CHALLENGING PROBLEMS WORTH SOLVING

ROBERT KAPLINSKY

robert@robertkaplinsky.com

robertkaplinsky.com

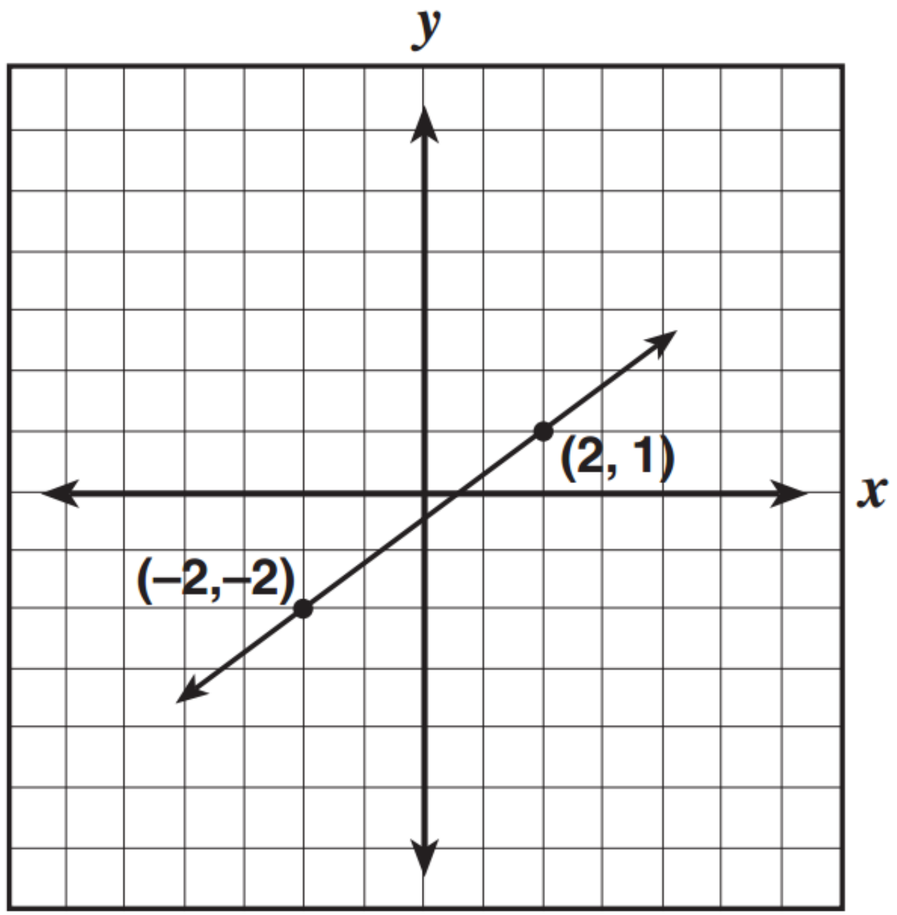
@robertkaplinsky

GOALS

- ❑ WHY DO WE NEED THEM?
- ❑ WHY ARE THEY DIFFERENT?
- ❑ HOW DO YOU IMPLEMENT THEM?
- ❑ HOW DO YOU CREATE YOUR OWN?
- ❑ WHERE DO YOU GET OTHERS?

