

# CHALLENGING PROBLEMS

# WORTH SOLVING

**ROBERT KAPLINSKY**

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# GOALS

WHY DO WE NEED THEM?

WHY ARE THEY DIFFERENT?

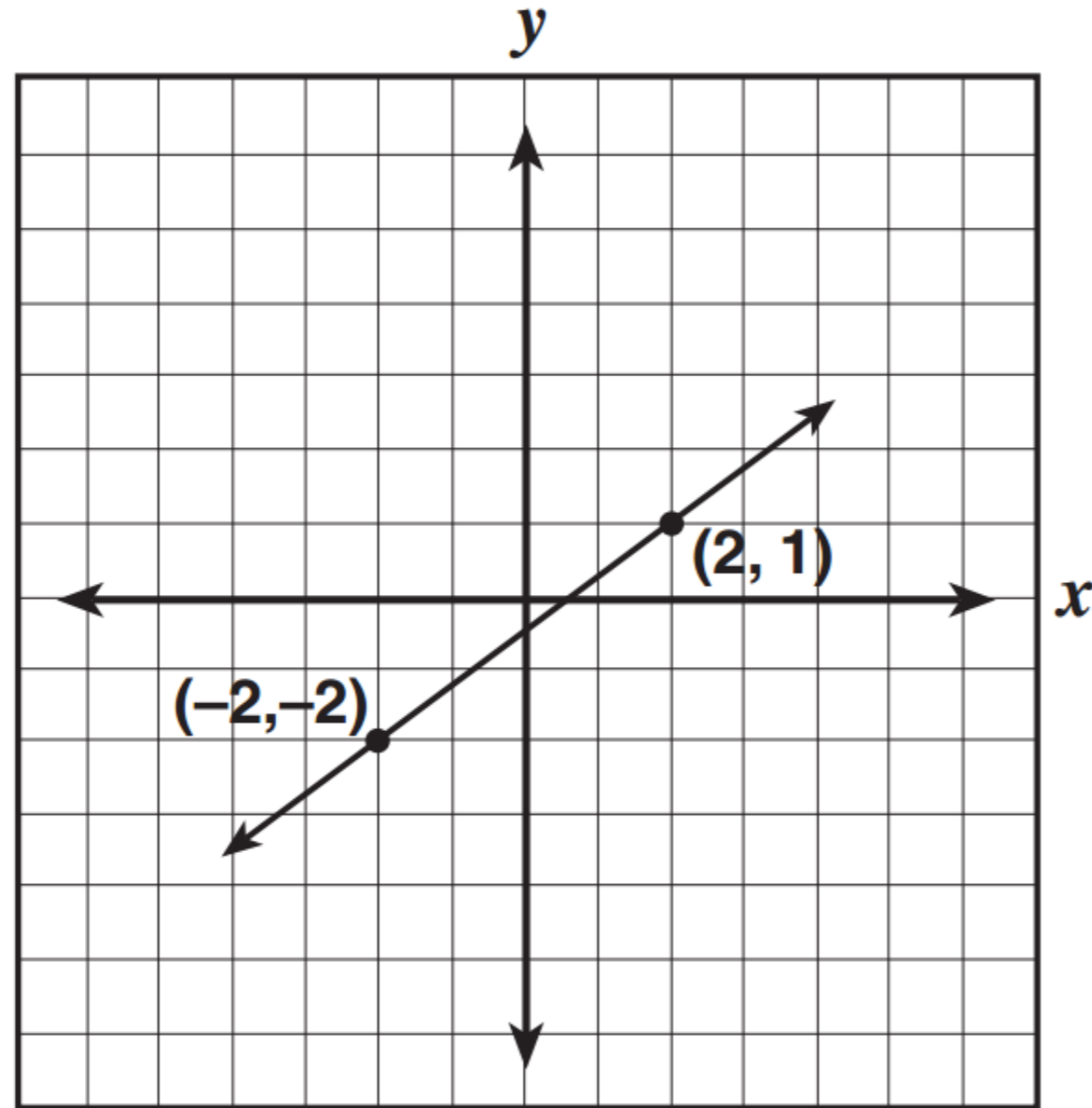
HOW DO YOU IMPLEMENT THEM?

HOW DO YOU CREATE YOUR OWN?

WHERE DO YOU GET OTHERS?

