

# WHY WE SHOULD RECONSIDER USING WORKSHEETS AND WORD PROBLEMS (AND WHAT WE SHOULD BE DOING INSTEAD)

**ROBERT KAPLINSKY**

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robertkaplinsky.com

@robertkaplinsky

**WANT THE RESOURCES?**

Download them at

robertkaplinsky.com/svmi













# paradigm shift



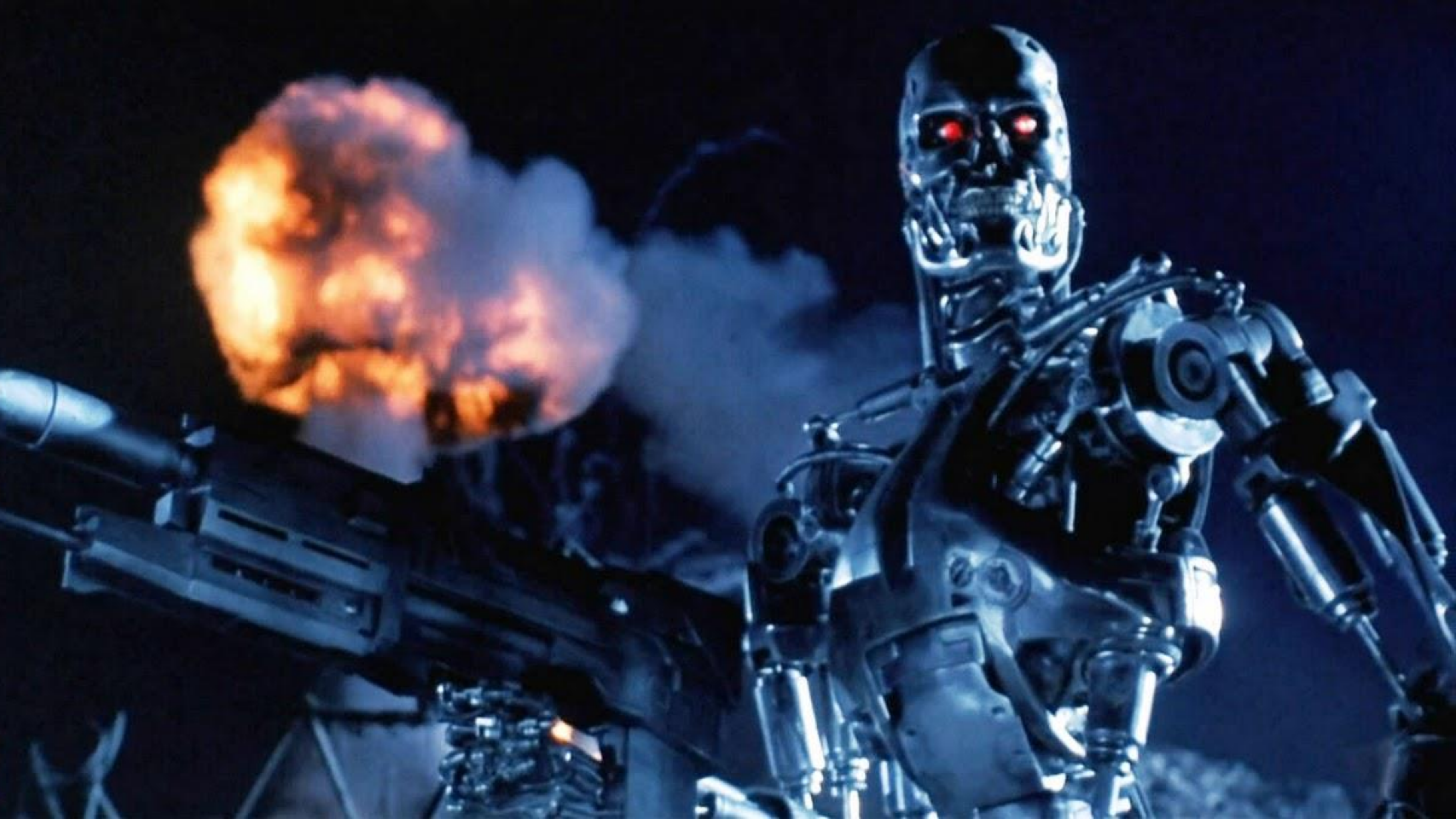
# GOALS

☐ **CORRECT ANSWERS = UNDERSTANDING?**