Student Name	ID Number	Perf. Level	Scaled Score	Mathematics Clusters (Clusters where the percent correct is shown in bold represent proficiency for that cluster.)											
				Rational numbers		Exponents, powers, and roots		Quantitative relationships and evaluating expressions		Multi-step problems, graphing, and functions		Measurement and geometry		Statistics, data analysis, and probability	
				Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct
[redacted]	[redacted]	ADV	476	13	93%	8	100%	8	80%	14	93%	12	92%	5	100%
[redacted]	[redacted]	ADV	464	13	93%	7	88%	8	80%	15	100%	11	85%	5	100%
[redacted]	[redacted]	ADV	453	10	71%	8	100%	10	100%	14	93%	11	85%	5	100%
[redacted]	[redacted]	ADV	453	13	93%	8	100%	9	90%	12	80%	11	85%	5	100%
[redacted]	[redacted]	ADV	444	14	100%	7	88%	8	80%	13	87%	10	77%	5	100%
[redacted]	[redacted]	ADV	444	12	86%	8	100%	8	80%	15	100%	10	77%	4	80%
[redacted]	[redacted]	ADV	444	13	93%	8	100%	8	80%	14	93%	9	69%	5	100%
[redacted]	[redacted]	ADV	435	12	86%	6	75%	9	90%	14	93%	10	77%	5	100%
[redacted]	[redacted]	ADV	435	12	86%	6	75%	8	80%	14	93%	11	85%	5	100%
[redacted]	[redacted]	ADV	435	13	93%	7	88%	9	90%	12	80%	10	77%	5	100%
[redacted]	[redacted]	ADV	427	13	93%	6	75%	9	90%	12	80%	10	77%	5	100%
[redacted]	[redacted]	ADV	427	13	93%	7	88%	6	60%	13	87%	11	85%	5	100%
[redacted]	[redacted]	ADV	427	14	100%	5	63%	7	70%	14	93%	10	77%	5	100%
[redacted]	[redacted]	ADV	421	13	93%	6	75%	6	60%	14	93%	10	77%	5	100%
[redacted]	[redacted]	ADV	421	11	79%	5	63%	9	90%	13	87%	11	85%	5	100%
[redacted]	[redacted]	ADV	414	12	86%	6	75%	8	80%	11	73%	11	85%	5	100%
[redacted]	[redacted]	ADV	414	12	86%	8	100%	8	80%	13	87%	8	62%	4	80%
[redacted]	[redacted]	PRO	408	11	79%	6	75%	9	90%	11	73%	10	77%	5	100%
[redacted]	[redacted]	PRO	402	12	86%	8	100%	9	90%	8	53%	11	85%	3	60%
[redacted]	[redacted]	PRO	402	8	57%	7	88%	8	80%	13	87%	10	77%	5	100%
[redacted]	[redacted]	PRO	402	13	93%	6	75%	7	70%	13	87%	8	62%	4	80%
[redacted]	[redacted]	PRO	402	11	79%	5	63%	7	70%	11	73%	12	92%	5	100%
[redacted]	[redacted]	PRO	402	13	93%	7	88%	9	90%	10	67%	7	54%	5	100%
[redacted]	[redacted]	PRO	402	13	93%	7	88%	7	70%	11	73%	8	62%	5	100%
[redacted]	[redacted]	PRO	396	10	71%	6	75%	9	90%	14	93%	7	54%	4	80%
[redacted]	[redacted]	PRO	396	12	86%	8	100%	6	60%	9	60%	11	85%	4	80%

52 What is the slope of this line?



A $\frac{1}{2}$

B $\frac{3}{4}$

C 1

D $\frac{4}{3}$

Source: California Released Test Questions (7th Grade Math)


X-RAY
VISION

Student Name	ID Number	Perf. Level	Scaled Score	Mathematics Clusters (Clusters where the percent correct is shown in bold represent proficiency for that cluster.)											
				Rational numbers		Exponents, powers, and roots		Quantitative relationships and evaluating expressions		Multi-step problems, graphing, and functions		Measurement and geometry		Statistics, data analysis, and probability	
				Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct	Number Correct	Percent Correct
[redacted]	[redacted]	ADV	476	13	93%	8	100%	8	80%	14	93%	12	92%	5	100%
[redacted]	[redacted]	ADV	464	13	93%	7	88%	8	80%	15	100%	11	85%	5	100%
[redacted]	[redacted]	ADV	453	10	71%	8	100%	10	100%	14	93%	11	85%	5	100%
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	8	100%	[redacted]	[redacted]	[redacted]	[redacted]	11	85%	5	100%
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	10	77%	5	100%
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	10	77%	4	80%
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	1[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	9	69%	5	100%
[redacted]	[redacted]	[redacted]	[redacted]	12	[redacted]	75[redacted]	[redacted]	[redacted]	[redacted]0%	[redacted]	[redacted]	10	77%	5	100%
[redacted]	[redacted]	[redacted]	[redacted]	12	[redacted]	75[redacted]	[redacted]	[redacted]	[redacted]0%	[redacted]	[redacted]	11	85%	5	100%
[redacted]	[redacted]	A[redacted]	[redacted]	[redacted]	[redacted]	[redacted]7	88%	[redacted]	[redacted]	[redacted]	[redacted]%	10	77%	5	100%
[redacted]	[redacted]	A[redacted]	[redacted]	[redacted]	[redacted]	6	75%	[redacted]	[redacted]	[redacted]	[redacted]%	10	77%	5	100%
[redacted]	[redacted]	AD[redacted]	[redacted]	[redacted]	[redacted]	7	88%	[redacted]	[redacted]	[redacted]	[redacted]7%	11	85%	5	100%
[redacted]	[redacted]	ADV	[redacted]	[redacted]	[redacted]	5	63%	[redacted]	[redacted]	[redacted]	93%	10	77%	5	100%
[redacted]	[redacted]	ADV	42[redacted]	[redacted]	93%	6	75%	6	[redacted]	[redacted]	93%	10	77%	5	100%
[redacted]	[redacted]	ADV	421	11	79%	5	63%	9	90%	13	87%	11	85%	5	100%
[redacted]	[redacted]	ADV	414	12	86%	6	75%	8	80%	11	73%	11	85%	5	100%
[redacted]	[redacted]	ADV	414	12	86%	8	100%	8	80%	13	87%	8	62%	4	80%
[redacted]	[redacted]	PRO	408	11	79%	6	75%	9	90%	11	73%	10	77%	5	100%
[redacted]	[redacted]	PRO	402	12	86%	8	100%	9	90%	8	53%	11	85%	3	60%
[redacted]	[redacted]	PRO	402	8	57%	7	88%	8	80%	13	87%	10	77%	5	100%
[redacted]	[redacted]	PRO	402	13	93%	6	75%	7	70%	13	87%	8	62%	4	80%
[redacted]	[redacted]	PRO	402	11	79%	5	63%	7	70%	11	73%	12	92%	5	100%
[redacted]	[redacted]	PRO	402	13	93%	7	88%	9	90%	10	67%	7	54%	5	100%
[redacted]	[redacted]	PRO	402	13	93%	7	88%	7	70%	11	73%	8	62%	5	100%
[redacted]	[redacted]	PRO	396	10	71%	6	75%	9	90%	14	93%	7	54%	4	80%
[redacted]	[redacted]	PRO	396	12	86%	8	100%	6	60%	9	60%	11	85%	4	80%