Student Name	ID Number	Perf. Level	Scaled Score	Mathematics Clusters											
				(Clusters where the percent correct is shown in bold represent proficiency for that cluster.)											
				Rational numbers		Exponents, powers, and roots		Quantitative relationships and evaluating expressions		Multi-step problems, graphing, and functions		Measurement and geometry		Statistics, data analysis, and probability	
Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct		
ALAN, ALAN	11111	ADV	476	13	93%	8	100%	8	80%	14	93%	12	92%	5	100%
ALAN, ALAN	11111	ADV	464	13	93%	7	88%	8	80%	15	100%	11	85%	5	100%
ALAN, ALAN	11111	ADV	453	10	71%	8	100%	10	100%	14	93%	11	85%	5	100%
ALAN, ALAN	11111	ADV	453	13	93%	8	100%	9	90%	12	80%	11	85%	5	100%
ALAN, ALAN	11111	ADV	444	14	100%	7	88%	8	80%	13	87%	10	77%	5	100%
ALAN, ALAN	11111	ADV	444	12	86%	8	100%	8	80%	15	100%	10	77%	4	80%
ALAN, ALAN	11111	ADV	444	13	93%	8	100%	8	80%	14	93%	9	69%	5	100%
ALAN, ALAN	11111	ADV	435	12	86%	6	75%	9	90%	14	93%	10	77%	5	100%
ALAN, ALAN	11111	ADV	435	12	86%	6	75%	8	80%	14	93%	11	85%	5	100%
ALAN, ALAN	11111	ADV	435	13	93%	7	88%	9	90%	12	80%	10	77%	5	100%
ALAN, ALAN	11111	ADV	427	13	93%	6	75%	9	90%	12	80%	10	77%	5	100%
ALAN, ALAN	11111	ADV	427	13	93%	7	88%	6	60%	13	87%	11	85%	5	100%
ALAN, ALAN	11111	ADV	427	14	100%	5	63%	7	70%	14	93%	10	77%	5	100%
ALAN, ALAN	11111	ADV	421	13	93%	6	75%	6	60%	14	93%	10	77%	5	100%
ALAN, ALAN	11111	ADV	421	11	79%	5	63%	9	90%	13	87%	11	85%	5	100%
ALAN, ALAN	11111	ADV	414	12	86%	6	75%	8	80%	11	73%	11	85%	5	100%
ALAN, ALAN	11111	ADV	414	12	86%	8	100%	8	80%	13	87%	8	62%	4	80%
ALAN, ALAN	11111	PRO	408	11	79%	6	75%	9	90%	11	73%	10	77%	5	100%
ALAN, ALAN	11111	PRO	402	12	86%	8	100%	9	90%	8	53%	11	85%	3	60%
ALAN, ALAN	11111	PRO	402	8	57%	7	88%	8	80%	13	87%	10	77%	5	100%
ALAN, ALAN	11111	PRO	402	13	93%	6	75%	7	70%	13	87%	8	62%	4	80%
ALAN, ALAN	11111	PRO	402	11	79%	5	63%	7	70%	11	73%	12	92%	5	100%
ALAN, ALAN	11111	PRO	402	13	93%	7	88%	9	90%	10	67%	7	54%	5	100%
ALAN, ALAN	11111	PRO	402	13	93%	7	88%	7	70%	11	73%	8	62%	5	100%
ALAN, ALAN	11111	PRO	396	10	71%	6	75%	9	90%	14	93%	7	54%	4	80%
ALAN, ALAN	11111	PRO	396	12	86%	8	100%	6	60%	9	60%	11	85%	4	80%

**52** What is the slope of this line?



- A  $\frac{1}{2}$
- B  $\frac{3}{4}$
- C 1
- D  $\frac{4}{3}$



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...	...	...	...	...	...	8	100%	...	...	...	...	11	85%	5	100%
...	...	...	...	...	...	...	...	...	...	...	...	10	77%	5	100%
...	...	...	...	...	...	...	...	...	...	...	...	10	77%	4	80%
...	...	...	...	...	...	...	...	...	...	...	...	9	69%	5	100%
...	...	...	...	12	75%	...	...	...	...	...	...	10	77%	5	100%
...	...	...	...	12	75%	...	...	...	...	...	...	11	85%	5	100%
...	...	...	...	...	...	7	88%	...	...	...	...	10	77%	5	100%
...	...	...	...	...	...	6	75%	...	...	...	...	10	77%	5	100%
...	...	...	...	...	...	7	88%	...	...	...	...	11	85%	5	100%
...	...	...	...	...	...	5	63%	...	...	...	...	10	77%	5	100%
...	...	...	421	...	93%	6	75%	6	...	...	...	10	77%	5	100%
...	...	...	421	11	79%	5	63%	9	90%	13	87%	11	85%	5	100%
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...	...	...	402	12	86%	8	100%	9	90%	8	53%	11	85%	3	60%
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# GOALS

