Exploring One-On-One Teacher-Student Conversations During Mathematical Problem Solving[1]

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We offer a framework to identify interaction patterns between a teacher and a child engaged in conversation during problem-solving interviews and, by proxy, in classrooms. The framework highlights the extent to which engagement with children's mathematical thinking is central to the conversation. We analyzed video data from 129 teachers who each worked one-on-one with 3 students. Teachers were drawn from 4 groups, differing in their amount of experience with children's thinking. Findings indicated that expertise in eliciting and building on children's thinking can be learned, especially when professional development is sustained over multiple years, and our cross-sectional data led to the initial mapping of a developmental trajectory. Also striking was the lack of conversation to extend children's thinking after correct answers were reached.

> Effective teaching involves observing students, listening carefully to their ideas and explanations, having mathematical goals, and using information to make instructional decisions. (National Council of Teachers of Mathematics [NCTM], 2000, p. 19)

This quote depicts the ambitious vision being promoted for mathematics classrooms, including new roles for teachers (NCTM, 2000; National Research Council [NRC], 2001). Guided by advances in researchers' knowledge of how children develop mathematical understandings (Lester, 2007) and recognition of the importance of attending to the different ways children make sense of mathematics, advocates have argued that teachers must prioritize eliciting and responding to children's ideas in the midst of instruction (Ball, Lubienski, & Mewborn, 2001). This approach to teaching has been described in the literature in several ways,

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for example, as an *adaptive style of teaching* (Sherin, 2002b), *discovery teaching* (Hammer, 1997), and *teaching with an inquiry stance* (Ball & Cohen, 1999; Cochran-Smith & Lytle, 1999, 2009), and researchers have documented benefits for children and teachers in these instructional settings (Lester, 2007; NRC, 2001).

Unfortunately, however, the expertise needed for this instructional approach has proven challenging to develop. In particular, teachers often struggle with how to be active (but not too directive) in advancing children's learning. A common recommendation is that teachers should "not tell" students how to solve problems, but this recommendation is insufficient because it is about what *not* to do instead of what to do (Chazan & Ball, 1999). Researchers have begun to address this issue by identifying ways of telling that are productive and consistent with reform goals (Chazan & Ball, 1999; Lobato, Clarke, & Ellis, 2005) and by specifying other teacher actions (e.g., developing discourse communities) that actively promote the types of classrooms envisioned (Franke, Kazemi, & Battey, 2007; Hufferd-Ackles, Fuson, & Sherin, 2004; Kazemi & Stipek, 2001; Mathematics Classroom Practice Study Group, 2007; McClain & Cobb, 2001; Moyer & Milewicz, 2002; Sherin, 2002a; Wood, 1998; Wood, Nelson, & Warfield, 2001).

Much work remains to be done, however, in helping teachers redefine their roles so that they are actively promoting their students' learning by (a) eliciting and making sense of children's thinking and (b) identifying and executing follow-up instructional moves that are based on that thinking and that advance the child's understanding. In focusing on teachers' participation *in relation to* children's thinking, we contribute to a growing body of research in which this interdependency is emphasized (see, e.g., Fraivillig, Murphy, & Fuson, 1999; Mathematics Classroom Practice Study Group, 2007; Wood, Williams, & McNeal, 2006). In this paper, we use this lens as the foundation for a framework to characterize one-on-one teacher-student conversations during mathematical problem solving. We then apply this framework to a cross-sectional study of 129 teachers who differed in their amount of experience with children's mathematical thinking. Before presenting the framework and empirical data, we describe our rationale for examining patterns of teacher-student interactions in one-on-one conversations during problem-solving interviews.

Attention to One-on-One Conversations in Problem-Solving Interviews

We define a *problem-solving interview* as a series of one-on-one interactions that a teacher has with a child about a set of mathematical problems outside the classroom setting. We recognize that interview settings are less complex than classroom settings, but we argue that the expertise needed for productive interviewing is foundational to the expertise needed for creating rich mathematical discussions in classrooms—in one-on-one, small-group, and whole-group situations. In all settings, teachers must observe, listen, question, design follow-up tasks, and so on, and engagement with the thinking of individual children is critical. We do not claim that all teachers who show expertise while interviewing will be able to show the same expertise in a classroom, where they often must not only interact with one child but also manage a discussion with several children. Nonetheless, we feel that teachers who cannot elicit and build on individual children's thinking during their interactions in interviews will most likely be unable to do so in a classroom setting.

Three main factors influenced our decision to study one-on-one interactions during interviews rather than in classrooms. First, interviews isolate teacher-student conversations about mathematics from other aspects of classroom life, making them more accessible for study. Second, this study involved both prospective and practicing teachers, and we thought it most appropriate to engage our prospective teachers (undergraduates just beginning their studies) with individual children. Third, interviews are more likely than other activities to capture a teacher's expertise in eliciting and responding to an individual child's thinking. Interviews require a teacher to focus exclusively on an individual child, whereas in classroom situations, this expertise can be masked when other children or classroom events "rescue" a teacher from having to respond (Thompson & Thompson, 1994, p. 300).

In using this approach, we join others in highlighting the importance of teachers' engagement with individual children's thinking. We draw explicitly from one project, Cognitively Guided Instruction (CGI). CGI is a research and professional development project in which teachers are given (a) access to research-based knowledge about how children develop understandings in particular mathematical domains and (b) help in thinking about how they can use that knowledge to support and extend their own students' understandings (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Carpenter, Franke, & Levi, 2003). Teachers' engagement with CGI has proven beneficial for their students (Carpenter, Fennema, Peterson, Chang, & Loef, 1989; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Villaseñor & Kepner, 1993), and CGI is one of the few projects to connect teacher and student learning (Fennema et al., 1996; Sykes, 1999; Wilson & Berne, 1999). Furthermore, CGI research has shown important benefits for teachers as well: When teachers have developed the ability to learn from the thinking of children in their classrooms, they can continue learning even after formal professional development support ends (Franke, Carpenter, Levi, & Fennema, 2001).

Of particular interest for this study are the efforts CGI researchers have made to understand and document changes in teachers' beliefs and practices over time. Specifically, CGI researchers have connected these changes with different levels of teachers' engagement with children's mathematical thinking, finding that teachers who demonstrated the most expertise used individual children's thinking (versus the thinking of their students as a group) to drive their instruction (Fennema et al., 1996; Franke, Carpenter, Fennema, Ansell, & Behrend, 1998; Franke, Fennema, & Carpenter, 1997). Thus, the most advanced teachers were responsive to children's thinking in that they were able to tailor their interactions to the thinking of the individuals in their classrooms.

We build on the CGI work by also closely examining teachers' engagement with children's mathematical thinking and the capabilities needed to be responsive to that thinking. We differ from the earlier research in terms of the instructional setting (and, thus, grain size) investigated. Prior research characterized teachers' engagement with children's thinking in their overall instructional practices by integrating analyses of classroom observations with interview data about teachers' knowledge and beliefs (see, e.g., Fennema et al., 1996). In contrast, we conducted a more narrowly focused analysis of teachers' engagement with individual children's thinking in one-on-one conversations.

In this paper, we offer a framework to identify patterns of interaction between a teacher and a child engaged in conversation during a problem-solving interview (and, by proxy, in classrooms). Of particular interest were the ways in which a teacher elicited and responded to a child's thinking, and we sought to characterize interactions reflecting a range of expertise. Through a cross-sectional design, we also explored the development of this expertise. Our broader goal in developing and using this framework was to provide a greater understanding of and appreciation for the active role teachers can play when working toward the vision of mathematics classrooms presented earlier—classrooms characterized by rich discussions in which teachers elicit and respond to individual children’s thinking.

Methods

We analyzed video of 129 teachers who each engaged in one-on-one problem-solving interviews with three children. These data were drawn from a larger cross-sectional study entitled "Studying Teachers Evolving Perspectives" (STEP), in which our overall goal was to map a trajectory for the changing needs and perspectives of teachers engaged in sustained professional development focused on children's mathematical thinking.

