

# CHALLENGING PROBLEMS

# WORTH SOLVING

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# GOALS

WHY DO WE NEED THEM?

WHY ARE THEY DIFFERENT?

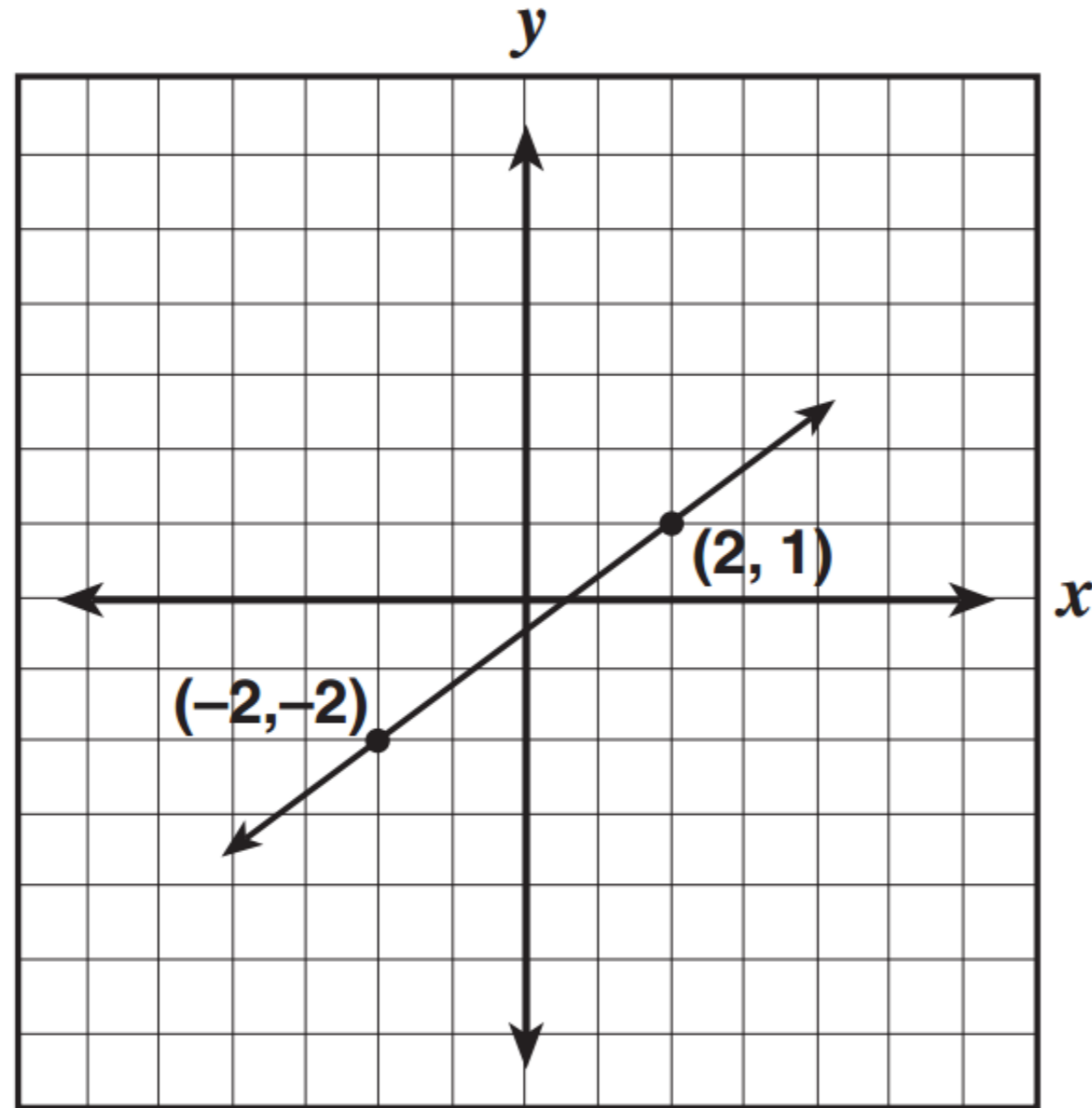
HOW DO YOU IMPLEMENT THEM?

HOW DO YOU CREATE YOUR OWN?

WHERE DO YOU GET OTHERS?