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# PROBLEM ONE

Solve for  $x$ .

$$21 + x = 70$$



# PROBLEM TWO

Using the digits 1 to 9, at most one time each, create two equations: one where  $x$  has a positive value and one where  $x$  has a negative value.

$$\boxed{\phantom{00}} + x = \boxed{\phantom{00}}$$

# PROBLEM THREE

Using the digits 1 to 9, at most one time each, create an equation where  $x$  has the greatest possible value.

$$\square\square + x = \square\square$$



Robert Kaplinsky

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MS & HS #MTBoS Ts, please ask your Ss these 3 ?s and put the % who answered correctly here:

[docs.google.com/forms/d/e/1FAI](https://docs.google.com/forms/d/e/1FAI...) .... Answers at top of form.

**PROBLEM ONE**  
Solve for x.  
 $21 + x = 7$

**PROBLEM TWO**  
Using the digits 1 to 9, at most one time each, create two equations: one where x has a positive value and one where x has a negative value.  
[ ][ ] + x = [ ][ ]

**PROBLEM THREE**  
Using the digits 1 to 9, at most one time each, create an equation where x has the greatest possible value.  
[ ][ ] + x = [ ][ ]

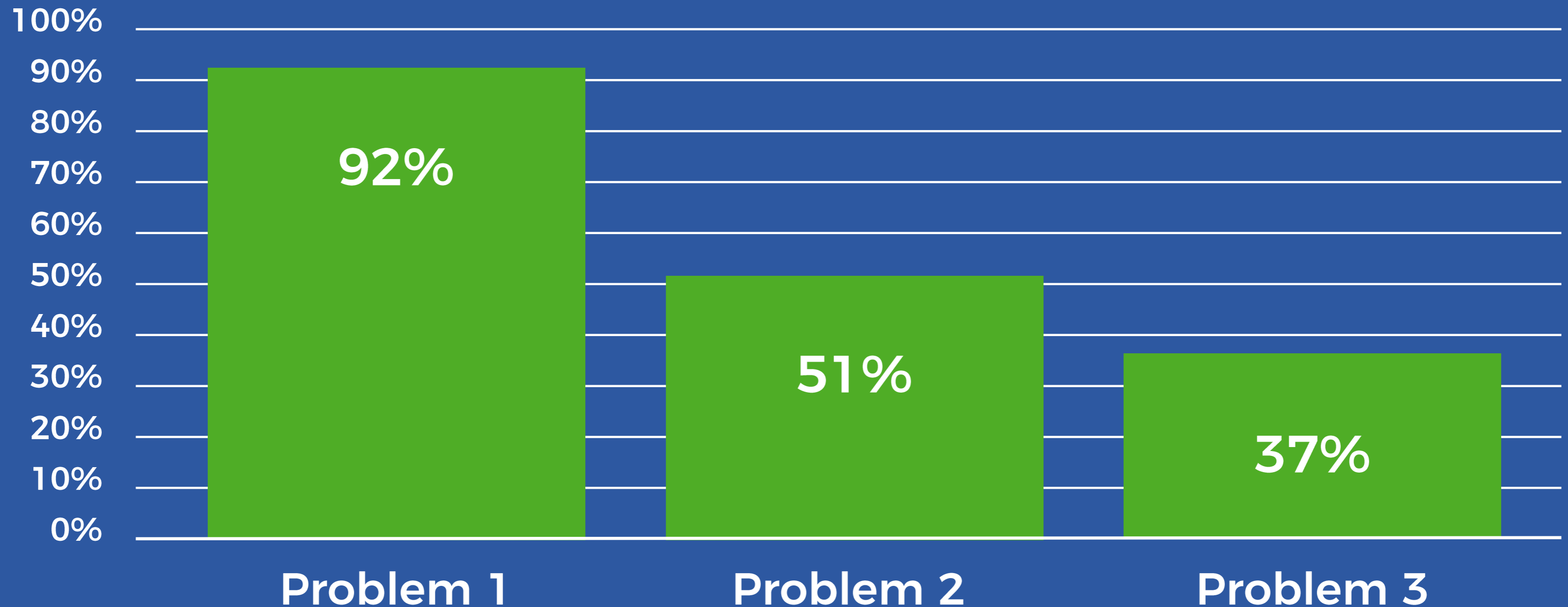
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RETWEETS  
36