GOALS

- [x] WHY DO WE NEED THEM?
- [ ] WHY ARE THEY DIFFERENT?
- [ ] HOW DO YOU IMPLEMENT THEM?
- [ ] HOW DO YOU CREATE YOUR OWN?
- [ ] WHERE DO YOU GET OTHERS?

PROBLEM ONE

Solve.

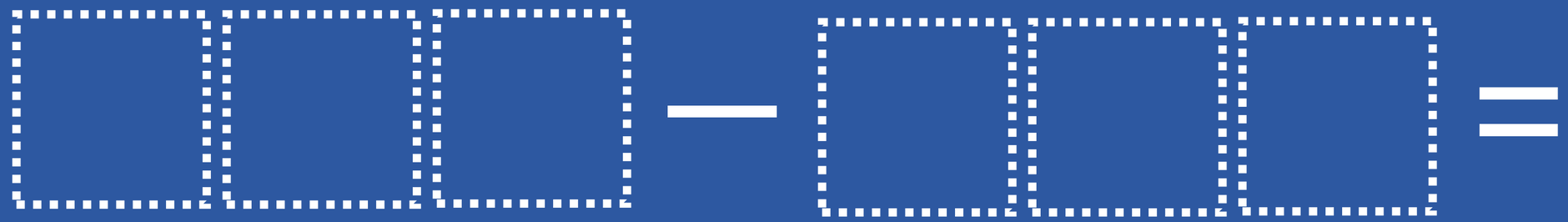
$$812 - 357 =$$

PROBLEM TWO

Use the digits 1 to 9, at most one time each, to fill in the boxes to make two sets of three-digit numbers that form a true number sentence. You can reuse numbers for each set.