Participants

The 129 participants (118 females and 11 males) included three groups of practicing K–3 teachers with similar amounts of teaching experience and one group of prospective teachers who were just beginning their studies to become elementary school teachers (see Table 1). Participant groups differed in their experience with children's mathematical thinking. Specifically, Prospective Teachers, by virtue of their lack of teaching experience and professional development, had the least experience with children's thinking, followed by Initial Participants, who had teaching experience but no sustained professional development, and then by Advancing Participants, who had teaching experience and 2 years of professional development. Emerging Teacher Leaders had the most experience with children's thinking because they had not only teaching experience coupled with 4 or more years of professional development but also engagement in at least a few formal or informal leadership activities to support other teachers (e.g., visiting classrooms or sharing problems).

Practicing teachers were drawn from three Southern California districts that were similar in demographics, with one third to one half of the students classified as Hispanic, about one fourth classified as English Language Learners, and one fourth to one half receiving free or reduced-cost lunch. Prospective teachers were undergraduates, generally in their first 2 years of study, in a nearby comprehensive urban university, and they had just begun their first mathematics content course for teachers.

Table 1
Participant Groups

Participant group	Description
Prospective Teachers (n = 35)	Undergraduates enrolled in a first mathematics content course for teachers
Experienced Practicing Teachers	
Initial Participants (n = 31)	Experienced K–3 teachers who were about to begin sustained professional development focused on children's mathematical thinking
Advancing Participants (n = 31)	Experienced K–3 teachers who had engaged with sustained professional development focused on children's mathematical thinking for 2 years
Emerging Teacher Leaders (n = 32)	Experienced K–3 teachers who had engaged with sustained professional development focused on children's mathematical thinking for at least 4 years and were beginning to engage in formal or informal leadership activities to support other teachers

Note. All practicing teachers had at least 4 years of teaching experience (with a range of 4–33 years), and the number of years of teaching experience in each group averaged 14–16 years.

Sustained Professional Development

The three groups of practicing teachers were volunteer participants in professional development focused on children's mathematical thinking, although the Initial Participants had yet to begin.[2] The professional development drew heavily from the research and professional

[2] We initially worried that, because of participant dropout, those teachers who persisted 2 years as Advancing Participants or 4-or-more years as Emerging Teacher Leaders were different from the teachers who chose to leave the professional development early. If so, the Initial Participants (who had yet to begin professional development) might differ in important ways from the Advancing Participants and Emerging Teacher Leaders for reasons other than the number of years of professional development. We examined the enrollment records and concluded that dropout was not an issue in our study because fewer than 10% of the Advancing Participants or Emerging Teacher Leaders dropped out by choice, for programmatic reasons. Furthermore, even though some individuals were forced to discontinue their participation for reasons outside their control (e.g., funding), they overwhelmingly chose to re-enroll when the opportunity

development project Cognitively Guided Instruction (CGI) (Carpenter et al., 1999; Carpenter et al., 2003), and the overarching goals were to help teachers learn about children's mathematical thinking and learn to use this knowledge to inform their instruction.

The professional development occurred prior to the study and was generally facilitated by the same experienced mathematics program specialist. It included about 5 full days of workshops per year (in half- or full-day increments), and during these workshops, teachers analyzed classroom artifacts (video and written student work), explored underlying mathematical concepts and children's understandings of those concepts, and considered how those understandings could be used to inform instruction. Between meetings, teachers were asked to pose problems to their students and bring their student work to the next meeting for discussion and reflection. (See Lamb, Philipp, Jacobs, & Schappelle [2009] for more details about the professional development.)

Measures

Teachers were asked to videotape one-on-one interviews with three children of differing achievement levels—below, at, and above grade level. Each child was presented with four or more story problems that involved whole-number operations; teachers posed (a) three problems from a mandatory list for their grade level and (b) one or more problems from an optional list. Teachers were encouraged to adapt problems (e.g., numbers or context) or pose additional problems to make the mathematical conversation appropriate for each child. Note that children were generally successful with these problems, with more than 80% of the 1798 problems being solved correctly.

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became available.

An interview with a child typically lasted 15–30 minutes. Teachers read each problem aloud and gave the child an opportunity to solve that problem before moving on to the next problem. A variety of tools were generally available, and, at times, the teacher also provided the child with a written version of the problem.

Teachers were told that their goal in the interviews was to explore their children's mathematical thinking; an excerpt from the instructions follows:

> We think of an interview as a one-on-one problem-solving session. We have included a set of problems to ask each child during the interview, but we are not concerned with how many problems are solved correctly. Instead, think about the interview as a learning opportunity for you—we want you to learn about each child's thinking by exploring that child's strategies and understandings.

This broad goal was designed to provide teachers with freedom to pursue the child's thinking, and we recognize that this broad goal may have different meanings for teachers with different levels of knowledge and appreciation of children's mathematics thinking. We argue that their different interpretations are encapsulated in the different types of teacher-student interactions we identified during analysis.

Analyses

We analyzed the teacher-student interviews using our *Framework of Engagement With Children's Mathematical Thinking in One-On-One Conversations* (see Table 2). This framework was initially developed prior to the current study (Jacobs & Ambrose, 2003) and then refined for use here.

Table 2. *Framework of Engagement With Children's Mathematical Thinking in One-on-One Conversations*

		Prioritizing Children's Thinking	
		Minimally	*Actively*
Prioritizing Teachers' Thinking	*Actively*	Imposing Teachers' Thinking	Interrupted Exploring of Children's Thinking
	Minimally	Limited Interacting	Exploring Children's Thinking

We designed our framework to describe teachers' engagement with children's mathematical thinking in one-on-one conversations. We began with the assumption that all teachers have good intentions in that they want to help children succeed and grow. We also recognized that, at any given moment, teachers are juggling many pressures (e.g., curricular goals, time constraints, the child's willingness to engage with the mathematics, and so on) and thus, decision making is multifaceted and complicated. In our framework, we chose to highlight one particularly critical issue: the extent to which engagement with the child's thinking is central to the conversation. We argue that for teachers to move toward the ambitious vision of teaching described at the beginning of the paper, engagement with the child's thinking needs to be the defining feature of the interaction. In presenting this argument, we do not suggest that the teachers' ideas can never be introduced. As Ball (1993), Lampert (2001), and others have articulated, mathematics teaching involves a constant struggle to respect both children's thinking and the discipline of mathematics, and the introduction of teachers' ideas—ideas that generally reflect deeper mathematical understandings, more sophisticated strategies, knowledge of curricular goals, and so on—can substantially enhance mathematical conversations. Thus, we agree that the dichotomy between teachers' telling and not telling is too simplistic (see, e.g., Lobato et al., 2005), but we also maintain that, regardless of which ideas are being discussed, the child's thinking in relation to those ideas must be prioritized.

On the basis of this assumption, our framework is built on two dimensions (prioritizing children's thinking and prioritizing teachers' thinking), and for each, we identify two levels that reflect the strength of the activity level of the teacher (actively prioritizing and minimally prioritizing). When actively prioritizing children's thinking, teachers explore children's ideas, and when they also share their own ideas, they do so in ways that build on the children's existing ideas and leave room for the children's subsequent thinking. In contrast, when actively prioritizing teachers' thinking, teachers' ideas are advanced regardless of their compatibility with the children's thinking and the children's abilities to benefit from them, having the effect of the children's ideas being ignored or even suppressed. Below we provide an overview of the four major categories of teacher-student interactions that resulted from crossing these two dimensions. Specific examples will be provided in the Findings section.

- ***Exploring Children's Thinking.*** In these interactions, the teacher actively prioritizes the child's thinking while minimally prioritizing his or her own thinking. Specifically, the teacher elicits, makes sense of, and values the child's *existing* thinking and at the same time finds ways to further stimulate this thinking. Note that teachers' moves do not always lead to a correct answer (or even a desired comment or action), but they do honor and try to build on the child's ways of reasoning. Exploring children's thinking is particularly challenging to enact because there is never a single best move, responsive moves cannot be preplanned, and children's unclear or incomplete explanations can make their thinking difficult to interpret. Nonetheless, engagement with the child's ideas, even when the ideas are incomplete or inaccurate, is the driving force for these conversations; the teacher's moves build on the child's ideas and are informed by what is known about that particular child and children's mathematical development in general.