Student Name	ID Number	Perf. Level	Scaled Score	Mathematics Clusters											
				(Clusters where the percent correct is shown in bold represent proficiency for that cluster.)											
				Rational numbers		Exponents, powers, and roots		Quantitative relationships and evaluating expressions		Multi-step problems, graphing, and functions		Measurement and geometry		Statistics, data analysis, and probability	
Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct		
ALYSSA, ARIANNA	117000	ADV	476	13	93%	8	100%	8	80%	14	93%	12	92%	5	100%
ALYSSA, ARIANNA	117000	ADV	464	13	93%	7	88%	8	80%	15	100%	11	85%	5	100%
ALYSSA, ARIANNA	117000	ADV	453	10	71%	8	100%	10	100%	14	93%	11	85%	5	100%
ALYSSA, ARIANNA	117000	ADV	453	13	93%	8	100%	9	90%	12	80%	11	85%	5	100%
ALYSSA, ARIANNA	117000	ADV	444	14	100%	7	88%	8	80%	13	87%	10	77%	5	100%
ALYSSA, ARIANNA	117000	ADV	444	12	86%	8	100%	8	80%	15	100%	10	77%	4	80%
ALYSSA, ARIANNA	117000	ADV	444	13	93%	8	100%	8	80%	14	93%	9	69%	5	100%
ALYSSA, ARIANNA	117000	ADV	435	12	86%	6	75%	9	90%	14	93%	10	77%	5	100%
ALYSSA, ARIANNA	117000	ADV	435	12	86%	6	75%	8	80%	14	93%	11	85%	5	100%
ALYSSA, ARIANNA	117000	ADV	435	13	93%	7	88%	9	90%	12	80%	10	77%	5	100%
ALYSSA, ARIANNA	117000	ADV	427	13	93%	6	75%	9	90%	12	80%	10	77%	5	100%
ALYSSA, ARIANNA	117000	ADV	427	13	93%	7	88%	6	60%	13	87%	11	85%	5	100%
ALYSSA, ARIANNA	117000	ADV	427	14	100%	5	63%	7	70%	14	93%	10	77%	5	100%
ALYSSA, ARIANNA	117000	ADV	421	13	93%	6	75%	6	60%	14	93%	10	77%	5	100%
ALYSSA, ARIANNA	117000	ADV	421	11	79%	5	63%	9	90%	13	87%	11	85%	5	100%
ALYSSA, ARIANNA	117000	ADV	414	12	86%	6	75%	8	80%	11	73%	11	85%	5	100%
ALYSSA, ARIANNA	117000	ADV	414	12	86%	8	100%	8	80%	13	87%	8	62%	4	80%
ALYSSA, ARIANNA	117000	PRO	408	11	79%	6	75%	9	90%	11	73%	10	77%	5	100%
ALYSSA, ARIANNA	117000	PRO	402	12	86%	8	100%	9	90%	8	53%	11	85%	3	60%
ALYSSA, ARIANNA	117000	PRO	402	8	57%	7	88%	8	80%	13	87%	10	77%	5	100%
ALYSSA, ARIANNA	117000	PRO	402	13	93%	6	75%	7	70%	13	87%	8	62%	4	80%
ALYSSA, ARIANNA	117000	PRO	402	11	79%	5	63%	7	70%	11	73%	12	92%	5	100%
ALYSSA, ARIANNA	117000	PRO	402	13	93%	7	88%	9	90%	10	67%	7	54%	5	100%
ALYSSA, ARIANNA	117000	PRO	402	13	93%	7	88%	7	70%	11	73%	8	62%	5	100%
ALYSSA, ARIANNA	117000	PRO	396	10	71%	6	75%	9	90%	14	93%	7	54%	4	80%
ALYSSA, ARIANNA	117000	PRO	396	12	86%	8	100%	6	60%	9	60%	11	85%	4	80%

**52** What is the slope of this line?



- A  $\frac{1}{2}$
- B  $\frac{3}{4}$
- C 1
- D  $\frac{4}{3}$



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		ADV	476	13	93%	8	100%	8	80%	14	93%	12	92%	5	100%
		ADV	464	13	93%	7	88%	8	80%	15	100%	11	85%	5	100%
		ADV	453	10	71%	8	100%	10	100%	14	93%	11	85%	5	100%
						8	100%					11	85%	5	100%
												10	77%	5	100%
												10	77%	4	80%
												9	69%	5	100%
						12	75%					10	77%	5	100%
						12	75%					11	85%	5	100%
		ADV				7	88%					10	77%	5	100%
		ADV				6	75%					10	77%	5	100%
		ADV				7	88%					11	85%	5	100%
		ADV				5	63%					10	77%	5	100%
		ADV	421		93%	6	75%	6	93%		93%	10	77%	5	100%
		ADV	421	11	79%	5	63%	9	90%	13	87%	11	85%	5	100%
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		PRO	402	12	86%	8	100%	9	90%	8	53%	11	85%	3	60%
		PRO	402	8	57%	7	88%	8	80%	13	87%	10	77%	5	100%
		PRO	402	13	93%	6	75%	7	70%	13	87%	8	62%	4	80%
		PRO	402	11	79%	5	63%	7	70%	11	73%	12	92%	5	100%
		PRO	402	13	93%	7	88%	9	90%	10	67%	7	54%	5	100%
		PRO	402	13	93%	7	88%	7	70%	11	73%	8	62%	5	100%
		PRO	396	10	71%	6	75%	9	90%	14	93%	7	54%	4	80%
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# GOALS

WHY DO WE NEED THEM?