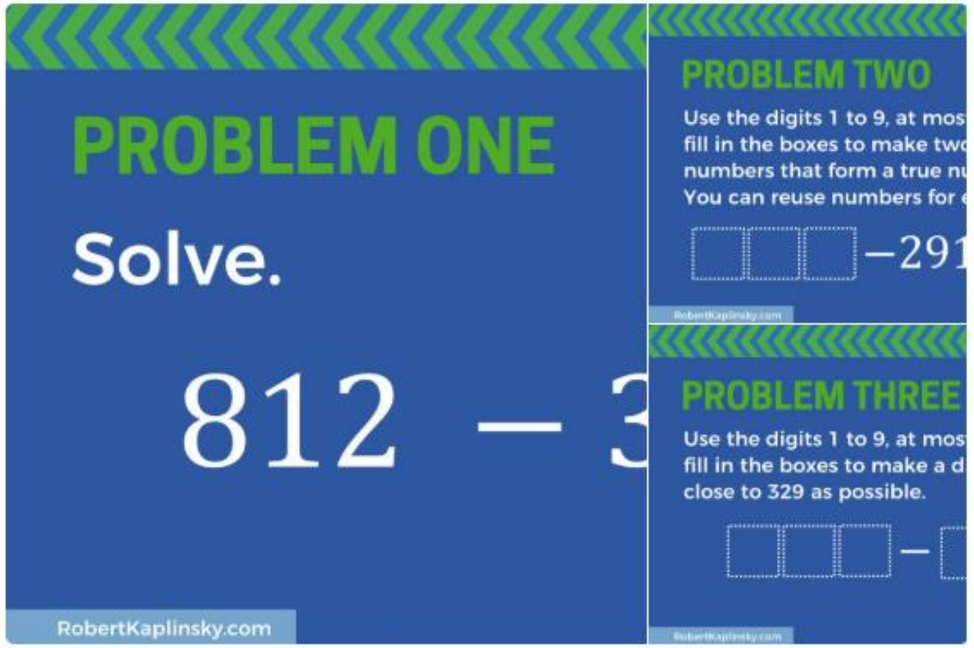
PROBLEM THREE

Use the digits 1 to 9, at most one time each, to fill in the boxes to make a difference that is as close to 329 as possible.



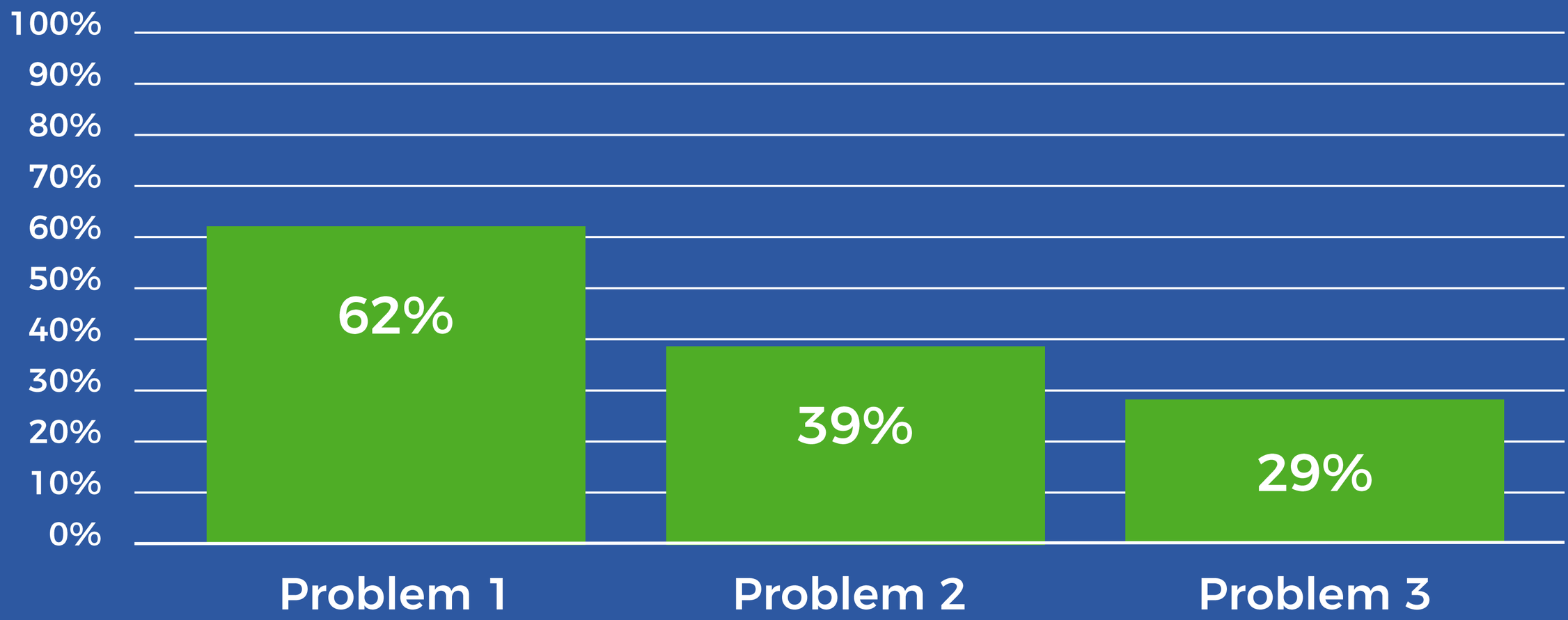
Robert Kaplinsky
@robertkaplinsky

Hey 3rd grade teachers, I need your help. Please ask your students these 3 questions and then let me know what percentage of them got the problems correct using this form. Thanks for sending this to your 3rd grade teacher friends too! goo.gl/forms/xZ5Ebknt… #MTBoS #iteachmath