☐ **RECONSIDER USING WORD PROBLEMS**

☐ **RECONSIDER USING WORKSHEETS**









11a 12a 13oi 14oi 15oi 16b 17f

21k 22g 23e 24oi 25oi 26f 27g

31f 32i 33oi 34f 35k 36k 37m

41w 42o 43o 44f 45g 46e 47f

54p 55p 56p 57l

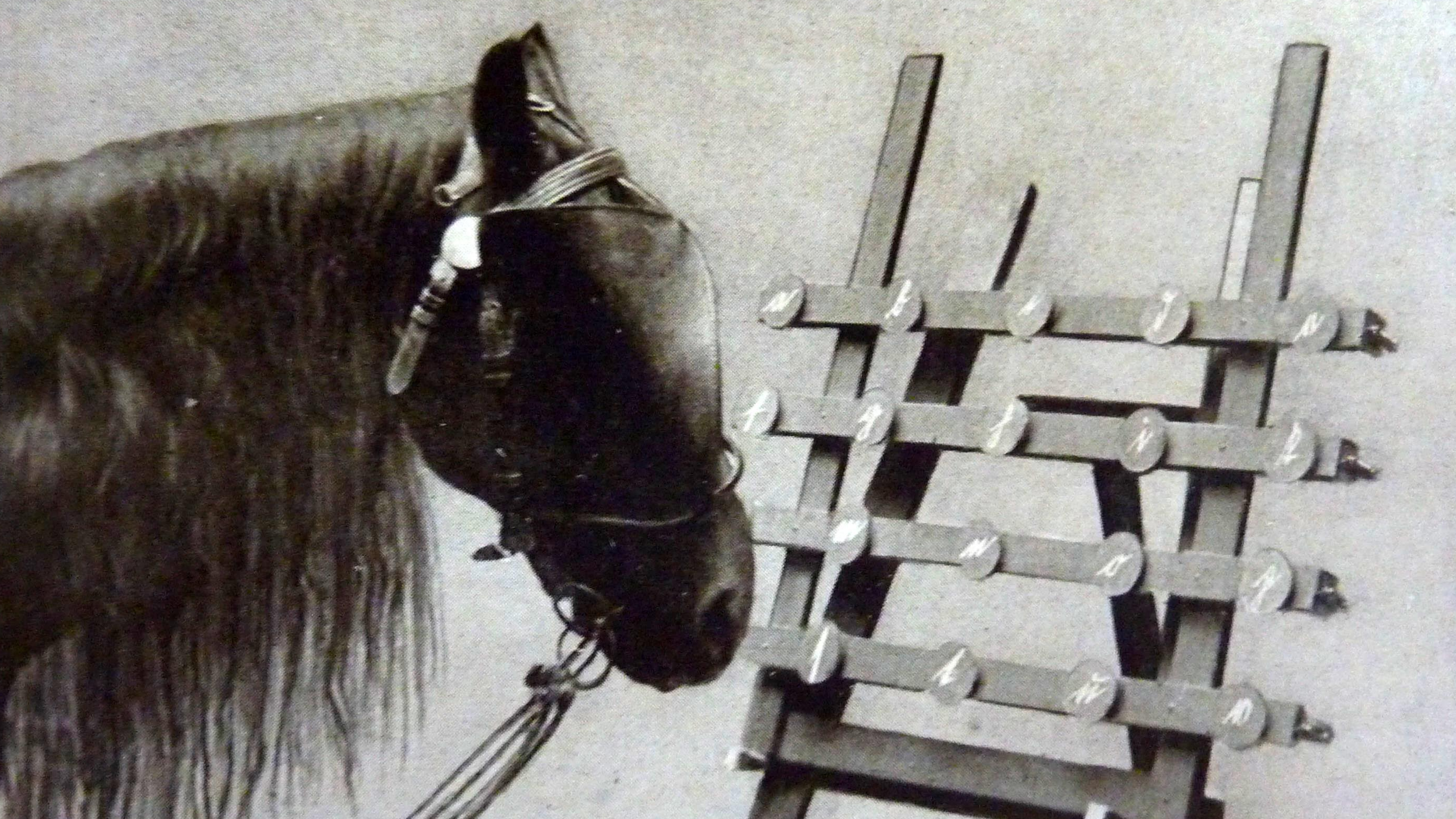
64o 65o 66g 67g

$$\frac{2}{3} + \frac{3}{4} =$$

$$26743:8=$$

$$712986 \times 3 =$$











Yes... no... uh...  
yes... maybe?