WHY ARE THEY DIFFERENT?

HOW DO YOU IMPLEMENT THEM?

HOW DO YOU CREATE YOUR OWN?

WHERE DO YOU GET OTHERS?

# PROBLEM ONE

Solve for  $x$ .

$$21 + x = 70$$



# PROBLEM TWO

Using the digits 1 to 9, at most one time each, create two equations: one where  $x$  has a positive value and one where  $x$  has a negative value.

$$\boxed{\phantom{00}} + x = \boxed{\phantom{00}}$$

# PROBLEM THREE

Using the digits 1 to 9, at most one time each, create an equation where  $x$  has the greatest possible value.

$$\square\square + x = \square\square$$



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MS & HS #MTBoS Ts, please ask your Ss these 3 ?s and put the % who answered correctly here:

[docs.google.com/forms/d/e/1FAI](https://docs.google.com/forms/d/e/1FAI) .... Answers at top of form.

**PROBLEM ONE**  
Solve for x.  
 $21 + x = 7$

**PROBLEM TWO**  
Using the digits 1 to 9, at most one time each, create two equations: one where x has a positive value and one where x has a negative value.  
[ ][ ] + x = [ ][ ]

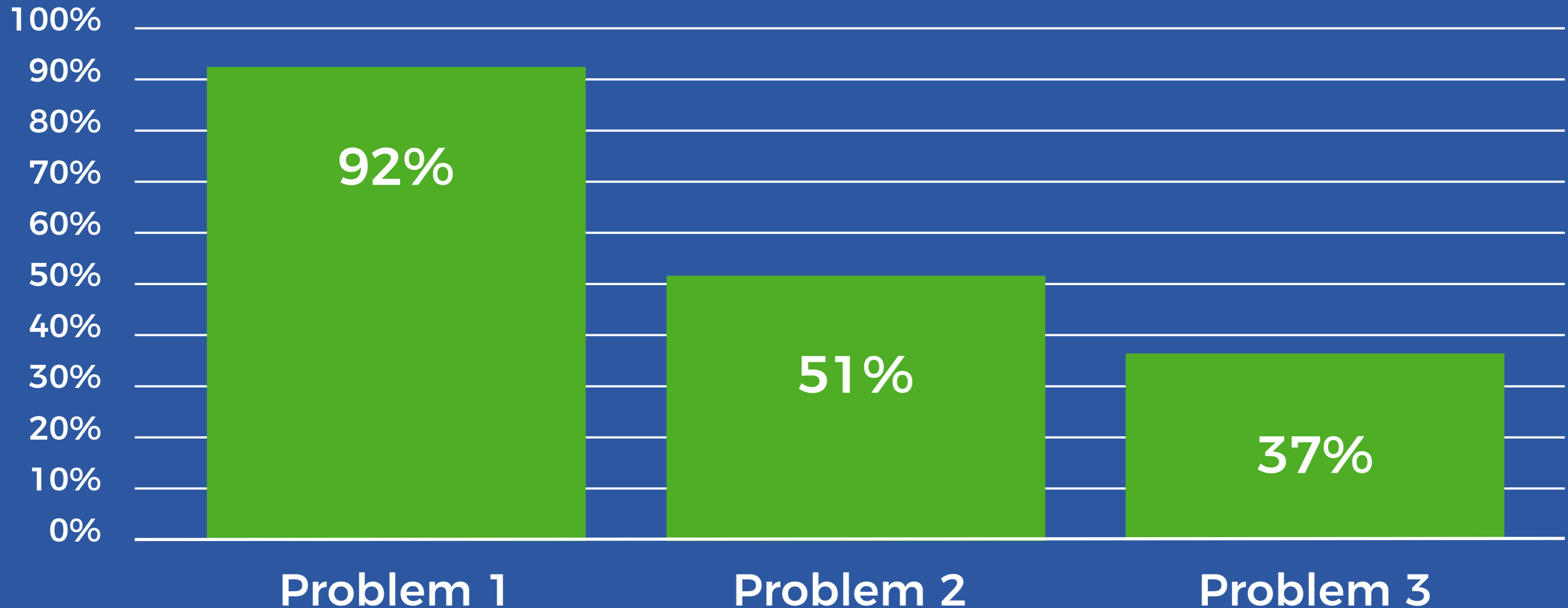
**PROBLEM THREE**  
Using the digits 1 to 9, at most one time each, create an equation where x has the greatest possible value.  
[ ][ ] + x = [ ][ ]