- ***Imposing Teachers' Thinking.*** In these interactions, the teacher consistently prioritizes his or her own mathematical thinking while only minimally considering the child's mathematical thinking. Thus, the teacher often guides the child through particular target strategies, generally ensuring a correct answer but often missing connections to the child's existing understandings. We reiterate that we do not mean to suggest that teachers should never share their knowledge. However, in these interactions, the child's thinking is largely obscured and, in many cases, the teacher's ideas are inconsistent with the child's initial attempts or what research has shown children of this age are inclined to do. Among the negative implications of this kind of interaction are several highlighted by Lobato and colleagues (2005) including emphasizing the teacher as mathematical authority, minimizing the cognitive engagement of the student, and conveying that there is a preferred solution path for mathematical problems.
- ***Interrupted Exploring of Children's Thinking.*** These interactions share similarities with the first two categories in that the teacher actively prioritizes both the child's thinking and his or her own thinking—but, in this case, in a competing way. The most common sequence is that a teacher begins the interaction prioritizing the child's thinking but eventually struggles to access or enhance the child's thinking and, consequently, resorts to advancing his or her own thinking without attention to how the child is engaging with those ideas. Thus, the core of this category is the inconsistency in the teacher's priorities. At times, the teacher actively prioritizes the child's thinking and, at other times, the teacher prioritizes his or her own thinking, often when the two ways of thinking are incompatible.

- ***Limited Interacting.*** These interactions are in contrast to the active nature of the interactions in the previous categories and involve minimally prioritizing the child's thinking and the teacher's thinking. The teacher adopts a more passive role as a spectator; the teacher steps back, watches, and listens to the child's problem solutions. The result is that sometimes the child's thinking is visible, but other times, it remains hidden or unclear because the teacher's elicitation of this thinking is so limited.

When employing the framework, we separately assessed the supporting and extending interactions related to each problem. A *supporting interaction* refers to the teacher-student interaction that takes place after a problem has been posed but before the child has arrived at a correct answer. Thus, teachers' supporting moves generally help a child to carry out a solution method and may include, for example, ensuring that a child understands the problem, suggesting a tool, or exploring what a child has already done. An *extending interaction* refers to the teacher-student interaction that takes place after a child arrives at a correct answer. A teacher's extending moves generally challenge the child to reflect on his or her strategy and to think in new ways about problems and strategies. These moves may include, for example, asking for a second strategy, connecting a child's thinking to symbolic notation, or posing a new (related) problem for the child to solve.[3] (See Jacobs & Ambrose [2008] for more discussion of potentially productive supporting and extending moves.)

By analyzing supporting and extending separately, we build on Fraivillig and colleagues' (1999) attention to the different aspects of teachers' activities. We also recognize the risk of overemphasizing correct answers by choosing the correct answer as the dividing line between

[3] By these definitions, supporting is sometimes unnecessary (e.g., when a child immediately provides a correct answer) and extending is sometimes impossible (e.g., when a child never reaches a correct answer).

supporting and extending. However, we purposefully chose this dividing line because conversations before and after an answer is given often differ in terms of teachers' goals and interventions, and we wanted to capture those differences. Furthermore, we wanted to explore the potential of conversations that continue beyond the correct answer. Teachers' work is sometimes considered to be finished once a correct answer emerges (as in the traditional initiation-response-evaluation [I-R-E] form of classroom discourse [Mehan, 1979]), but we join others in believing that important mathematics can often be more easily and effectively discussed after a child understands a problem situation and has successfully engaged with the problem (see also, Schleppenbach, Perry, Miller, Sims, & Fang, 2007).

Coding. Each interview was blinded in terms of participant group membership, and coding was done primarily from the video (with the transcript as a backup). Coding occurred on two levels. First, we coded at the problem level by categorizing the supporting and extending interaction for each problem (N = 1798 problems). Coding was holistic, and we were not seeking particular teacher moves. Instead, teacher moves were considered in conjunction with children's comments and actions because the same teacher move can be responsive to a child's thinking in one situation but not in another. Specifically, when looking for evidence of teachers' actively prioritizing children's thinking, we sought moves that elicited, built on, and left room for the child's ways of reasoning. Such moves might not lead to correct answers or even desired comments or actions by the child, but they needed to be consistent with the specific situation and, more generally, the research on children's thinking.

Second, we coded at the teacher level by synthesizing the codes across all problems (and across all three children) to generate an overall supporting code and an overall extending code for each teacher. This synthesis was not a simple count but was again holistic. In particular, we considered the opportunities teachers had to demonstrate their supporting and extending moves and sometimes weighted the interaction on one problem more heavily than another. For example, opportunities to support varied across children and problems because when a child correctly solved a problem quickly, there was little opportunity (or need) for support. Similarly, when a child spent 10–15 minutes struggling through a problem, expecting lengthy extending moves was unreasonable. Thus, we did not expect (or desire) lengthy supporting and extending on every problem. Instead, for the category of exploring children's thinking, we looked for evidence that the teacher could, *when appropriate*, actively explore a child's thinking without also imposing his or her own thinking in ways that resulted in the child's thinking being suppressed. When this evidence did not exist, we considered which of the other three categories best reflected the degree to which the teacher prioritized children's thinking versus his or her own thinking. In this paper, we focus only on the coding done at the teacher level. Our goal was to capture patterns within and across our participant groups to characterize teacher performance in the development of expertise in eliciting and building on individual children's thinking.

Four individuals coded all the interviews, and each interview was coded by at least two coders. Agreement was more than 80%, and disagreements were resolved through discussion and involvement of additional coders when necessary. To ensure consistency, (a) 10% of the interviews were coded by a third coder, and (b) one individual coded all the interviews.

FINDINGS AND DISCUSSION

We used our Framework of Engagement With Children's Mathematical Thinking in One-on-One Conversations to characterize teachers' engagement with children's thinking in supporting and extending interactions, and we provide examples to illustrate the framework categories. Note that each example was drawn from our data set and, thus, represents an actual interaction that took place between a teacher and a child around a problem. In sharing each example, we try to convey a sense of what happened in the entire conversation because our characterization of interactions was holistic and could not be reduced to, for example, a particular teacher move or count of teacher moves. As such, we were limited in the number of examples we could provide. The danger in presenting limited examples is that overly narrow conceptualizations of categories may be conveyed. We therefore urge readers to attend to the underlying principles of each category as well as to the specifics in the examples.

To begin to map a trajectory for the development of expertise in eliciting and building on children's thinking, we also share the distribution of teachers in the four framework categories for each participant group. We remind readers that this data reflects coding done at the teacher level—across problems and children.

Supporting Children's Mathematical Thinking

When a child struggles or solves a problem incorrectly, a teacher must determine how and when to intervene. A single question presented at the right time can be all a child needs for self-redirection. Other times, a more extensive conversation is necessary. Still other times, moving to a new problem is productive. We do not believe that a single best move or series of moves exists in response to a particular situation, and teachers must constantly weigh the advantages and disadvantages of various moves when they make in-the-moment decisions about how to respond. Each interaction category in our framework reflects this decision making in relation to the degree to which teachers prioritize children's thinking versus their own thinking. Figure 1 provides the distribution of teachers in each participant group according to their categorization for supporting interactions. Given the widespread distribution across the framework, the following sections provide examples for all four interaction categories, beginning with an interaction characterized by exploring children’s thinking. After these illustrations, we return to the distribution findings and synthesize the patterns reflected both overall and within each participant group.

		Prioritizing Children’s Thinking	
		Minimally	*Actively*
Prioritizing Teachers’ Thinking	*Actively*	**Imposing Teachers’ Thinking** Prospective Teachers 57% Initial Participants 7% Advancing Participants 23% Emerging Teacher Leaders 3%	**Interrupted Exploring of Children’s Thinking** Prospective Teachers 17% Initial Participants 19% Advancing Participants 19% Emerging Teacher Leaders 16%
	Minimally	**Limited Interacting** Prospective Teachers 20% Initial Participants 48% Advancing Participants 26% Emerging Teacher Leaders 9%	**Exploring Children’s Thinking** Prospective Teachers 6% Initial Participants 26% Advancing Participants 32% Emerging Teacher Leaders 72%

Figure 1. Percentage of teachers in each participant group classified in the four interaction categories for supporting children in solving problems.