**MANY STUDENTS**



# CHINESE ROOM



见体配字母的常套



见体配字母的常套



# DISCUSSION TIME

- How is it possible for students to get correct answers yet not understand what they did?
- How can we tell if the problems we use are Chinese room and horse proof?



# GOALS

- ☒ **CORRECT ANSWERS = UNDERSTANDING?**
- ☐ **RECONSIDER USING WORD PROBLEMS**
- ☐ **RECONSIDER USING WORKSHEETS**



# SAME OR DIFFERENT?

Describe at least three ways in which the problems are the same and three ways they are different:

- A. How many pizzas do you need to buy?
- B. You and your seven friends want to have pizza for dinner. Each person will eat three slices of pizza. Each pizza has eight slices. How many pizzas do you need to buy?





**Robert Kaplinsky**  
@robertkaplinsky



Which of these are word problems:

A) How many pizzas do you need to buy?

B) You and your seven friends want to have pizza for dinner. Each person will eat three slices of pizza. Each pizza has eight slices. How many pizzas do you need to buy?

[#MTBoS](#) [#iteachmath](#)

A

13%

B

36%

**Both**


**44%**

Neither

8%

709 votes · Final results





Why do we  
have word  
problems?



MILNE'S  
INDUCTIVE ALGEBRA

Milne's Inductive Algebra © 1881



**183. DIRECTIONS FOR SOLVING.**—*Represent one of the unknown quantities by  $x$ , and from the conditions of the problem find an expression for each of the other quantities given.*

*Find from the problem two expressions that are equal, and express them as an equation.*

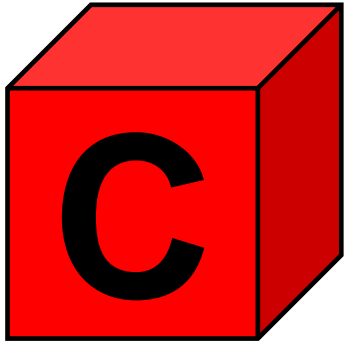
*Solve the equation.*

51. When the half of a certain number is added to the number, the sum is as much more than 60 as the number is less than 65. What is the number? *50 ans*

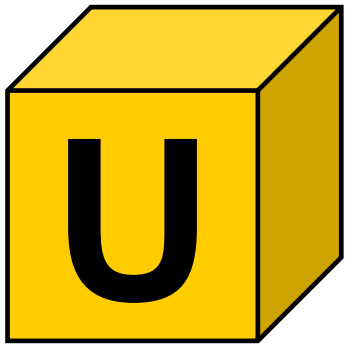
52. The difference between two numbers is 8, and the quotient arising from dividing the greater by the less is 3. What are the numbers?

53. A man left one-half of his property to his wife, one-sixth to his children, a twelfth to his brother, and the rest, which was \$600, to charitable purposes. How much property had he?

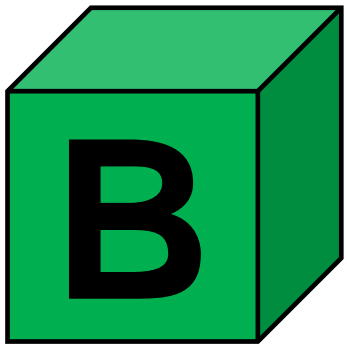




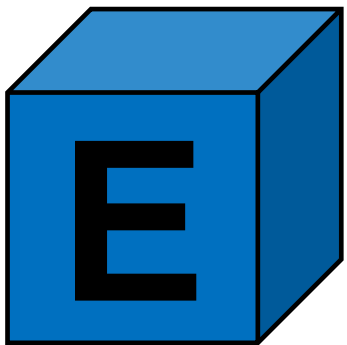
CIRCLE the numbers



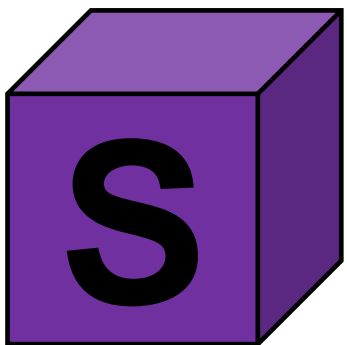
UNDERLINE the question



BOX the key words



~~ELIMINATE~~ info not needed



SOLVE and check ✓



In a class of 30 children, there are 3 girls for every 2 boys. How many girls are there altogether?



**Source: Marilyn Burns**



There are 125  
sheep and 5 dogs  
in a flock. How old  
is the shepherd?