RETWEETS  
36

LIKES  
54



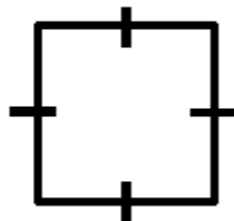
# PROBLEM RESULTS



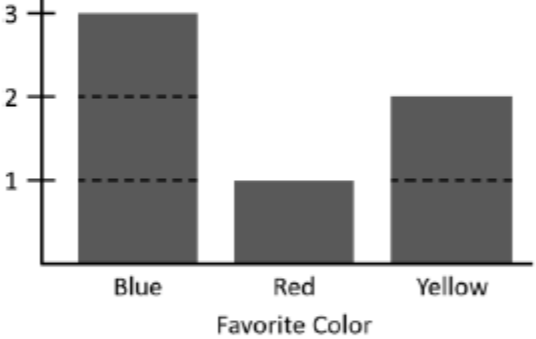
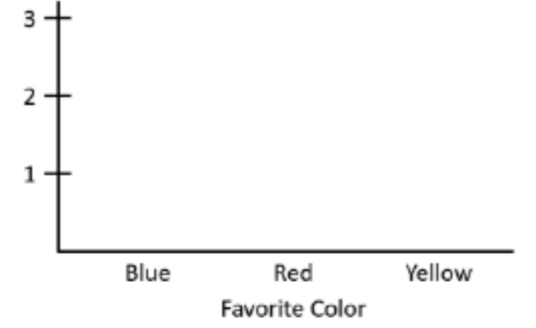
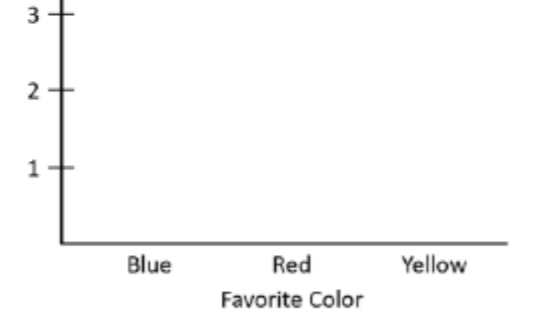
# Depth of Knowledge Matrix - Secondary Math

Topic	Dividing Fractions	Solving Two-Step Equations	Exponents	Solving Equations with Variables on Both Sides
CCSS Standard(s)	<ul style="list-style-type: none"> <li>6.NS.1</li> </ul>	<ul style="list-style-type: none"> <li>7.EE.4a</li> </ul>	<ul style="list-style-type: none"> <li>8.EE.1</li> </ul>	<ul style="list-style-type: none"> <li>8.EE.8</li> <li>A-REI.3</li> </ul>
DOK 1 Example	Evaluate. $\frac{4}{9} \div \frac{2}{5}$	Solve for x. $2x + 3 = 9$	Evaluate. $3^4$	Solve for x. $3x + 2 = -2x + 4$
DOK 2 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two different pairs of fractions that have a quotient of $\frac{2}{3}$ . $\frac{\square}{\square} \div \frac{\square}{\square} = \frac{2}{3}$	Use the digits 1 to 9, at most one time each, to create two equations: one where x has a positive value and one where x has a negative value. $\square x + \square = \square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two true number sentences. $\square^{\square} = 64$	Use the digits 1 to 9, at most <u>two</u> times each, to fill in the boxes to make an equation with no solutions. $\square x + \square = \square x + \square$
DOK 3 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two fractions that have a quotient that is as close to $\frac{4}{11}$ as possible. $\frac{\square}{\square} \div \frac{\square}{\square}$	Use the digits 1 to 9, at most one time each, to create an equation where x has the greatest possible value. $\square x + \square = \square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a result that has the greatest value possible. $\square^{\square} = \square\square\square$	Use the digits 1 to 9, at most one time each, to fill in the boxes so that the solution is closest to zero. $\square x + \square = \square x + \square$

# Depth of Knowledge Matrix - Secondary Math

Topic	Geometric Proofs	Complex Numbers	Trigonometric Functions	Definite Integral
CCSS Standard(s)	<ul style="list-style-type: none"> <li>G-CO.11</li> </ul>	<ul style="list-style-type: none"> <li>N-CN.2</li> </ul>	<ul style="list-style-type: none"> <li>F-TF.3</li> </ul>	<ul style="list-style-type: none"> <li>N/A</li> </ul>
DOK 1 Example	Add one geometric marking to demonstrate the quadrilateral is a square. 	Multiply the binomials. $(3 + 4i)(2 + 3i)$	Evaluate. $\sin \frac{\pi}{3}$	Solve. $\int_2^6 x^3 dx$
DOK 2 Example	Use exactly 5 geometric markings to show that a quadrilateral is a square.	Use the integers -9 to 9, at most one time each, to fill in the boxes twice: once to make a positive real number product and once to make a negative real number product. $(\square + \square i)(\square + \square i)$	Use the digits 1 to 9, at most one time each, to fill in the boxes and make two true number sentences. $\sin \frac{\square \pi}{\square} = 0$	Use the digits 1 to 9, at most one time each, to fill in the boxes and make a positive and a negative solution. $\int_{\square}^{\square} x^{\square} dx$
DOK 3 Example	What is the least number of geometric markings needed to demonstrate that a quadrilateral is a square?	Use the integers -9 to 9, at most one time each, to fill in the boxes and make a real number product with the greatest value. $(\square + \square i)(\square + \square i)$	Use the digits 1 to 9, at most one time each, so that the function has the greatest possible value. $\sin \frac{\square \pi}{\square} = \frac{\sqrt{\square}}{\square}$	Use the digits 1 to 9, at most one time each, to fill in the boxes and make a solution that is as close to 100 as possible. $\int_{\square}^{\square} x^{\square} dx$