LIKES  
54



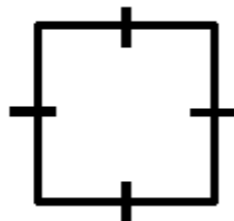
# PROBLEM RESULTS



# Depth of Knowledge Matrix - Secondary Math

Topic	Dividing Fractions	Solving Two-Step Equations	Exponents	Solving Equations with Variables on Both Sides
CCSS Standard(s)	<ul style="list-style-type: none"> <li>6.NS.1</li> </ul>	<ul style="list-style-type: none"> <li>7.EE.4a</li> </ul>	<ul style="list-style-type: none"> <li>8.EE.1</li> </ul>	<ul style="list-style-type: none"> <li>8.EE.8</li> <li>A-REI.3</li> </ul>
DOK 1 Example	Evaluate. $\frac{4}{9} \div \frac{2}{5}$	Solve for x. $2x + 3 = 9$	Evaluate. $3^4$	Solve for x. $3x + 2 = -2x + 4$
DOK 2 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two different pairs of fractions that have a quotient of $\frac{2}{3}$ . $\frac{\square}{\square} \div \frac{\square}{\square} = \frac{2}{3}$	Use the digits 1 to 9, at most one time each, to create two equations: one where x has a positive value and one where x has a negative value. $\square x + \square = \square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two true number sentences. $\square^{\square} = 64$	Use the digits 1 to 9, at most <u>two</u> times each, to fill in the boxes to make an equation with no solutions. $\square x + \square = \square x + \square$
DOK 3 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two fractions that have a quotient that is as close to $\frac{4}{11}$ as possible. $\frac{\square}{\square} \div \frac{\square}{\square}$	Use the digits 1 to 9, at most one time each, to create an equation where x has the greatest possible value. $\square x + \square = \square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a result that has the greatest value possible. $\square^{\square} = \square\square\square$	Use the digits 1 to 9, at most one time each, to fill in the boxes so that the solution is closest to zero. $\square x + \square = \square x + \square$

# Depth of Knowledge Matrix - Secondary Math

Topic	Geometric Proofs	Complex Numbers	Trigonometric Functions	Definite Integral
CCSS Standard(s)	<ul style="list-style-type: none"> <li>G-CO.11</li> </ul>	<ul style="list-style-type: none"> <li>N-CN.2</li> </ul>	<ul style="list-style-type: none"> <li>F-TF.3</li> </ul>	<ul style="list-style-type: none"> <li>N/A</li> </ul>
DOK 1 Example	Add one geometric marking to demonstrate the quadrilateral is a square. 	Multiply the binomials. $(3 + 4i)(2 + 3i)$	Evaluate. $\sin \frac{\pi}{3}$	Solve. $\int_2^6 x^3 dx$
DOK 2 Example	Use exactly 5 geometric markings to show that a quadrilateral is a square.	Use the integers -9 to 9, at most one time each, to fill in the boxes twice: once to make a positive real number product and once to make a negative real number product. $(\square + \square i)(\square + \square i)$	Use the digits 1 to 9, at most one time each, to fill in the boxes and make two true number sentences. $\sin \frac{\square \pi}{\square} = 0$	Use the digits 1 to 9, at most one time each, to fill in the boxes and make a positive and a negative solution. $\int_{\square}^{\square} x^{\square} dx$
DOK 3 Example	What is the least number of geometric markings needed to demonstrate that a quadrilateral is a square?	Use the integers -9 to 9, at most one time each, to fill in the boxes and make a real number product with the greatest value. $(\square + \square i)(\square + \square i)$	Use the digits 1 to 9, at most one time each, so that the function has the greatest possible value. $\sin \frac{\square \pi}{\square} = \frac{\sqrt{\square}}{\square}$	Use the digits 1 to 9, at most one time each, to fill in the boxes and make a solution that is as close to 100 as possible. $\int_{\square}^{\square} x^{\square} dx$

# DOK ONE

$$6.9 + a = 46$$

# DOK TWO

- 11.** Anton walked 8.9 miles of his 13.5-mile goal for this week. Use the equation  $m + 8.9 = 13.5$  to find which path Anton should walk so that he meets his goal for the week.