Making sense: 8  
Not making sense: 24

$$\begin{array}{r} 5 \overline{) 125} \\ \underline{10} \phantom{0} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

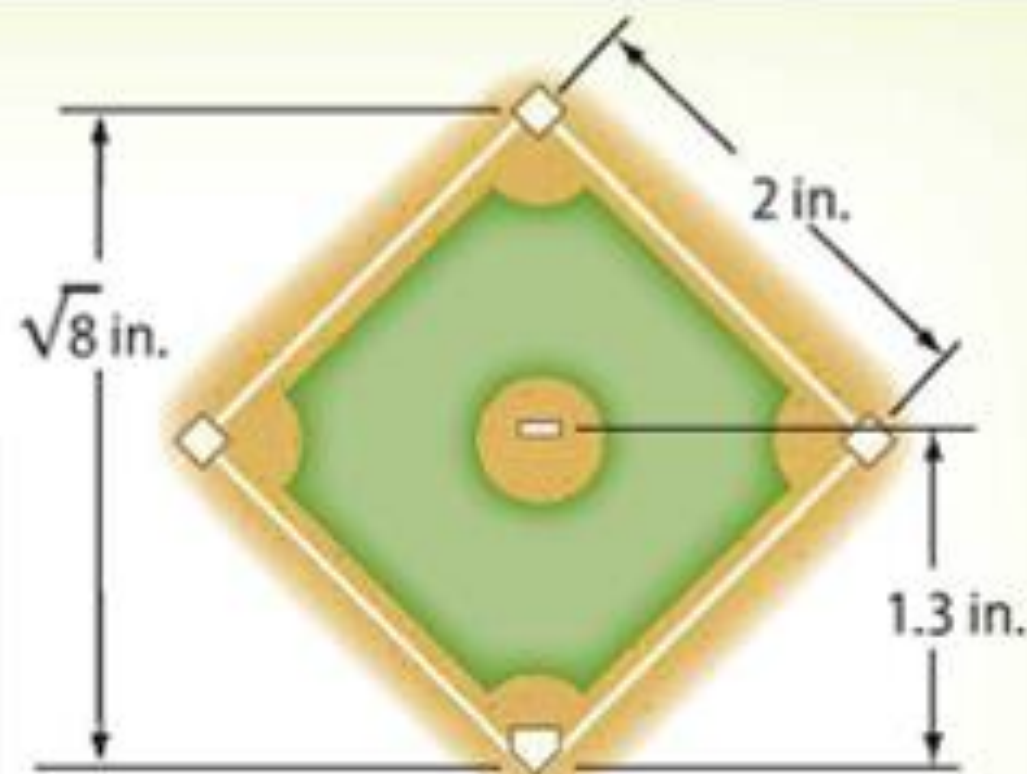




## Real-World Link

**Sports** Major League baseball has rules for the dimensions of the baseball diamond. A model of the diamond is shown.

1. On the model, the distance from the pitching mound to home plate is 1.3 inches. Is 1.3 a rational number? Explain.



2. On the model, the distance from first base to second base is 2 inches. Is 2 a rational number? Explain.

3. The distance from home plate to second base is  $\sqrt{8}$  inches. Using a calculator, find  $\sqrt{8}$ . Does it appear to terminate or repeat?



## Common Core State Standards

### Content Standards


8.NS.1, 8.NS.2, 8.EE.2

### Mathematical Practices

1, 3, 4, 6







When you remove a  
problem's context and it's  
still solvable, it's nothing  
more than a worksheet.





# Doritos® & Cheetos® Mix

**20**  
Singles

DORITOS® Nacho Cheese Flavored Tortilla Chips 1 OZ. EA. DORITOS® COOL RANCH® Flavored Tortilla Chips 1 OZ. EA. CHEETOS® Puffs Cheese Flavored Snacks  $\frac{7}{8}$  OZ. EA. CHEETOS® Crunchy Cheese Flavored Snacks 1 OZ. EA.

20 INDIVIDUAL BAGS:  $\frac{7}{8}$  OZ. EACH, 1 OZ. EACH, TOTAL NET WT.  $19\frac{5}{8}$  OZ. (1 LB.  $3\frac{5}{8}$  OZ.) 556.3 g

⚠ WARNING: PREVENT ENTANGLEMENT AND STRANGULATION. KEEP THIS BAG AWAY FROM YOUNG CHILDREN. IT IS NOT A TOY.