# Depth of Knowledge Matrix - Elementary Math


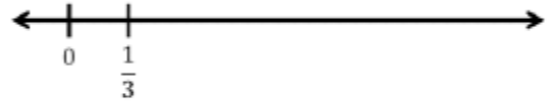

Topic	Adding 1-Digit Numbers (< 5)	Equality	Interpreting Data	Money
CCSS Stand.	<ul style="list-style-type: none"> <li>K.OA.5</li> </ul>	<ul style="list-style-type: none"> <li>1.OA.7</li> </ul>	<ul style="list-style-type: none"> <li>1.MD.4</li> </ul>	<ul style="list-style-type: none"> <li>2.MD.8</li> </ul>
DOK 1 Example	Solve.  $3 + 1 =$	Determine whether the number sentence is true or false.  $4 + 1 = 5 - 2$	How many people were surveyed? 	If you have 1 quarter, 4 dimes, 2 nickels, and 3 pennies, how many cents do you have?
DOK 2 Example	Use the digits 1 to 5, at most one time each, to fill in the boxes to create two true number sentences.  $\square + \square = \square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to create two true number sentences.  $\square + \square = \square - \square$	Make a graph that shows a possible result of 7 students' favorite color. 	Make 72¢ in two different ways with either quarters, dimes, nickels, or pennies.
DOK 3 Example	Use the digits 1 to 5, at most one time each, to fill in the boxes to create a true number sentence with the greatest possible sum.  $\square + \square = \square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to create a true number sentence with the greatest possible value.  $\square + \square = \square - \square$	Make a graph that shows a possible result of 7 students' favorite color with red being the most popular color. 	Make 72¢ using exactly 9 coins that are either quarters, dimes, nickels, or pennies.

# Depth of Knowledge Matrix - Elementary Math

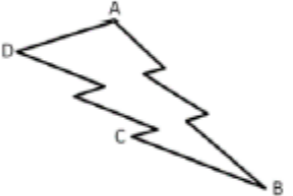
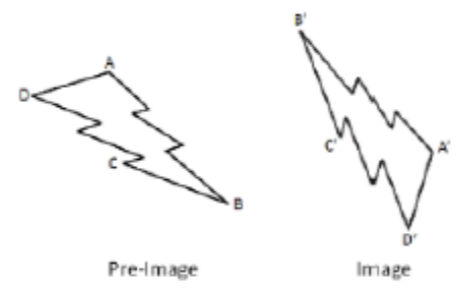
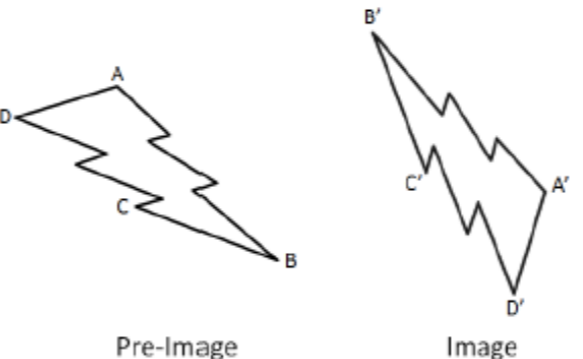
Topic	Subtracting 3-Digit Numbers	Operations with Time	Comparing Fractions	Multiplying Decimals
CCSS Stand.	<ul style="list-style-type: none"> <li>3.NBT.2</li> </ul>	<ul style="list-style-type: none"> <li>3.MD.1</li> </ul>	<ul style="list-style-type: none"> <li>4.NF.2</li> </ul>	<ul style="list-style-type: none"> <li>5.NBT.7</li> </ul>
DOK 1 Example	Solve.  $821 - 357 =$	What time will it be 14 minutes after 1:27 pm?	Place a < or > between the two fractions to make a true number sentence.  $\frac{4}{7}$ $\frac{3}{5}$	Solve.  $3.4 \times 2.5 =$
DOK 2 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two different pairs of three-digit numbers that form a true number sentence.  $\square\square\square - 291 = \square\square\square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a time that is 4:37 pm.  $\square\square$ minutes after $\square:\square\square$ pm	Use the digits 1 to 9, at most one time each, to fill in the boxes to create two different fractions: one that is less than one half and one that is more than one half.  $\frac{\square}{\square} < \frac{1}{2}$ and $\frac{\square}{\square} > \frac{1}{2}$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a true number sentence.  $\square.\square \times 3.2 = \square.\square$
DOK 3 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a difference that is as close to 329 as possible.  $\square\square\square - \square\square\square =$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make the latest possible time.  $\square\square$ minutes after $\square:\square\square$ pm	Use the digits 1 to 9, at most one time each, to fill in the boxes to create a fraction that is as close to 5/11 as possible.  $\frac{\square}{\square}$	Use the digits 1 to 9, at most one time each, so that the product is as close to 50 as possible.  $\square.\square \times \square.\square =$