Supporting Through Exploring Children’s Thinking

When a teacher supports through exploring children’s thinking, the child’s ideas are the starting point for the conversation, and the teacher’s moves are designed to elicit and build on the child’s existing understandings. These moves can take a variety of forms and focus on different parts of the problem-solving process (Jacobs & Ambrose, 2008). For example, with some moves, the teacher can elicit the child’s understanding of the problem and work to ensure that the child understands the problem. With other moves, the teacher can ask specific questions to explore the child’s partial strategies, hoping to help the child recognize and move beyond his or her error or stopping point. In the following example, we illustrate these two types of moves as well as several others that serve to explore the child’s thinking.

In this 8-minute interaction, a teacher asked a second grader, Miguel, to solve this problem: *Miguel had 24 chocolate chips to decorate four cupcakes. If he wants to put the same number of chocolate chips on each cupcake, how many chocolate chips would he put on each cupcake?* Miguel drew four circles (cupcakes) and then drew various lines linked to the circles to indicate chocolate chips. However, he struggled to divide the chocolate chips evenly and instead put 10 on the first two cupcakes and 2 on the last two cupcakes. The teacher then helped Miguel revisit the problem conditions:

> (T)eacher: Did you put the same number of chocolate chips on each cupcake?
> (S)tudent: I couldn't, because this would be 10 and that would be 20. And I had 4 left—2, 1. And 1, 2.
> T: But, how could you do it so that you had the same number on each cupcake?
> S: Well I had—this was 10 and 20—and I could have subtracted 5 to equal that, but no, it's gotta' be 10 and 10.
> T: Why does it have to be 10 and 10?
> S: Because there's 20—there's 24 cup—chocolate chips and I wanna decorate four, but I have 10 and 20. Those are the only tens I had, so I have to have add 4 right here— 1, 2 and 1, 2.
> T: Okay, what if you were this kid that got this cupcake (points to a cupcake with 2 chocolate chips)? Would you think it was very fair if Adam got 10 and you got 2? (Miguel shakes his head no.) Could you make it so that it's fair?
> S: Yes.
> T: Show me how it could be fair.

The teacher first explored Miguel's existing understandings, even using his language ("Why does it have to be 10 and 10?") to probe his reasoning. When Miguel seemed unable to move beyond his initial distribution, the teacher personalized the problem and used his representation to highlight that Miguel would be unhappy with this distribution if he received one of the cupcakes with only 2 chocolate chips.

Miguel subsequently worked on changing the distribution but eventually became confused and declared, "I don't know what I did." The teacher built on this comment, suggesting that Miguel start over because *he* did not know what he just did. Sometimes suggesting even the possibility of a new strategy can help children move past a sticking point (Jacobs & Ambrose, 2008).

Miguel then pursued an idea that he could "split it up like into halves," but his idea was unclear and he seemed unable to execute it. Again, the teacher first tried to explore Miguel's ideas by asking, "Split what up into halves?" and then tried to follow this reasoning. When discussion did not lead to more clarity, she again encouraged Miguel to start over but first revisited his understanding of the problem—this time personalizing and elaborating the story. Specifically, she engaged Miguel in identifying three friends who wanted to share fairly the four cupcakes with him. Miguel was visibly excited about including his friends in the story and offered names of specific friends he wanted included. After hearing the elaborated story, Miguel—almost immediately—drew four new circles (cupcakes), wrote *5* next to each cupcake, and then *1* next to each cupcake. At this point, Miguel was easily able to conclude that each cupcake should have 6 chocolate chips to be fair. According to our definitions of *supporting* and *extending*, the supporting interaction concluded after this correct answer was presented. (The conversation did continue, however, and in the extending interaction, the teacher asked Miguel to further explain his reasoning and reflect on why he initially struggled but was eventually able to be successful.)

We characterized this supporting interaction as exploring children's thinking, not because Miguel ultimately gave a correct answer but because the teacher engaged Miguel in a mathematical conversation about a challenging problem by continually working to elicit, make sense of, and build on his thinking. This interaction showcases a variety of moves the teacher used to assist Miguel in (a) understanding the problem situation and (b) reflecting on his partial strategies. Furthermore, the moves were timed such that they consistently built on what Miguel said and did. At no time did the teacher propose her own idea for solving the problem or try to lead him toward a particular strategy. Instead, in her interactions with Miguel, she probed some ideas he proposed and helped him to keep track of his goal of equally distributing the chocolate chips. Substantial expertise was reflected in this type of interaction, and our cross-sectional data revealed the uneven distribution of this expertise across our participant groups.

Distribution findings. Figure 1 showed that not all individuals demonstrated the expertise needed to explore children's thinking during supporting, and those demonstrating this expertise tended to be in the Emerging Teacher Leaders group. Specifically, although almost 3/4 of the Emerging Teacher Leaders were classified as exploring children's thinking, only 1/3 or fewer of the participants in the other groups were similarly classified. These data suggest that development of this expertise requires substantial time and support. Interestingly, neither teaching experience alone (for Initial Participants) nor teaching experience plus 2 years of professional development (for Advancing Participants) was sufficient for a substantial number of teachers to consistently demonstrate this expertise. However, after experiencing 4 or more years of professional development (and some leadership activities), the majority of the Emerging Teacher Leaders were classified as exploring children's thinking.

Supporting in Ways Other Than Exploring Children's Thinking

When teachers did not focus on exploring children's thinking, their supporting was classified as one of three other interaction types: supporting through imposing teachers' thinking, supporting through interrupted exploring of children's thinking, or supporting with limited interacting. We provide examples of each type of interaction and then discuss the distribution across participant groups.

Supporting through imposing teachers' thinking. When teachers support through imposing their thinking, their moves promote particular solution paths instead of eliciting and building on children's ideas. Teachers orchestrate these interactions so that children almost always solve problems correctly but with limited opportunity to think independently. For example, in this 3-minute interaction, a teacher asked a third grader, Griffen, to solve this problem: *Adam has already read 20 pages in his favorite book. How many more does he need to read to finish the book if it has 75 pages?* Griffen wrote (vertically) 20 + 75 = 95. The teacher did not explore Griffen's reasoning about this problem setup, but instead (implicitly) offered the idea of subtraction.

> T: Well, there's 75 in the whole book, right? The front cover to the back cover is 75 pages, but he's already read 20, so there's going to be less.
> S: Oh, so I'm subtracting?
> T: Subtracting. There you go!

Griffen again wrote (vertically) 20 + 75, but this time he subtracted (incorrectly) and answered 65. Specifically, Griffen crossed out the *2*, wrote *1* above it, and then wrote another *1* next to the *0* in his minuend (20). When computing in the one's column, he wrote *5* (presumably from 10 – 5), and in the ten's column, he wrote *6* (presumably from reversing the subtrahend and minuend and computing 7 – 1) (see Figure 2).

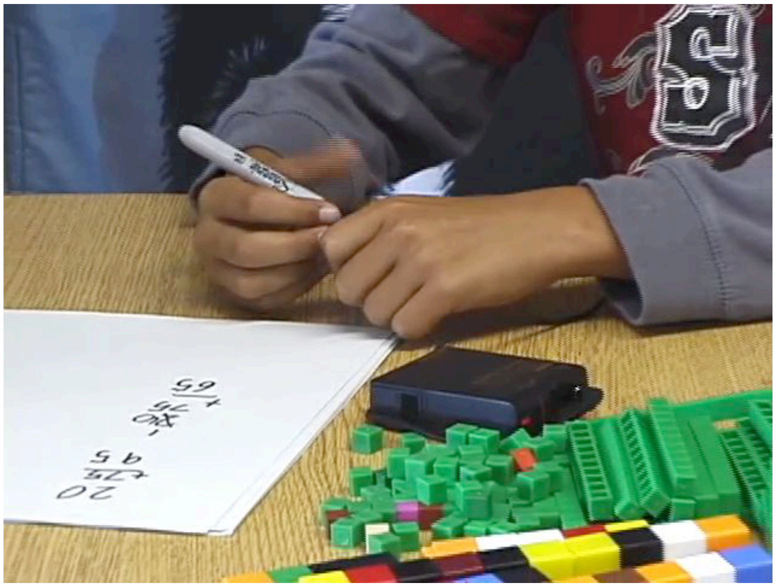

Figure 2. Griffen's written work on the problem about the number of pages to finish a book.