# THINKING TIME

- Why did many of you expect there to be five of each?
- Why was it not five of each?
- How might they decide on this combination?





# Classic Mix

**20**  
Singles

LAY'S® Classic Potato Chips. DORITOS® Nacho Cheese Flavored Tortilla Chips. DORITOS® COOL RANCH® Flavored Tortilla Chips. CHEETOS® Crunchy Cheese Flavored Snacks. SUNCHIPS® Original Multigrain Snacks. FRITOS® Original Corn Chips (All 1 OZ. Each)

20 INDIVIDUAL BAGS: 1 OZ. EACH, TOTAL NET WT. 20 OZ. (1 LB. 4 OZ.) 567 g

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# MATH MODELING

☐ HOW DO WE MAKE SENSE OF MATH MODELING?

☐ IS IT JUST ANSWERING QUESTIONS?

☐ HOW IS MATH MODELING USED IN REAL LIFE?

☐ HOW DO WE HELP OUR STUDENTS IMPROVE?

















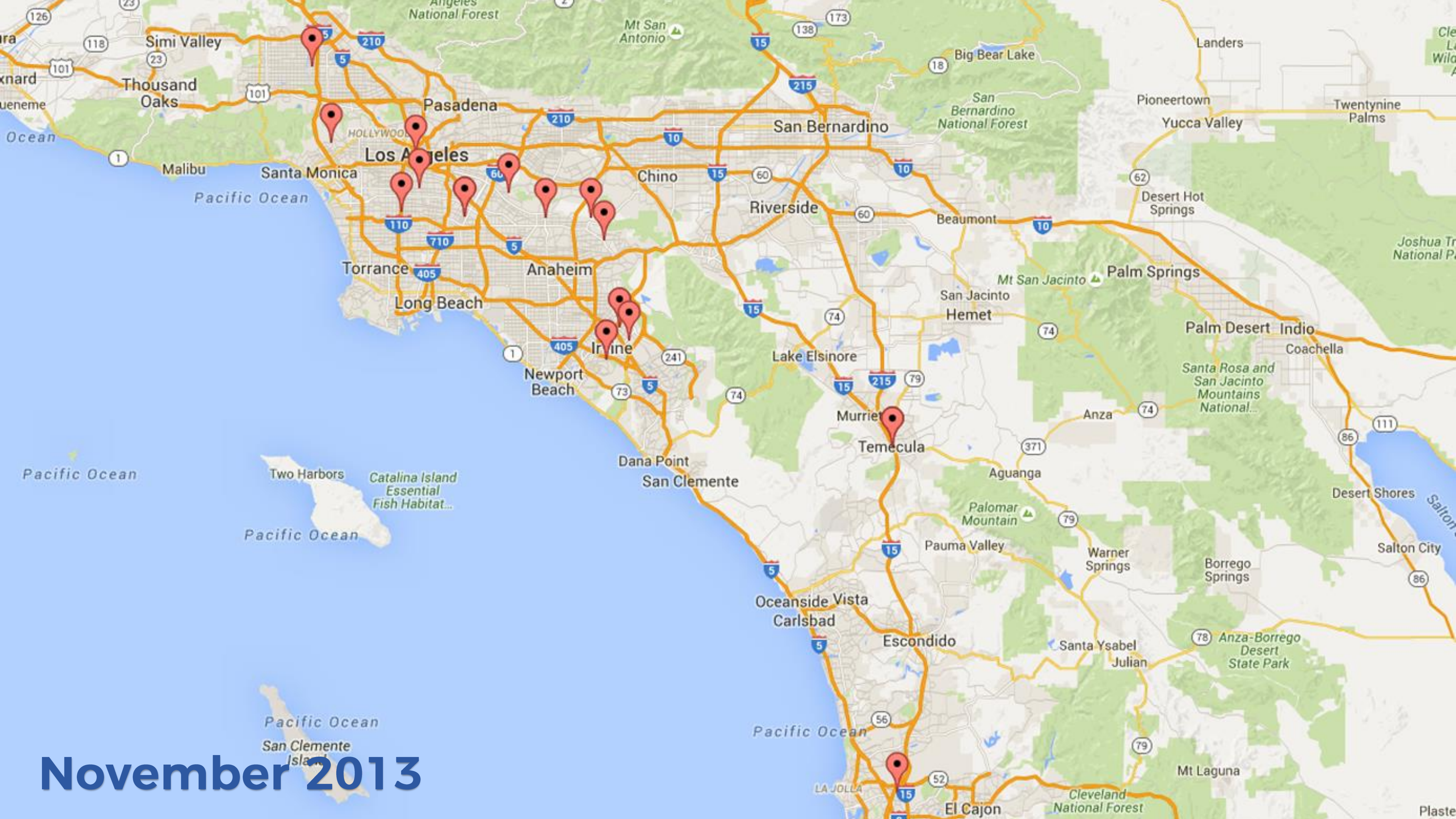
```
graph TD; Spies --> Analysts; Analysts --> Model; Model --> Spies;
```