# Depth of Knowledge Matrix - Elementary & Secondary Math

Topic	Adding Whole Numbers	Money	Fractions on a Number Line	Area and Perimeter	Subtracting Mixed Numbers
CCSS Standard(s)	<ul style="list-style-type: none"> <li>1.NBT.4</li> <li>2.NBT.5</li> </ul>	<ul style="list-style-type: none"> <li>2.MD.8</li> </ul>	<ul style="list-style-type: none"> <li>3.NF.2</li> </ul>	<ul style="list-style-type: none"> <li>3.MD.8</li> <li>4.MD.3</li> </ul>	<ul style="list-style-type: none"> <li>5.NF.1</li> </ul>
DOK 1 Example	<p>Find the sum.</p> $44 + 27 =$	<p>If you have 2 dimes and 3 pennies, how many cents do you have?</p>	<p>Which point is located at <math>\frac{7}{12}</math> below?</p> 	<p>Find the perimeter of a rectangle that measures 4 units by 8 units.</p>	<p>Find the difference.</p> $5\frac{1}{2} - 4\frac{2}{3} =$
DOK 2 Example	<p>Fill in the boxes below using the whole numbers 1 through 9, no more than one time each, so that you make a true equation.</p> $\square\square + 53 = \square\square$	<p>Make 47¢ in three different ways with either quarters, dimes, nickels, or pennies.</p>	<p>Label the point where <math>\frac{3}{4}</math> belongs on the number line below. Be as precise as possible.</p> 	<p>List the measurements of three different rectangles that each has a perimeter of 20 units.</p>	<p>Create three different mixed numbers that will make the equation true by using the whole numbers 1 through 9, no more than one time each. You may reuse the same whole numbers for each of the three mixed numbers.</p> $5\frac{4}{5} - \square\frac{\square}{\square} = 3\frac{1}{20}$
DOK 3 Example	<p>Make the largest sum by filling in the boxes below using the whole numbers 1 through 9, no more than one time each.</p> $\square\square + \square\square =$	<p>Make 47¢ using exactly 6 coins with either quarters, dimes, nickels, or pennies.</p>	<p>Create 5 fractions using the whole numbers 0 through 9, exactly one time each as numerators and denominators, and place them all on a number line.</p>	<p>What is the greatest area you can make with a rectangle that has a perimeter of 24 units?</p>	<p>Make the smallest difference by filling in the boxes below using the whole numbers 1 through 9, no more than one time each.</p> 

# Depth of Knowledge Matrix - Elementary & Secondary Math

Topic	Surface Area and Volume	Probability	Transformations	Factoring Quadratics	Quadratics in Vertex Form
CCSS Standard(s)	<ul style="list-style-type: none"> <li>6.G.4</li> <li>7.G.6</li> </ul>	<ul style="list-style-type: none"> <li>7.SP.5</li> <li>7.SP.7</li> </ul>	<ul style="list-style-type: none"> <li>8.G.1</li> <li>G-CO.5</li> </ul>	<ul style="list-style-type: none"> <li>A-SSE.3a</li> </ul>	<ul style="list-style-type: none"> <li>F-IF.7a</li> </ul>
DOK 1 Example	Find the surface area of a rectangular prism that measures 3 units by 4 units by 5 units.	What is the probability of rolling a sum of 5 using two 6-sided dice?	Rotate the image below 90° counterclockwise about point D and reflect it across a horizontal line. 	Find the factors: $2x^2 + 7x + 3$	Find the roots and maximum of the quadratic equation below. $y = -3(x - 4)^2 - 3$
DOK 2 Example	List the measurements of three different rectangular prisms that each have a surface area of 20 square units.	What value(s) have a 1/12 probability of being rolled as the sum of two 6-sided dice?	List three sequences of transformations that take pre-image ABCD to image A'B'C'D'. 	Find three different integers to put in the blank that will make the quadratic expression factorable. $x^2 + \_\_x + 4$	Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values.
DOK 3 Example	What is the greatest volume you can make with a rectangular prism that has a surface area of 20 square units?	Fill in the blanks to complete this sentence using the whole numbers 1 through 9, no more than one time each.  Rolling a sum of $\_\_$ on two $\_\_$ -sided dice is the same probability as rolling a sum of $\_\_$ on two $\_\_$ -sided dice.	What is the fewest number of transformations needed to take pre-image ABCD to image A'B'C'D'? 	Fill the blank by finding the largest and smallest integers that will make the quadratic expression factorable. $2x^2 + 3x + \_\_$	Create a quadratic equation with the largest maximum value using the whole numbers 1 through 9, no more than one time each. $y = -\square(x - \square)^2 + \square$