In short, Griffen appeared to use the traditional subtraction algorithm with a bug. However, because his thinking was never probed, we are unsure about his calculating procedure, why he wrote an addition sign, or why he wrote the larger number (75) below the smaller number (20). Instead of exploring these types of issues, the teacher further promoted her suggestion of subtraction, but this time she was more explicit in her suggestions.

> T: What if we tried the 75, 'cause that's the total number, right? So, if you have 75 and then you take away 20 from that (using her fingers to show how to set up the new subtraction problem)?
> S: So, I'm supposed to switch it?
> T: Yeah, there you go. 'Cause this is how many there's altogether (pointing to the 75) and this is how many he's read (pointing to the 20).

Griffen then wrote (vertically) 75 – 20 and solved to get 55 (presumably computing using the traditional subtraction algorithm). The teacher confirmed the correct answer, praised Griffen, and the interaction concluded.

We characterized this supporting interaction as imposing teachers' thinking because the teacher did not explore Griffen's thinking and made suggestions that were likely inconsistent with his reasoning. Specifically, she did not ask about his initial strategy attempts, including why he set up the problem as he did or how he computed. Furthermore, her suggestions to use subtraction were noteworthy. Research has shown that this interpretation of the problem situation is inconsistent with how children typically view problems with missing addends. Most children view (and solve) missing-addend problems as addition problems because the joining action (in this case, reading more pages) is visible (Carpenter et al., 1999). Griffen also initially approached the problem though addition although we do not have access to his exact reasoning. Thus, not only was the exploration of Griffen's thinking missing but the imposition of the teacher's thinking was likely inconsistent with how Griffen (and, in fact, most children) interpret the problem.

A final comment is in order. The decision to categorize this interaction as imposing teachers' thinking was not simply because the teacher offered a suggestion. As discussed earlier, a teacher's ideas can productively be incorporated into conversations in ways that still honor and build on the child's thinking. For example, the teacher in the previous interaction suggested that Miguel stop what he was doing and start over with his strategy. This suggestion was in response to Miguel's stating that he was confused. In contrast, attention to Griffen's thinking was missing when the teacher initially suggested subtraction. When Griffen *asked* whether he should subtract, the teacher did not respond to this question by exploring his thinking. Instead, she encouraged him to go ahead with the operation. Note that Griffen's teacher (and other teachers who engage in these types of interactions) have good intentions and are likely trying to ensure success (correct answers) or help children to use efficient strategies. However, these goals—when devoid of any substantial exploration of children's thinking—can undermine children's sense making in mathematics and are unlikely to be of any benefit when children encounter similar problems in the future.

Supporting through interrupted exploring of children's thinking. When a teacher supports through interrupted exploring of children's thinking, his or her moves both (a) elicit and build on the child's ideas and (b) promote particular solution paths (including ones that are often inconsistent with the child's existing ideas). For example, in this 6-minute interaction, a teacher asked a third grader, Eloise, to solve this problem: *The teacher wants to pack 36 books in boxes. If 2 books can fit in each box, how many boxes does she need to pack all the books?* The teacher began the interaction by exploring Eloise's understanding of the problem situation and problem quantities. Specifically, she asked, "What is the problem asking you to do? What is it saying?" When Eloise responded, "Adding," the teacher pursued this idea by inquiring how the problem was asking her to add and by reviewing the quantities in the problem. Eloise then began drawing tally marks and after she had drawn 38 tally marks, recounted, and paused, the teacher explored her partial strategy.

> T: What are those lines for?
> S: Boxes.
> T: They're boxes or they're books?
> S: Boxes.
> T: Okay, how many boxes are you making?
> S: Thirty-six.
> T: All right, does she have 36 boxes or 36 books?
> S: Books.
> T: Okay, so are those the boxes or are those the books?
> S: Books.
> T: Those are the books, yeah. (Eloise recounts the tally marks and crosses out 2 to correct the total to 36.) Okay, now how many books does she want to put in each box?
> S: Two.

In probing Eloise's partial strategy, the teacher tried to clarify how her representation linked with the problem situation. This line of inquiry was not successful, however, as evidenced by Eloise's next move, which was to put 2 dots over each tally mark; she was still conceptualizing the 36 tally marks as boxes and was putting 2 books (represented by dots) in each box. Nonetheless, the teacher allowed Eloise to complete the activity of assigning 2 dots to each tally mark and then inquired about the meaning of the dots and how they related to the problem situation and quantities. Eloise, still confused, concluded that the teacher needed 36 boxes.

Up to this point in the interaction, the teacher had persisted in prioritizing Eloise's thinking, trying to understand her interpretation of the problem and her partial strategies. She also tried to correct Eloise's misinterpretation of the problem (i.e., assuming that there were 2 books in 36 boxes instead of 36 books that needed to go into boxes that each held 2 books). However, these efforts were unsuccessful, and when Eloise continued to demonstrate confusion (e.g., answering 36 boxes), the teacher began to prioritize her own thinking, resulting in the teacher's strategy being imposed on Eloise who did not understand that way of reasoning.

> T: So can we try this? How about if that's one box, and that's one box, and that's one box.
>
> (Teacher takes the marker and draws a square around the first two tally marks, another square around the third and fourth tally marks, and another square around the fifth and sixth tally marks. The teacher then gives the marker to Eloise who continues drawing squares until all the tally marks are used. Eloise missed some tally marks thereby drawing only 17 (instead of 18) squares [see Figure 3].)
>
> T: Okay, so how many boxes did you just make?
> S: (immediately responding without counting) Thirty-six.
> T: (points to the squares that Eloise is to count) Count your boxes.
> S: (immediately responding without counting) Two.
> T: (more emphatically points to the squares that Eloise is to count) Count *all* your boxes.
> S: (counts the squares silently) Seventeen.
> T: You got 17? Okay, but did you see what we did? We had 36 books and we put 2 books in a box, okay? So, basically she just wanted to be able to put—store her books and put 2 books per box. And that's how many boxes she would need.

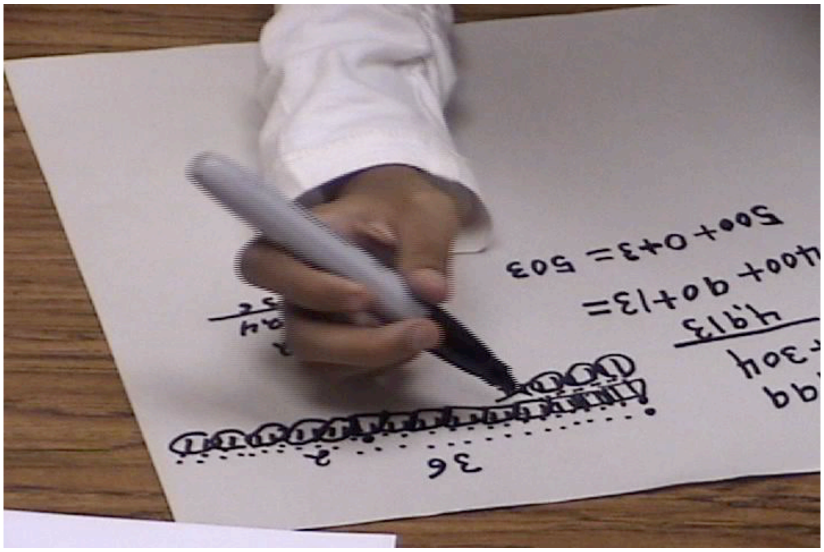

Figure 3. Eloise's written work on the packing-books problem after the teacher encouraged her to draw boxes around each group of two tally marks.