Path Lengths	
Meadow Path	3.2 miles
Circle Path	4.2 miles
Oak Tree Path	4.6 miles



# DOK THREE

- 14. Reasoning** Kyle bought a movie ticket for \$8.45 and a drink for \$1.80. He had just enough money remaining to buy a large popcorn. How much money did Kyle start with? Write an equation to show your reasoning. © MP.2

Cost of Popcorn	
Small	\$2.85
Medium	\$3.75
Large	\$4.75
Extra Large	\$4.85

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# IMPLEMENTATION

- Open Middle Worksheet

First attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Name: \_\_\_\_\_ Period: \_\_\_\_\_ Date: \_\_\_\_\_

First attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Second attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation

# IMPLEMENTATION

- Open Middle Worksheet
- Classwork
  - Single problem for entire class
  - Extensions menu

### QUESTION #1

Use the digits 1 to 9, at most one time each, to create an equation where  $x$  has the greatest possible value.

$$\square\square + x = \square\square$$

4 points

### QUESTION #2

Solve for  $x$ .

$$3x + 7 = 19$$

1 point

### QUESTION #3

Use the digits 1 to 9, at most one time each, to create two equations: one where  $x$  has a positive value and one where  $x$  has a negative value.

$$\square\square + x = \square\square$$

2 points

### QUESTION #4

Use the digits 1 to 9, at most one time each, to make each equation true.

$$\square + a = \square$$

$$\square - \square = \square$$

## SOLVING EQUATIONS EXTENSION MENU

You must earn at least 12 points by doing the problems of your choice. Circle the questions you

### QUESTION #5

Use the digits 1 to 9, at most one time each, to create an equation where  $x$  has the greatest possible value.

# IMPLEMENTATION

- Open Middle Worksheet
- Classwork
  - Single problem for entire class
  - Extensions menu
- Homework
- Assessments



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# STEP ONE

- **Find a One-Operation Problem**
  - Addition
  - Subtraction
  - **Multiplying**
  - Dividing
  - Exponents (including square root)
  - Trigonometric functions

# ADDING 2-DIGIT NUMBERS

Solve.

$$41 + 36 =$$

# MULTIPLYING FRACTIONS

Solve.

$$\frac{3}{7} \times \frac{2}{9} =$$

**THINKING TIME**

# STEP TWO

- **Go from DOK 1 to DOK 2**
  - **Strategically remove some information from the problem to prevent immediate calculation**
  - **Increase the quantity of solutions needed to increase the need to look for patterns**

# ADDING 2-DIGIT NUMBERS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of two-digit numbers that have a sum of 71.

$$\boxed{\phantom{00}} \boxed{\phantom{00}} + \boxed{\phantom{00}} \boxed{\phantom{00}} = 71$$

# MULTIPLYING FRACTIONS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of fractions that have a product of  $\frac{2}{3}$ .

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \times \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{2}{3}$$



**THINKING TIME**

# STEP THREE

- Go from DOK 2 to DOK 3
  - Introduce the need to optimize the solution by making the greatest or least product / sum / difference / quotient / answer.
  - Another optimization option is make the answer closest to a specific value.

# ADDING 2-DIGIT NUMBERS

Using the digits 1 to 9, at most one time each, fill in the boxes to make the smallest sum.

$$\boxed{\phantom{00}} \boxed{\phantom{00}} + \boxed{\phantom{00}} \boxed{\phantom{00}} = \boxed{\phantom{00}} \boxed{\phantom{00}}$$

# MULTIPLYING FRACTIONS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two fractions that have a product that is as close to  $\frac{4}{11}$  as possible.