# DOK ONE

$$6.9 + a = 46$$

# DOK TWO

- 11.** Anton walked 8.9 miles of his 13.5-mile goal for this week. Use the equation  $m + 8.9 = 13.5$  to find which path Anton should walk so that he meets his goal for the week.

Path Lengths	
Meadow Path	3.2 miles
Circle Path	4.2 miles
Oak Tree Path	4.6 miles

# DOK THREE

- 14. Reasoning** Kyle bought a movie ticket for \$8.45 and a drink for \$1.80. He had just enough money remaining to buy a large popcorn. How much money did Kyle start with? Write an equation to show your reasoning. © MP.2

Cost of Popcorn	
Small	\$2.85
Medium	\$3.75
Large	\$4.75
Extra Large	\$4.85

# GOALS

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# IMPLEMENTATION

- Open Middle Worksheet

First attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?



Name: \_\_\_\_\_ Period: \_\_\_\_\_ Date: \_\_\_\_\_

First attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Second attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation

# IMPLEMENTATION

- Open Middle Worksheet
- Classwork
  - Single problem for entire class
  - Extensions menu

### QUESTION #1

Use the digits 1 to 9, at most one time each, to create an equation where  $x$  has the greatest possible value.

$$\square\square + x = \square\square$$

4 points

### QUESTION #2

Solve for  $x$ .

$$3x + 7 = 19$$

1 point

### QUESTION #3

Use the digits 1 to 9, at most one time each, to create two equations: one where  $x$  has a positive value and one where  $x$  has a negative value.

$$\square\square + x = \square\square$$

2 points

### QUESTION #4

Use the digits 1 to 9, at most one time each, to make each equation true.

$$\square + a = \square$$

$$\square - \square = \square$$

## SOLVING EQUATIONS EXTENSION MENU

You must earn at least 12 points by doing the problems of your choice. Circle the questions you

### QUESTION #5

Use the digits 1 to 9, at most one time each, to create an equation where  $x$  has the greatest possible value.

# IMPLEMENTATION

- Open Middle Worksheet
- Classwork
  - Single problem for entire class
  - Extensions menu
- Homework
- Assessments

# GOALS

WHY DO WE NEED THEM?

WHY ARE THEY DIFFERENT?

HOW DO YOU IMPLEMENT THEM?

HOW DO YOU CREATE YOUR OWN?

WHERE DO YOU GET OTHERS?

# STEP ONE

- Find a One-Operation Problem
  - Addition
  - Subtraction
  - Multiplying
  - Dividing
  - Exponents (including square root)
  - Trigonometric functions

# ADDING 2-DIGIT NUMBERS

Solve.

$$41 + 36 =$$

# MULTIPLYING FRACTIONS

Solve.

$$\frac{3}{7} \times \frac{2}{9} =$$



**THINKING TIME**

# STEP TWO

- Go from DOK 1 to DOK 2
  - Strategically remove some information from the problem to prevent immediate calculation
  - Increase the quantity of solutions needed to increase the need to look for patterns

# ADDING 2-DIGIT NUMBERS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of two-digit numbers that have a sum of 71.

$$\boxed{\phantom{00}} \boxed{\phantom{00}} + \boxed{\phantom{00}} \boxed{\phantom{00}} = 71$$

# MULTIPLYING FRACTIONS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of fractions that have a product of  $\frac{2}{3}$ .

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \times \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{2}{3}$$

# THINKING TIME

- Go from DOK 1 to DOK 2
  - Strategically remove some information from the problem to prevent immediate calculation
  - Increase the quantity of solutions needed to increase the need to look for patterns

# STEP THREE

- Go from DOK 2 to DOK 3
  - Introduce the need to optimize the solution by making the greatest or least product / sum / difference / quotient / answer.
  - Another optimization option is make the answer closest to a specific value.

# ADDING 2-DIGIT NUMBERS

Using the digits 1 to 9, at most one time each, fill in the boxes to make the smallest sum.

$$\boxed{\phantom{00}} \boxed{\phantom{00}} + \boxed{\phantom{00}} \boxed{\phantom{00}} = \boxed{\phantom{00}} \boxed{\phantom{00}}$$

# MULTIPLYING FRACTIONS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two fractions that have a product that is as close to  $\frac{4}{11}$  as possible.