In this portion of the interaction, the teacher introduced the idea of grouping tally marks (drawing a square) as a way of indicating 2 books per box. To execute this strategy, the teacher used Eloise's representation but had the tally marks represent books instead of boxes (as Eloise thought) without mentioning that change. She also involved Eloise in the drawing of the squares, but clearly Eloise did not understand this representation or strategy; when asked how many boxes were needed, Eloise guessed 36 and then 2 without even counting. After explicit prompting, Eloise eventually did count the squares, but was off by one because she had missed some of the tally marks when drawing squares. Furthermore, what problem Eloise thought she was solving in counting the squares is unclear. At this point, the teacher summarized the strategy—her own strategy—and then ended the interaction.

We characterized this supporting interaction as interrupted exploring of children's thinking because the teacher both actively explored Eloise's thinking and actively imposed her own thinking (in a way that was inconsistent with Eloise's understandings). This episode illustrates the most common sequence in this interaction category—prioritizing the child's thinking followed by prioritizing the teacher's thinking. We hypothesized that teachers with this sequence may have worked to prioritize the child's thinking until they encountered difficulty in eliciting and building on the child's thinking and then resorted to prioritizing their own thinking. A few interactions in this category had other sequences. For example, some teachers began prioritizing their own thinking by introducing a strategy but then backed off to pursue how the child was engaging with the mathematics, thus prioritizing the child's thinking. Still other interactions had a more back and forth feel. The common thread in these interactions was that, for some substantial amount of time, the teacher prioritized his or her thinking regardless of the child's understanding of that thinking and, at some other point, the teacher prioritized the child's thinking.

Supporting with limited interacting. When a teacher supports with limited interacting, his or her moves neither (a) elicit and build on the child's ideas nor (b) promote particular solution paths. Instead, the teacher asks only a few, often general, questions and rarely follows up if the child's answer is incomplete. In some cases, these minimal questions are sufficient to help the child solve the problem successfully, but in many cases they fail to help the child engage with the problem. For example, in this 1-minute interaction, a teacher asked a second grader, Max, to solve this problem: *The teacher wants to pack 120 books in a box. If 10 books fit in each box, how many boxes does she need to pack all the books?* Max counted on his fingers and then answered (incorrectly).

> S: Four.
> T: Four boxes. Okay and can you tell me how you got that?
> S: I counted with my fingers.
> T: You counted with your fingers, okay.

In this short interaction, the teacher began to probe Max's partial strategy ("can you tell me how you got that?") but did not persist when the child's response was unclear, naming only the tool used (“fingers”). After this response, the interaction ended. Contrast this type of general probing with the exploration of Miguel’s and Eloise’s thinking in which the teachers posed specific questions (e.g., [to Miguel] “Why does it have to be 10 and 10?” or [to Eloise] “What are those lines for?”) and persisted when the responses were unclear or the child remained confused. With Max, the teacher neither posed specific questions (e.g., inquiring about what he did with his fingers) nor persisted and, consequently, did not gain access to Max’s thinking.

We characterized this supporting interaction as limited interacting because the teacher neither prioritized Max’s thinking nor her own thinking. Instead, Max had an opportunity to solve the problem independently and the teacher had an opportunity to observe, but there was little active engagement with Max’s thinking and little imposition of the teacher’s ideas.

Distribution findings. Participant groups varied in their distribution across the three categories not focused on exploration of children’s thinking (see Figure 1), and our cross-sectional design allows us to speculate about developmental trajectories. We recognize the limitations of cross-sectional designs (versus longitudinal designs) but the patterns across our cross-sectional groups provide strong indicators of how growth may occur.

First, across the four participant groups, 16%–19% of each group were categorized as interrupted exploring of children’s thinking. It is unclear from this distribution whether teachers move through a phase of interacting with children by vacillating between prioritizing their own and the child’s thinking before figuring out a way to consistently prioritize the child’s thinking or whether this style of interacting persists over time.

Second, and in contrast, the distribution of teachers in the limited interacting category varied from group to group suggesting that this may be a phase that teachers go through. Almost half of the Initial Participants were assigned to this category suggesting that as they began their professional development, they were cautious in their interactions with children, perhaps because they were unsure how to engage in a mathematical conversation with an individual child when the goal was to explore the child's thinking. After two years of professional development, however, the picture was different as the Advancing Participants were almost equally divided across the four interaction categories. Although some Advancing Participants remained cautious in their interactions, some had developed the skills needed to prioritize children's thinking, and some had switched to prioritizing their own thinking. After four (or more) years of professional development (and some leadership activities) almost 90% of the Emerging Teacher Leaders had switched to exploring children's thinking either exclusively or in an interrupted way.

Third, the distribution of the imposing teachers' thinking category reflected the reverse pattern of what was seen with exploring children's thinking; more than half of the Prospective Teachers were classified as imposing teachers' thinking whereas only 3% of the Emerging Teacher Leaders were similarly classified. Of particular note is that even after 2 years of professional development focused on children's mathematical thinking, almost one fourth of the Advancing Participants were classified as imposing teachers' thinking. We believe that this performance may be reflective of teachers who were in transition and still coordinating the knowledge, beliefs, and skills needed not only to believe that children can generate strategies on their own but also to determine the teachers' roles in supporting children's thinking (vs. imposing their own thinking) during problem solving. This finding confirms conclusions drawn from other data we gathered in our cross-sectional study. Specifically, in assessing teachers' professional noticing of children's mathematical thinking, we used a video task to examine teachers' abilities to attend to a child's strategies and decide how to respond on the basis of the understandings reflected in those strategies (Jacobs, Lamb, Philipp, & Schappelle, 2011). We found that almost half of the Advancing Participants suggested sharing particular strategies that reflected the teachers' thinking without attention to how the child might make sense of this reasoning. In contrast—and similar to the findings here—this emphasis on the teachers' thinking was minimal after 4 or more years of professional development (and some leadership activities) in that fewer than one fifth of the Emerging Teacher Leaders generated responses focused on teachers' thinking, instead suggesting moves that explored or built on the child's thinking.

Together, these findings indicate that the development of expertise in eliciting and building on children's thinking takes substantial time as teachers must learn about children's thinking, learn to value that thinking, learn to access that thinking in the moment, and learn to use that thinking in determining and executing next steps. Furthermore, change is not always linear, and teachers may revert to less beneficial practices in the transition.

Extending Children's Mathematical Thinking

Solving a problem correctly with a valid strategy is an important mathematical endeavor. However, we view problem solving as a context for having mathematical conversations, and this conversation need not end when the correct answer is reached. Instead, the teacher can pose additional questions to ask the child to, for example, try additional strategies, reflect on the mathematics underlying various strategies, or solve related problems. We believe that these extending interactions can lead to the child's deepened understanding of both the mathematics in the problem solved and its connections to other mathematical ideas.

Similar to our view of supporting moves, we believe that no single best extending move or series of extending moves exists in response to a situation. Furthermore, when determining and executing next steps—in the moment—teachers again must weigh the advantages and disadvantages of various moves. Each interaction category in our framework reflects this decision making in relation to the degree to which teachers prioritize children's thinking versus their own thinking. Figure 4 provides the distribution of teachers in each participant group according to their categorization for extending interactions. Note that actively prioritizing teachers' thinking was essentially absent in all participant groups. Theoretically, teachers could have advanced their own thinking after the correct answer was reached, but they did not tend to do so, perhaps because they most often viewed the interaction as completed once the problem was solved. Thus, in the following sections, we provide examples for the two interaction categories that were prevalent in extending: exploring children's thinking and limited interacting.

		Prioritizing Children's Thinking	
		Minimally	*Actively*
Prioritizing Teachers' Thinking	*Actively*	**Imposing Teachers' Thinking** Prospective Teachers 6% Initial Participants 3% Advancing Participants 6% Emerging Teacher Leaders 0%	**Interrupted Exploring of Children's Thinking** Prospective Teachers 0% Initial Participants 3% Advancing Participants 0% Emerging Teacher Leaders 0%
	Minimally	**Limited Interacting** Prospective Teachers 94% Initial Participants 81% Advancing Participants 71% Emerging Teacher Leaders 41%	**Exploring Children's Thinking** Prospective Teachers 0% Initial Participants 13% Advancing Participants 23% Emerging Teacher Leaders 59%

Figure 4. Percentage of teachers in each participant group classified in the four interaction categories for extending children's understandings after children reached a correct answer.