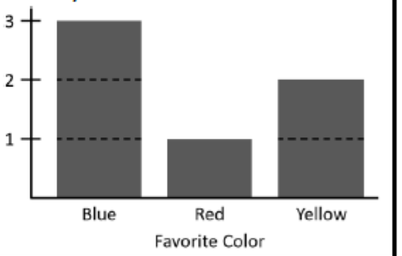
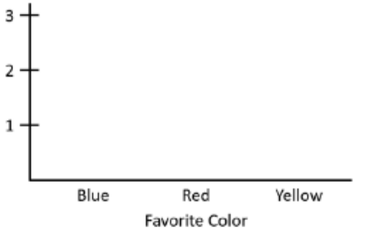
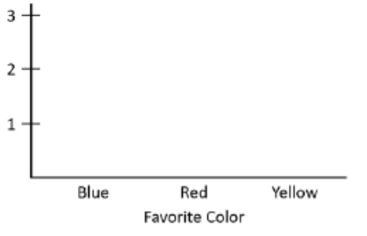


10:45 AM - 22 May 2018

PROBLEM RESULTS



Depth of Knowledge Matrix - Elementary Math

Topic	Adding 1-Digit Numbers (< 5)	Equality	Interpreting Data	Money
CCSS Stand.	• K.OA.5	• 1.OA.7	• 1.MD.4	• 2.MD.8
DOK 1 Example	Solve. $3 + 1 =$	Determine whether the number sentence is true or false. $4 + 1 = 5 - 2$	How many people were surveyed? 	If you have 1 quarter, 4 dimes, 2 nickels, and 3 pennies, how many cents do you have?
DOK 2 Example	Use the digits 1 to 5, at most one time each, to fill in the boxes to create two true number sentences. $\square + \square = \square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to create two true number sentences. $\square + \square = \square - \square$	Make a graph that shows a possible result of 7 students' favorite color. 	Make 72¢ in two different ways with either quarters, dimes, nickels, or pennies.
DOK 3 Example	Use the digits 1 to 5, at most one time each, to fill in the boxes to create a true number sentences with the greatest possible sum. $\square + \square = \square$	Use the digits 1 to 9, at most one time each, to fill in the boxes to create a true number sentence with the greatest possible value. $\square + \square = \square - \square$	Make a graph that shows a possible result of 7 students' favorite color with red being the most popular color. 	Make 72¢ using exactly 9 coins that are either quarters, dimes, nickels, or pennies.

Version 1.2

Depth of Knowledge Matrix - Elementary Math

Topic	Subtracting 3-Digit Numbers	Operations with Time	Comparing Fractions	Multiplying Decimals
CCSS Stand.	• 3.NBT.2	• 3.MD.1	• 4.NF.2	• 5.NBT.7
DOK 1 Example	Solve. $821 - 357 =$	What time will it be 14 minutes after 1:27 pm?	Place a < or > between the two fractions to make a true number sentence. $\frac{4}{7} \quad \frac{3}{5}$	Solve. $3.4 \times 2.5 =$
DOK 2 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make two different pairs of three-digit numbers that form a true number sentence. □□□-291=□□□	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a time that is 4:37 pm. □□ minutes after □:□□ pm	Use the digits 1 to 9, at most one time each, to fill in the boxes to create two different fractions: one that is less than one half and one that is more than one half. $\frac{\square}{\square} < \frac{1}{2}$ and $\frac{\square}{\square} > \frac{1}{2}$	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a true number sentence. □.□ × 3.2=□.□
DOK 3 Example	Use the digits 1 to 9, at most one time each, to fill in the boxes to make a difference that is as close to 329 as possible. □□□-□□□=	Use the digits 1 to 9, at most one time each, to fill in the boxes to make the latest possible time. □□ minutes after □:□□ pm	Use the digits 1 to 9, at most one time each, to fill in the boxes to create a fraction that is as close to 5/11 as possible. $\frac{\square}{\square}$	Use the digits 1 to 9, at most one time each, so that the product is as close to 50 as possible. □.□ × □.□ =

GOALS

- [x] WHY DO WE NEED THEM?
- [x] WHY ARE THEY DIFFERENT?
- [ ] HOW DO YOU IMPLEMENT THEM?
- [ ] HOW DO YOU CREATE YOUR OWN?
- [ ] WHERE DO YOU GET OTHERS?