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} \times \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$



# THINKING TIME

- Go from DOK 2 to DOK 3
  - Introduce the need to optimize the solution by making the greatest or least product / sum / difference / quotient / answer.
  - Another optimization option is make the answer closest to a specific value.

# 3 Steps to Increase Math DOK Levels

## Step 1: Find a One-Operation Problem

- Procedural problems with one operation are easiest to modify.
- Other problems may also be modified but may not be as easy.

Adding 2-Digit Numbers

Solve.

$$41 + 36 = \underline{\quad}$$

Multiplying Fractions

Solve.

$$\frac{3}{7} \times \frac{2}{9} = \underline{\quad}$$

Trigonometry

Solve.

$$\sin \frac{\pi}{3} = \underline{\quad}$$

## Step 2: Go from DOK 1 to DOK 2

- Strategically remove some information from the problem to prevent immediate calculation
- Increase the quantity of solutions needed to increase the need to look for patterns

Adding 2-Digit Numbers

Using the digits 1 to 9, at most one time each, fill in the boxes

Multiplying Fractions

Using the digits 1 to 9, at most one time each, fill in the boxes

Trigonometry

Using the digits 1 to 9, at most one time each, fill in the boxes

# TWO PROBLEMS

## Problem One

Find the sum:  $37 + 27$

## Problem Two

Using the digits 1 to 9, at most one time each, fill in the boxes to make the smallest sum.

$$\boxed{\phantom{00}} + \boxed{\phantom{00}} = \boxed{\phantom{00}}$$

# Problem Drives Sense Making

Yes

No

Teacher Drives Sense Making

Yes

Ideal

Valuable

No

Valuable

Not ideal

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# Open Middle

Challenging math problems worth solving

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Grade 1 ▾

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Grade 3 ▾

Grade 4 ▾

Grade 5 ▾

Grade 6 ▾

Grade 7 ▾

Grade 8 ▾

High School ▾

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## THE TOP 10 MOST POPULAR PROBLEMS OF 2016

1. Two-Step Equations by Robert Kaplinsky, Daniel Mendivil, Daniel Luevanos, and Robert Kaplinsky
2. Order of Operations by Robert Kaplinsky with answer from Michael Fenton and his students
3. Dot Card Counting by Dan Meyer
4. Rational and Irrational Numbers by Bryan Anderson
5. One Solution, No Solutions, Infinite Solutions by Bryan Anderson
6. Multiplying a Two-Digit Number by a Single-Digit Number by Robert Kaplinsky
7. Exponents and Order of Operations by Zack Miller
8. Converting Between Fractions and Decimals by Robert Kaplinsky
9. Interpreting Percentages by Robert Kaplinsky
10. Two-Step Equations 3 by Erick Lee

## WHAT ARE PEOPLE SAYING ABOUT OPEN MIDDLE?



Brian Mark



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## OPEN MIDDLE WORKSHEET

Download the Open Middle Worksheet (Regular):  
Version 1.2

Download the Open Middle Worksheet (Large):  
Version 1.1

## BROWSE BY DEPTH OF KNOWLEDGE LEVEL

DOK 2: Skills and Concepts

DOK 3: Strategic Thinking

## BROWSE BY COMMON CORE STATE STANDARDS

Kindergarten (10)

Counting & Cardinality (2)

Geometry (2)

Number & Operations in Base Ten (1)

Operations & Algebraic Thinking (5)

## EQUIDISTANT POINTS

Directions: How many points with integer coordinates are 5 units away from  $(-2, 3)$ ?

### Hint

Which methods are available to determine the answer to this problem? What shape is defined by *all* of the points that are 5 units away  $(-2, 3)$ ?

### Answer

12 points:  $(-5, 7)$ ,  $(-7, 3)$ ,  $(-5, -1)$ ,  $(-2, -2)$ ,  $(3, 3)$ ,  $(1, -1)$ ,  $(-2, 8)$ ,  $(1, 7)$ ,  $(2, 6)$ ,  $(-6, -6)$ ,  $(-6, 0)$ , and  $(2, 0)$

Source: [Dylan Kane](#)



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8



7



6



1



2





# GOALS

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# CALL TO ACTION

Action	Do Now	Start Planning	Yes & No	Don't Do
Incorporate higher DOK problems on assessments.		✓		
Replace all DOK 1 problems with higher DOK problems.				✓
Share these resources with colleagues to make them aware.	✓			
Find problems I can integrate on Open Middle.	✓			
Use the 3 steps process to strengthen existing problems.			✓	

# CHALLENGING PROBLEMS

# WORTH SOLVING

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