Extending Through Exploring Children's Thinking

When extending through exploring children's thinking, a teacher engages in a conversation with a child about his or her strategy, often investigating connections to broader mathematical concepts or related problems. Similar to supporting moves, these extending moves can take a variety of forms and focus on different parts of the problem-solving process (Jacobs & Ambrose, 2008). For example, teachers may ask children to reflect on their completed (or abandoned) strategies, generate (and compare) additional strategies, or make links to symbolic notation. In the following example, we illustrate these types of moves as well as several others that served to explore the child's thinking.

In this 4-minute interaction, a teacher asked a first grader, Sean, to solve this problem: *Melanie has four pockets, and in each pocket she has 5 rocks. How many rocks does Melanie have?* Sean was able to solve the problem successfully without any supporting moves by the teacher. Specifically, he put out four single cubes (as pockets) and then put a set of 5 connected cubes (as rocks) beneath each single cube. To create two of the sets of 5, he counted the cubes by ones, but for the other two sets, he broke a ten-stick of maroon cubes in half and needed to count only one set of 5. Sean then counted the four sets of 5 by 5s and correctly answered 20.

At this point, the supporting interaction ended and the extending interaction began. The teacher further explored Sean's thinking in a variety of ways that potentially helped to advance his thinking. She first asked Sean to describe his strategy.

T: Okay, I noticed you do a couple things. Why don't you tell me what you did first?
S: Well, first I counted four, which I know.
T: Four what?
S: Four pockets.
S: And then I put 5 in each one, just to get started.
T: Okay.
S: Then after I did that, I can count by fives—fives would go 5, 10, 15, 20.

T: Excellent job. And I noticed that when you used these sort of maroon colored ones [cubes], tell me about how that worked out when you used those?
S: Well, I know 5 + 5 is 10. So, if I just took them apart, I would have two 5s, and that's what I need.

In this part of the conversation, the teacher asked specific follow-up questions to help Sean reflect on his strategy. Specifically, she asked for clarification on what the single cubes represented (pockets) and highlighted his method of splitting a 10-stick into 5 and 5.

The teacher then helped Sean link his thinking to symbolic notation. She asked him to write an equation for the problem, and he wrote *4 x 5 = 20*. She had Sean read the equation ("4 times 5 equals 20") and, building on what he read, asked, "What does *times* mean?" Sean responded, "Times means groups—four groups, and then you just put as many as you have in each one and then you just count it, and then you just put equal, and then the answer." The teacher continued to press for specificity by asking "four groups of?" and Sean responded, "Five."

Finally, the teacher explored the possibility of Sean's using a more sophisticated strategy—one that built on his use of tens (when splitting the maroon ten-stick).

T: Is there any way you could have used your groups of 10 to solve it a different way? I noticed how you used this group of 10 here (points to the sets of maroon cubes). If you didn't make pockets, is there another way you could have done it using the groups of 10?
S: Yeah, just put these without pockets (points to the four sets of 5 connected cubes).
T: You could just do those without pockets?
S: (removes the four individual cubes that stand for pockets and replaces them by drawing dots for each cube) And then I can just use dot markers for the pockets. Or I can just use it without these dots.
T: Or without the dots?
S: Mmm-hmm (nods yes).
T: So, you could mark your pockets or not mark your pockets. Okay. Is there any way that you might think about this problem without using cubes?
S: Yes.
T: How would you do that?
S: (writing the number sentence in the air with his fingers) Well, I could do 4, and then after I do 4, I can remember times, and then I do 5 equals 20.
T: (chuckling) Okay, all right. Great job. Nice thinking.

The teacher asked Sean to consider solving the problem a different way, and she persisted with this line of thinking through several strategies. Sean initially made tool changes (e.g., changing cubes for dots) and then acknowledged that the pockets could or could not be represented. However, he did not move to a more sophisticated strategy based on tens. Thus, the teacher tried one more time for another strategy (by asking for a strategy without cubes), but when Sean wrote the equation in the air(!), she decided it was time to move on.

We characterized this extending interaction as exploring children's thinking because the teacher was actively building on Sean's initial strategy and understandings. She encouraged reflection, a link to symbolic notation, and a more sophisticated (but connected) strategy. The fact that her request for a more sophisticated strategy was unsuccessful does not make it problematic. The request was appropriate given Sean's initial work with the maroon ten-stick. Equally important is the attention that the teacher paid to Sean's thinking during this conversation about the alternative strategy; the teacher did not impose her thinking but instead provided opportunities for Sean to engage with these ideas and for the teacher to explore his related understandings. It was this attention to Sean's thinking that led the teacher to eventually abandon her quest for a more sophisticated strategy when she realized Sean was not ready to generate a more sophisticated strategy with understanding.

Distribution findings. Similar to the supporting data, the extending data in Figure 4 showed that a majority of the participants did not demonstrate the expertise needed to explore children's thinking during extending but that this expertise could be learned with continued professional development. Specifically, although almost 2/3 of the Emerging Teacher Leaders were classified as exploring children's thinking when extending, only 1/4 or fewer of participants in the other groups were similarly classified.

Extending With Limited Interacting

When extending with limited interacting, a teacher often appears to be following a script after a correct answer is generated: (a) Ask the child how he or she solved a problem; (b) accept any response (however unclear); and (c) move on. For example, in this 2-minute interaction, a teacher asked a first grader, Laura, to solve the same problem Sean solved: *Melanie has four pockets, and in each pocket, she has 5 rocks. How many rocks does Melanie have?* Laura was able to solve the problem successfully without any supporting moves by the teacher. Specifically, Laura drew four squares, and in each square, she drew 5 dots (in dice formation). She then counted all her dots by ones, wrote 20, and circled it. She also wrote 5 + 5 = 10 + 10 = 20. At this point, the supporting interaction ended and the extending interaction began. The teacher asked a few questions to help Laura reflect on her strategy.

> T: What'd you come up with?
> S: Twenty.
> T: Twenty, how did you get 20?
> S: Because there's four pockets and each one is 5 so 5 + 5 is 10 and then 10 + 10 is 20.
> T: All right. Can you hold up your paper so I can see your paper clearly? Well done. Good solving.

We characterized this extending interaction as limited interacting because, although the teacher did not impose her thinking, she also engaged only minimally with Laura's thinking. This teacher's questioning followed the typical script of asking only for a strategy description, but sometimes teachers in this category expanded the script to include one or two questions about the details of the child's strategy. For example, in this scenario, the teacher might have asked, "Why did you draw four boxes?" Thus, interactions in this category ranged from asking no questions to posing only general questions ("How did you figure it out?") to asking only a few specific questions about the child's strategy. In all cases, engagement with the child's thinking was minimal, and this thinking often remained hidden. In Laura's case, her written work was clear enough to provide some sense of her strategy, but in other cases, strategies were less accessible.

Distribution findings. One prominent feature of the results in Figure 4 is the large number of teachers classified in the limited interacting category. This finding shows that consistent use of extending was rare; teachers sometimes demonstrated a willingness to ask children how they solved problems but did not probe or explore their thinking further. This lack of extension has been found elsewhere (see, e.g., Fraivillig et al., 1999) and points to the need for professional development to address these missed opportunities.