IMPLEMENTATION

- Open Middle Worksheet

First attempt: Points: _____/2 attempt _____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Name: _____ Period: _____ Date: _____

First attempt: Points: ____/2 attempt ____/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Second attempt: Points: ____/2 attempt ____/2 explanation

IMPLEMENTATION

- Open Middle Worksheet
- Classwork
  - Single problem for entire class
  - Extensions menu

QUESTION #1

Use the digits 1 to 9, at most one time each, to create an equation where x has the greatest possible value.

$$\square\square + x = \square\square$$

4 points

QUESTION #2

Solve for x.

$$3x + 7 = 19$$

1 point

QUESTION #3

Use the digits 1 to 9, at most one time each, to create two equations: one where x has a positive value and one where x has a negative value.

$$\square\square + x = \square\square$$

2 points

QUESTION #4

Use the digits 1 to 9, at most one time each, to make each equation true.

$$\square + a = \square$$

$$\square - b = \square$$

3 points

SOLVING EQUATIONS EXTENSION MENU

You must earn <u>at least 12 points</u> by doing the problems of your choice. Circle the questions you

QUESTION #5

Use the digits 1 to 9, at most one time each, to create an equation where x has the greatest possible value.

IMPLEMENTATION

- Open Middle Worksheet
- Classwork
  - Single problem for entire class
  - Extensions menu
- Homework
- Assessments

GOALS

- [x] WHY DO WE NEED THEM?
- [x] WHY ARE THEY DIFFERENT?
- [x] HOW DO YOU IMPLEMENT THEM?
- [ ] HOW DO YOU CREATE YOUR OWN?
- [ ] WHERE DO YOU GET OTHERS?

STEP ONE

- Find a One-Operation Problem
  - Addition
  - Subtraction
  - Multiplying
  - Dividing
  - Exponents (including square root)
  - Trigonometric functions

ADDING 2-DIGIT NUMBERS

Solve.

$$41 + 36 =$$

MULTIPLYING FRACTIONS

Solve.

$$\frac{3}{7} \times \frac{2}{9} =$$

THINKING TIME

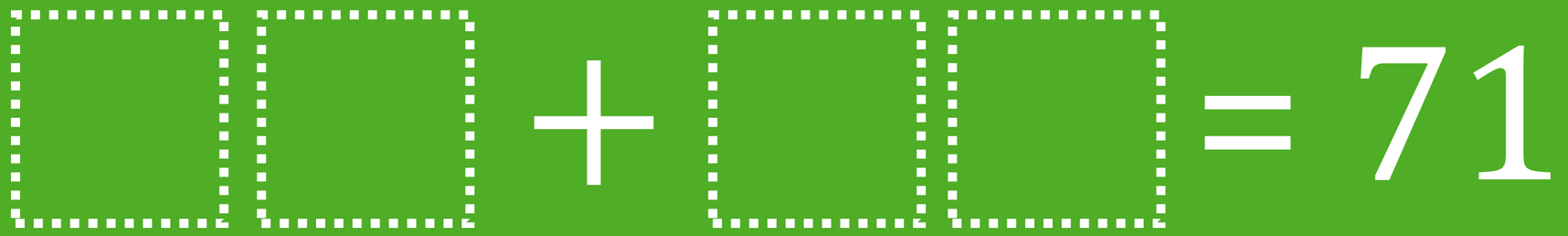
RobertKaplinsky.com

STEP TWO

- Go from DOK 1 to DOK 2
  - Strategically remove some information from the problem to prevent immediate calculation
  - Increase the quantity of solutions needed to increase the need to look for patterns

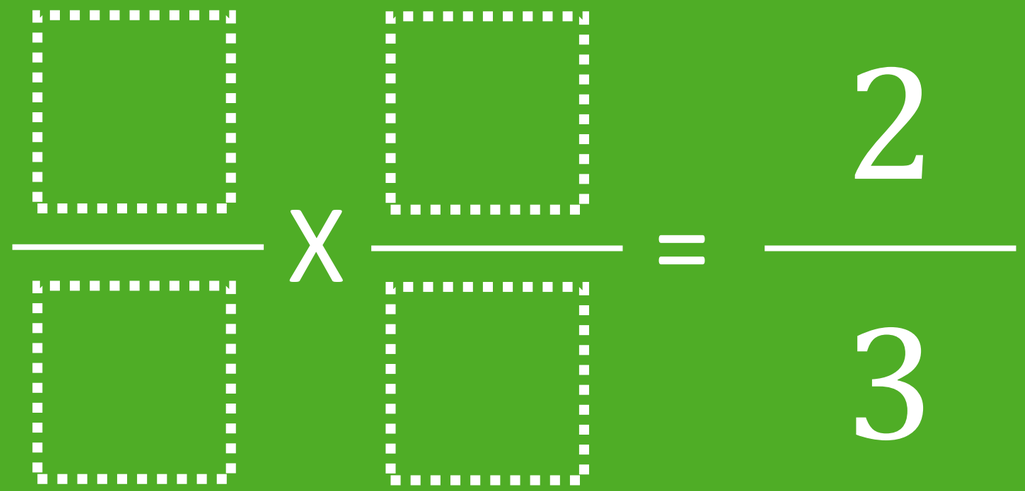
ADDING 2-DIGIT NUMBERS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of two-digit numbers that have a sum of 71.



