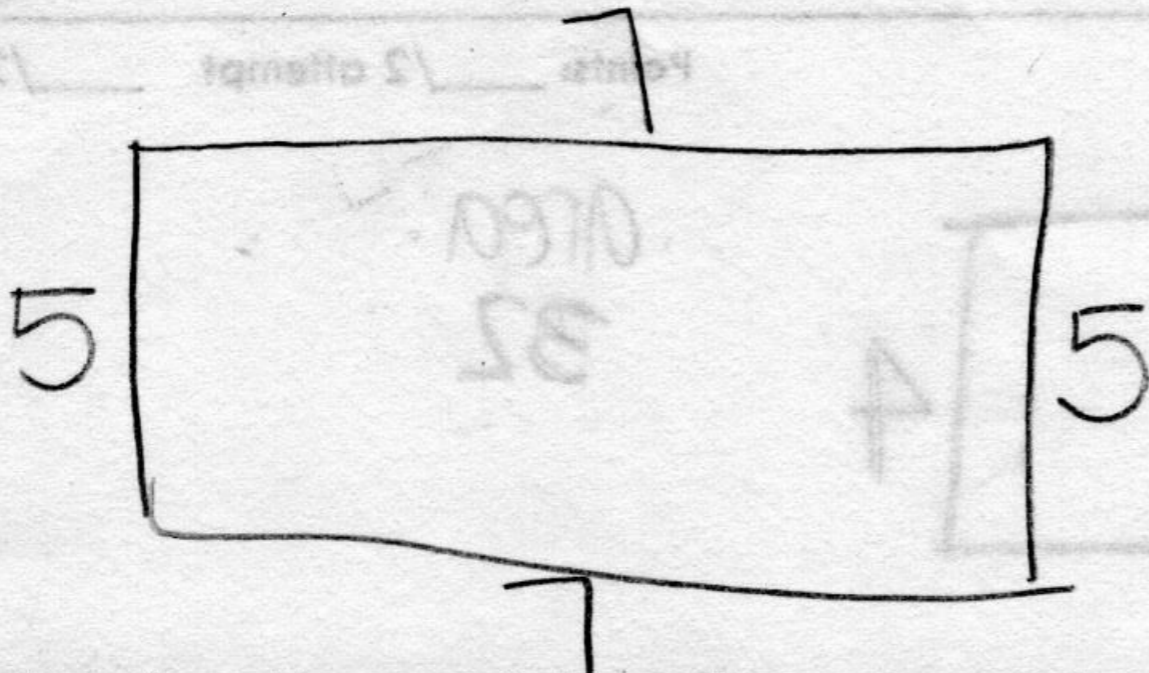


What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter is 24, but the area is 11 and attempt #2 the area is 32.
Strategy: Use #'s with more than one row.

Fifth attempt:

Points: ___/2 attempt ___/2 explanation



area:
35

What did you learn from this attempt? How will your strategy change on your next attempt?

Open Middle

Challenging math problems worth solving

- Home
- Kinder ▾
- Grade 1 ▾
- Grade 2 ▾
- Grade 3 ▾
- Grade 4 ▾
- Grade 5 ▾
- Grade 6 ▾
- Grade 7 ▾
- Grade 8 ▾
- High School ▾
- About
- Submit



THE TOP 10 MOST POPULAR PROBLEMS OF 2016

1. Two-Step Equations with One Variable by Daniel Luevanos, and Robert Kaplinsky
2. Order of Operations with answer from Michael Fenton and his students
3. Dot Card Counting by Dan Meyer
4. Rational and Irrational Numbers by Bryan Anderson
5. One Solution, No Solutions, Infinite Solutions by Bryan Anderson
6. Multiplying a Two-Digit Number by a Single-Digit Number by Robert Kaplinsky
7. Exponents and Order of Operations by Zack Miller
8. Converting Between Fractions and Decimals by Robert Kaplinsky
9. Interpreting Percentages by Robert Kaplinsky
10. Two-Step Equations 3 by Erick Lee

Search



OPEN MIDDLE WORKSHEET

Download the Open Middle Worksheet (Regular):
Version 1.2

Download the Open Middle Worksheet (Large):
Version 1.1

SUBSCRIBE

Receive emails every time a new problem is published.

Enter your e-mail address

Subscribe

WHAT ARE PEOPLE SAYING ABOUT OPEN MIDDLE?



Brian Marks

@Yummymath

Follow

Have you checked out openmiddle.com @openmiddle Should

BROWSE BY COMMON CORE STATE STANDARDS

- Kindergarten (6)
- Counting & Cardinality (2)
- Number & Operations in Base Ten (1)
- Operations & Algebraic Thinking (3)

Open Middle

Challenging math problems worth solving

Home Kinder ▾ Grade 1 ▾ Grade 2 ▾ Grade 3 ▾ Grade 4 ▾ Grade 5 ▾ Grade 6 ▾ Grade 7 ▾ **Grade 8 ▾** High School ▾ About Submit

Home > High School: Geometry > Expressing Geometric Properties with Equations > Equidistant Points

EQUIDISTANT POINTS

Directions: How many points with integer coordinates are 5 units away from $(-2, 3)$?

Hint

Which methods are available to determine the answer to this problem? What shape is defined by *all* of the points that are 5 units away $(-2, 3)$?

Answer

12 points: $(-5, 7)$, $(-7, 3)$, $(-5, -1)$, $(-2, -2)$, $(3, 3)$, $(1, -1)$, $(-2, 8)$, $(1, 7)$, $(2, 6)$, $(-6, -6)$, $(-6, 0)$, and $(2, 0)$

Source: [Dylan Kane](#)

 Print

SHARE

 Like 0

 Tweet

 G+ 0

 Share

 Pin it

Tags [8.G.8](#) [DOK 2: SKILL / CONCEPT](#) [DYLAN KANE](#) [G-GPE.1](#)

Search



OPEN MIDDLE WORKSHEET

Download the Open Middle Worksheet (Regular):
Version 1.2

Download the Open Middle Worksheet (Large):
Version 1.1

SUBSCRIBE

Receive emails every time a new problem is published.

Enter your e-mail address

Subscribe

BROWSE BY COMMON CORE STATE STANDARDS

-  Kindergarten (6)
-  Counting & Cardinality (2)
-  Number & Operations in Base Ten (1)
-  Operations & Algebraic Thinking (3)



Download the resources at:
robertkaplinsky.com/iowa