We recognize that, theoretically, extending could go on indefinitely for each problem, and decisions about when to stop for a particular session must be made. Nonetheless, given that so few teachers extended in an interview situation—when time is less of an issue than in classroom situations—we are concerned that teachers are missing key opportunities to continue mathematical conversations after the correct answer is given. For example, consider what Sean (and Sean's teacher) would have missed by not extending the conversation: articulation and consolidation of his strategy, highlighting of the use of halving a ten-stick in his strategy, discussion of the corresponding number sentence, exploration of the meaning of multiplication, and consideration of alternative (potentially more sophisticated) strategies. Even though Sean did not ultimately produce these alternative strategies, he engaged with the ideas and the teacher learned where Sean's existing understandings were. In contrast, we learned little about Laura's understandings, and opportunities for further discussion were missed. For example, a teacher might have wanted to explore (a) why Laura drew the dots in dice formation, (b) how she knew that she could double 10 rather than count an additional two 5s, or (c) whether Laura could have counted the total any other way (e.g., by 5s). The teacher may have also wanted to discuss the number sentence, which may have effectively showed Laura's thinking but also reflected a problematic use of the equal sign to represent a running total (e.g., $5 + 5 \neq 20$). We are not suggesting that all of these issues needed to be addressed. We are suggesting, however, that by omitting exploration of any of these (or other related) issues, the teacher (and Laura) missed important opportunities for mathematical conversation. Our data showed that most teachers regularly missed these opportunities.

CONCLUSIONS

Using data from a cross-sectional study, we analyzed videotaped problem-solving interviews in which 129 teachers each worked one-on-one with 3 students. We suggest that our study provides contributions in three areas: assessment of expertise, development of expertise, and attention to extending.

Assessment of Expertise

Our goal was to capture a snapshot of each teacher's practice to explore their one-on-one conversations with students, and, in particular their expertise in eliciting and building on children's thinking. We were particularly interested in capturing the extent to which the child's thinking was central to the conversation. Through our Framework of Engagement With Children's Mathematical Thinking in One-on-One Conversations, we provide a structured way to examine and assess teacher-student interactions, and, in particular, a way to draw attention to both the supporting and extending components of problem solving. Continuing analyses of our relatively large data set ($N = 129$ teachers and 1798 problems) will enable us to further identify the subtle variations that characterize the four major types of interactions reflected in the framework.

Our goal in developing the framework was to create a structure to provide meaningful distinctions for researchers but also of a grain size to be useful to professional developers and teachers. Researchers interested in capturing teacher change should appreciate that our framework was effective, when applied to our cross-sectional data, in capturing important distinctions among teachers with varying levels of experience with children's mathematical thinking. Professional development work is also underway to determine the usefulness of these categories as a reflection tool for professional developers and teachers, and preliminary conversations have been promising. Our hope is that our framework will provide not only a self-reflection tool but also language to characterize teachers' responsiveness to children's mathematical thinking. In doing so, we join others working to decompose teaching into core activities that can be discussed and practiced (Ball, Sleep, Boerst, & Bass, 2009; Grossman & McDonald, 2008; Lampert 2001; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010). Future research could explore whether this explicit attention to the ways that children's thinking is (and is not) central in conversations could help to accelerate teachers' acquisition of the skills, knowledge, and dispositions needed to explore children's mathematical thinking.

Development of Expertise

Our cross-sectional design provided evidence to help us begin to understand the development of this expertise. First, in both supporting and extending, we found evidence that not all teachers readily elicit and build on children's mathematical thinking—even when the interview directions explicitly asked them to do so. However, from our cross-sectional data, we also found that this expertise can develop over time.

Second, we found striking the fact that, throughout our analyses, the Initial Participants and Advancing Participants looked quite similar. One might have expected otherwise. Advancing

Participants had completed 2 years of professional development whereas the Initial Participants had yet to begin. Do these results indicate that individuals do not change during the first 2 years of professional development and that we must wait for 4 or more years to see growth? We have evidence to the contrary. Data from other measures in our cross-sectional study showed that Advancing Participants differed from Initial Participants in many important ways—in their beliefs, in their content knowledge (Philipp, Siegfried, Schappelle, & Jacobs, 2011), in their inquiry stances (Lamb et al., 2009), and in their abilities to notice children's mathematical thinking in instructional settings (Jacobs, Lamb, & Philipp, 2010). Furthermore, we have evidence from Advancing Participants' self-reports and corresponding case studies that teachers' practices were changing as well (e.g., increased use of story problems, increased opportunities for children to generate [rather than imitate] solution strategies, and so on).

Why did the two groups look so similar in the interviews? Eliciting and building on children's thinking—in the moment—requires a variety of elements to be in place and coordinated (e.g., belief in children's abilities to reason through problems without explicit instructions, skill in eliciting children's thinking, knowledge about mathematical connections, understanding of children's developmental trajectories, and so on.) Thus, we were not surprised to find that this expertise takes longer than other expertise to emerge. Because this expertise is critical for the vision of mathematics teaching depicted at the beginning of the paper, our data provide strong evidence for the importance of sustained professional development over multiple years. Specifically, teachers do not appear to have consolidated this expertise from teaching experience alone (i.e., average number of years of teaching for the Initial Participants was 14–16 years) or from 2 years of professional development focused on children's thinking (i.e., Advancing Participants).

Our data also helped us to develop a profile for each of our participant groups, providing insights for professional developers trying to meet the needs of individuals in their classes or workshops.

Prospective Teachers. These individuals generally imposed their own thinking when supporting children in the interviews and interacted in a limited way when extending. Notably, only 2 Prospective Teachers were classified as exploring children's thinking in either supporting or extending. We hypothesized that Prospective Teachers might benefit from exposure to student-generated strategies to gain an appreciation for children's abilities to generate strategies without being told step-by-step what to do. Sharing examples of productive extending interactions could be useful as well.

Initial Participants. These teachers were generally unlikely to actively prioritize either children's thinking or their own thinking in supporting or extending. However, almost one third of the Initial Participants were classified as exploring children's thinking in either supporting or extending, indicating that some individuals had developed expertise. We hypothesized that Initial Participants might benefit from opportunities to observe and discuss patterns in children's mathematical thinking and teacher moves that can help children progress both before and after the correct answer is given.

Advancing Participants. These teachers were the most mixed in their performance. In supporting, Advancing Participants were placed almost equally in the four categories of the framework, but in extending, they were unlikely to explore children's thinking. As discussed above, perhaps most noteworthy were the almost one-fourth who were classified as imposing teachers' thinking.[4] Nonetheless, some did demonstrate expertise as 42% were classified as exploring children's thinking in either supporting or extending. We hypothesized that Advancing Participants, like Initial Participants, might benefit from opportunities to observe and discuss patterns in children's mathematical thinking and teacher moves that can help children before and after the correct answer is given. They may also benefit from special attention to the distinction between exploring children's thinking and imposing teachers' thinking in supporting situations.

Emerging Teacher Leaders. These teachers generally actively prioritized children's thinking in both supporting and extending; more than three fourths of the Emerging Teacher Leaders were classified as exploring children's thinking in either supporting or extending. We concluded that Emerging Teacher Leaders had experienced sufficient time and support not only to value children's thinking but also to understand it and find ways to elicit and build on it, although they demonstrated this expertise more consistently in supporting than in extending. We hypothesized that they might benefit from opportunities to refine their questioning when eliciting and building on children's thinking, with special attention to the benefits of exploring children's thinking in extending interactions.

Attention to Extending

Our data highlight how teachers generally end mathematical conversations after the correct answer is reached—even in interview situations when practical time constraints are minimized. These truncated conversations are problematic because they preclude important learning

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[4] We are conducting additional analyses focused on the Prospective Teachers and Advancing Participants who were classified as imposing teachers' thinking in supporting. Preliminary analyses indicate subtle but important differences between the groups in terms of a stronger tendency by Prospective Teachers to not only impose their thinking but also to actively interrupt children when they were visibly engaged in problem solving and to be more likely to take over the moving of physical tools or the writing of algorithms. In contrast, Advancing Participants tended to use more subtle verbal guidance in imposing their thinking.

opportunities for children and limit what teachers can learn about children's understandings. We want to suggest another potential positive contribution of extending interactions. Given that teachers rarely prioritized their own thinking in extending interactions, we hypothesize that once the teacher's need for the student to reach a correct answer is satisfied, he or she may feel less pressure to direct the interaction down particular paths and more freedom to explore with the child. Thus, extending interactions may initially be more promising venues than supporting interactions for teachers to learn to value and to practice exploring children's mathematical thinking; helping teachers come to see the correct answer not as the end of a conversation but as a new beginning may enable them to see new opportunities for eliciting and building on children's mathematical thinking.

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