MULTIPLYING FRACTIONS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two different pairs of fractions that have a product of 2/3.



THINKING TIME

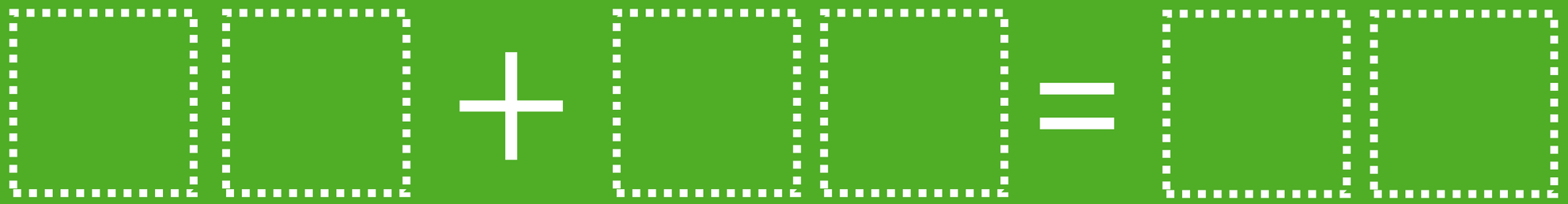
- Go from DOK 1 to DOK 2
  - Strategically remove some information from the problem to prevent immediate calculation
  - Increase the quantity of solutions needed to increase the need to look for patterns

STEP THREE

- Go from DOK 2 to DOK 3
  - Introduce the need to optimize the solution by making the greatest or least product / sum / difference / quotient / answer.
  - Another optimization option is make the answer closest to a specific value.

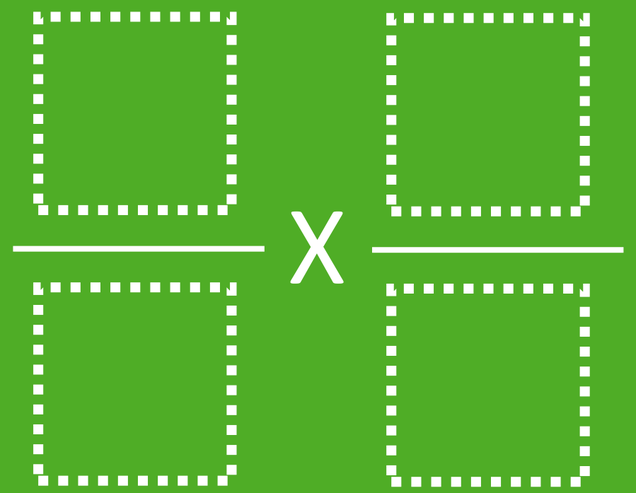
ADDING 2-DIGIT NUMBERS

Using the digits 1 to 9, at most one time each, fill in the boxes to make the smallest sum.



MULTIPLYING FRACTIONS

Using the digits 1 to 9, at most one time each, fill in the boxes to make two fractions that have a product that is as close to 4/11 as possible.



THINKING TIME

- Go from DOK 2 to DOK 3
  - Introduce the need to optimize the solution by making the greatest or least product / sum / difference / quotient / answer.
  - Another optimization option is make the answer closest to a specific value.

3 Steps to Increase Math DOK Levels

Step 1: Find a One-Operation Problem

- Procedural problems with one operation are easiest to modify.
- Other problems may also be modified but may not be as easy.

Adding 2-Digit Numbers
Solve.

$41 + 36 = \_\_\_$

Multiplying Fractions
Solve.

$\frac{3}{7} \times \frac{2}{9} = \_\_\_$

Trigonometry
Solve.

$\sin \frac{\pi}{3} = \_\_\_$

Step 2: Go from DOK 1 to DOK 2

- Strategically remove some information from the problem to prevent immediate calculation
- Increase the quantity of solutions needed to increase the need to look for patterns

Adding 2-Digit Numbers
Using the digits 1 to 9, at most one time each, fill in the boxes

Multiplying Fractions
Using the digits 1 to 9, at most one time each, fill in the boxes

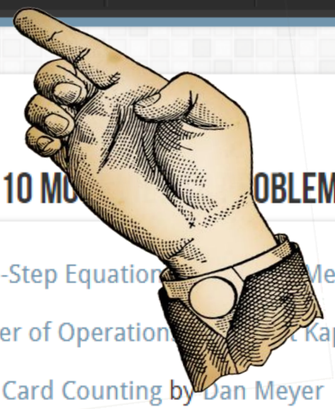
Trigonometry
Using the digits 1 to 9, at most one time each, fill in the boxes

GOALS

- [x] WHY DO WE NEED THEM?
- [x] WHY ARE THEY DIFFERENT?
- [x] HOW DO YOU IMPLEMENT THEM?
- [x] HOW DO YOU CREATE YOUR OWN?
- [ ] WHERE DO YOU GET OTHERS?