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \times \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

**THINKING TIME**

# 3 Steps to Increase Math DOK Levels

## Step 1: Find a One-Operation Problem

- Procedural problems with one operation are easiest to modify.
- Other problems may also be modified but may not be as easy.

Adding 2-Digit Numbers

Solve.

$$41 + 36 = \underline{\quad}$$

Multiplying Fractions

Solve.

$$\frac{3}{7} \times \frac{2}{9} = \underline{\quad}$$

Trigonometry

Solve.

$$\sin \frac{\pi}{3} = \underline{\quad}$$

## Step 2: Go from DOK 1 to DOK 2

- Strategically remove some information from the problem to prevent immediate calculation
- Increase the quantity of solutions needed to increase the need to look for patterns

Adding 2-Digit Numbers

Using the digits 1 to 9, at most one time each, fill in the boxes

Multiplying Fractions

Using the digits 1 to 9, at most one time each, fill in the boxes

Trigonometry

Using the digits 1 to 9, at most one time each, fill in the boxes

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# Open Middle

Challenging math problems worth solving

Home

Kinder ▾

Grade 1 ▾

Grade 2 ▾

Grade 3 ▾

Grade 4 ▾

Grade 5 ▾

Grade 6 ▾

Grade 7 ▾

Grade 8 ▾

High School ▾

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## THE TOP 10 MOST POPULAR PROBLEMS OF 2016

1. Two-Step Equations by Robert Kaplinsky, Daniel Mendivil, Daniel Luevanos, and Robert Kaplinsky
2. Order of Operations by Robert Kaplinsky with answer from Michael Fenton and his students
3. Dot Card Counting by Dan Meyer
4. Rational and Irrational Numbers by Bryan Anderson
5. One Solution, No Solutions, Infinite Solutions by Bryan Anderson
6. Multiplying a Two-Digit Number by a Single-Digit Number by Robert Kaplinsky
7. Exponents and Order of Operations by Zack Miller
8. Converting Between Fractions and Decimals by Robert Kaplinsky
9. Interpreting Percentages by Robert Kaplinsky
10. Two-Step Equations 3 by Erick Lee

## WHAT ARE PEOPLE SAYING ABOUT OPEN MIDDLE?



Brian Marko



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## OPEN MIDDLE WORKSHEET

Download the Open Middle Worksheet (Regular):  
Version 1.2

Download the Open Middle Worksheet (Large):  
Version 1.1

## BROWSE BY DEPTH OF KNOWLEDGE LEVEL

DOK 2: Skills and Concepts

DOK 3: Strategic Thinking

## BROWSE BY COMMON CORE STATE STANDARDS

Kindergarten (10)

Counting & Cardinality (2)

Geometry (2)

Number & Operations in Base Ten (1)

Operations & Algebraic Thinking (5)



## EQUIDISTANT POINTS

Directions: How many points with integer coordinates are 5 units away from  $(-2, 3)$ ?

### Hint

Which methods are available to determine the answer to this problem? What shape is defined by *all* of the points that are 5 units away  $(-2, 3)$ ?

### Answer

12 points:  $(-5, 7)$ ,  $(-7, 3)$ ,  $(-5, -1)$ ,  $(-2, -2)$ ,  $(3, 3)$ ,  $(1, -1)$ ,  $(-2, 8)$ ,  $(1, 7)$ ,  $(2, 6)$ ,  $(-6, -6)$ ,  $(-6, 0)$ , and  $(2, 0)$

Source: [Dylan Kane](#)



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Version 1.1

### BROWSE BY DEPTH OF KNOWLEDGE LEVEL

DOK 2: Skills and Concepts

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Hey @openmiddle fans, we want to hear from you. Why you use our problems



Open Middle

@openmiddle

Hey @openmiddle fans, we want to hear from you. Why do you use our problems with your students? Share your success stories or lessons learned.

RETWEETS

7

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6



2:10 PM - 11 Jan 2017



8



7



6



1



2



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# CALL TO ACTION

Action	Do Now	Start Planning	Yes & No	Don't Do
Incorporate higher DOK problems on assessments.		✓		
Replace all DOK 1 problems with higher DOK problems.				✓
Share these resources with colleagues to make them aware.	✓			
Find problems I can integrate on Open Middle.	✓			
Use the 3 steps process to strengthen existing problems.			✓	

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[@robertkaplinsky](https://www.instagram.com/robertkaplinsky)