Open Middle

Challenging math problems worth solving



THE TOP 10 MO OBLEMS OF 2016

1. Two-Step Equation Mendivil, Daniel Luevanos, and Robert Kaplinsky
2. Order of Operation Kaplinsky with answer from Michael Fenton and his students
3. Dot Card Counting by Dan Meyer
4. Rational and Irrational Numbers by Bryan Anderson
5. One Solution, No Solutions, Infinite Solutions by Bryan Anderson
6. Multiplying a Two-Digit Number by a Single-Digit Number by Robert Kaplinsky
7. Exponents and Order of Operations by Zack Miller
8. Converting Between Fractions and Decimals by Robert Kaplinsky
9. Interpretting Percentages by Robert Kaplinsky
10. Two-Step Equations 3 by Erick Lee

WHAT ARE PEOPLE SAYING ABOUT OPEN MIDDLE?

Search

OPEN MIDDLE WORKSHEET

Download the Open Middle Worksheet (Regular): Version 1.2

Download the Open Middle Worksheet (Large): Version 1.1

BROWSE BY DEPTH OF KNOWLEDGE LEVEL

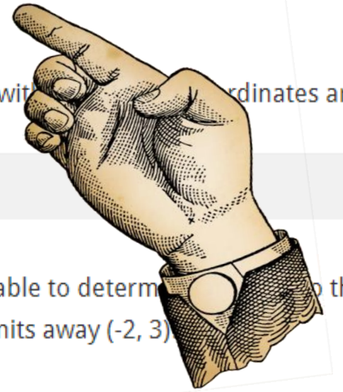
DOK 2: Skills and Concepts

DOK 3: Strategic Thinking

BROWSE BY COMMON CORE STATE STANDARDS

- Kindergarten (10)
  - Counting & Cardinality (2)
  - Geometry (2)
  - Number & Operations in Base Ten (1)
  - Operations & Algebraic Thinking (5)

EQUIDISTANT POINTS



Directions: How many points wit[illegible]rdinates are 5 units away from (-2, 3)?

Hint

Which methods are available to determ[illegible]o this problem? What shape is defined by *all* of the points that are 5 units away (-2, 3)

Answer

12 points: (-5, 7), (-7, 3), (-5, -1), (-2, -2), (3, 3), (1, -1), (-2, 8), (1, 7), (2,6), (-6, -6), (-6, 0), and (2, 0)

Source: Dylan Kane

Search

OPEN MIDDLE WORKSHEET

Download the Open Middle Worksheet (Regular): Version 1.2

Download the Open Middle Worksheet (Large): Version 1.1

BROWSE BY DEPTH OF KNOWLEDGE LEVEL

DOK 2: Skills and Concepts

DOK 3: Strategic Thinking

BROWSE BY COMMON CORE STATE STANDARDS

- Kindergarten (10)
  - Counting & Cardinality (2)
  - Geometry (2)
  - Number & Operations in Base Ten (1)
  - Operations & Algebraic Thinking (5)

Open Middle @openmiddle · Jan 11

Open Middle
@openmiddle

Hey @openmiddle fans, we want to hear from you. Why do you use our problems with your students? Share your success stories or lessons learned.

RETWEETS	LIKES
7	6

2:10 PM - 11 Jan 2017

8 7 6 •••

1 2 •••

GOALS

- [x] WHY DO WE NEED THEM?
- [x] WHY ARE THEY DIFFERENT?
- [x] HOW DO YOU IMPLEMENT THEM?
- [x] HOW DO YOU CREATE YOUR OWN?
- [x] WHERE DO YOU GET OTHERS?

CALL TO ACTION

Action	Do Now	Start Planning	Yes & No	Don't Do
Incorporate higher DOK problems on assessments.		✓		
Replace all DOK 1 problems with higher DOK problems.				✓
Share these resources with colleagues to make them aware.	✓			
Find problems I can integrate on Open Middle.	✓			
Use the 3 steps process to strengthen existing problems.			✓	

CHALLENGING PROBLEMS WORTH SOLVING

ROBERT KAPLINSKY

robert@robertkaplinsky.com

robertkaplinsky.com/bousd

@robertkaplinsky