Spies

Analysts

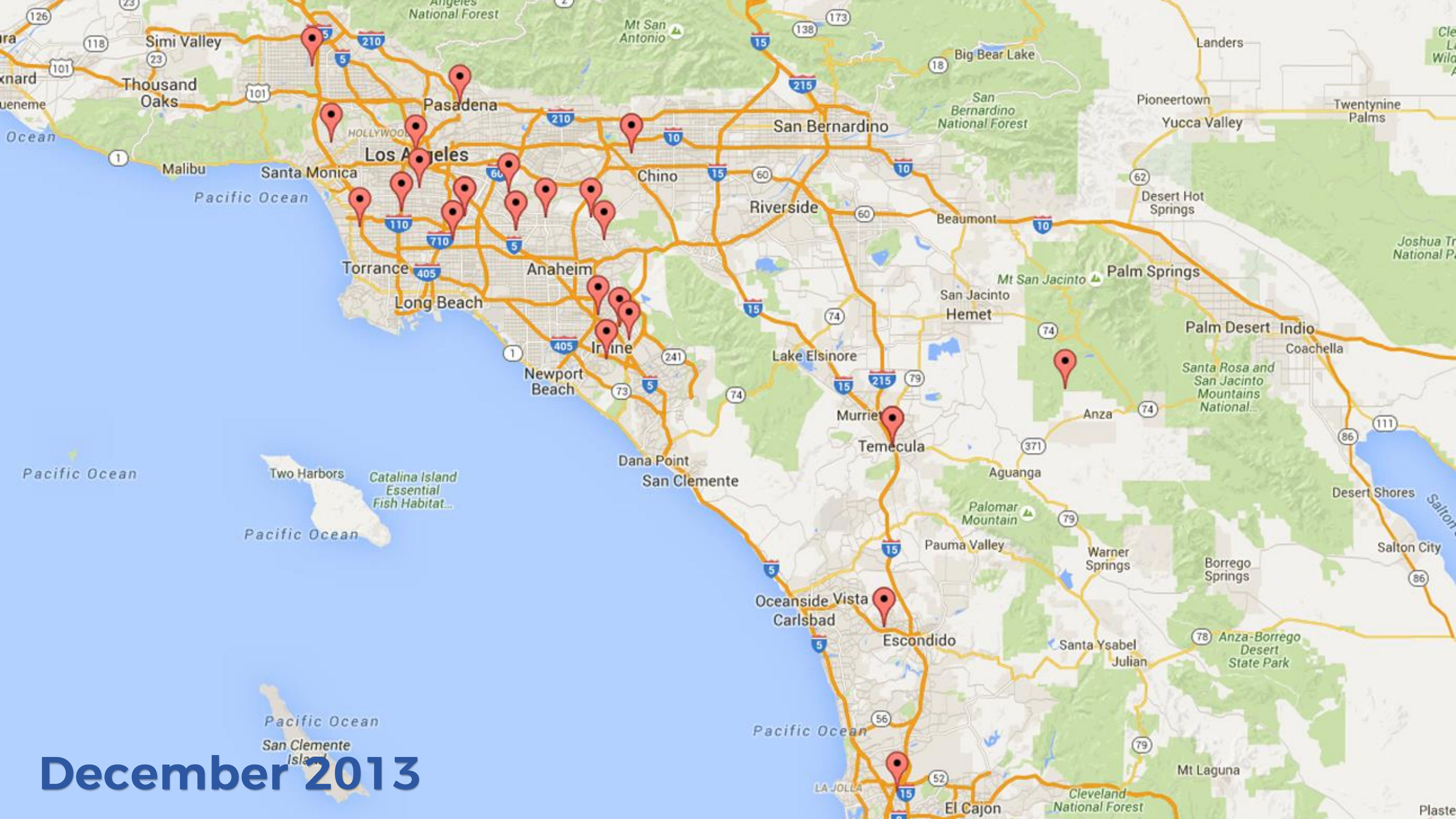
Model





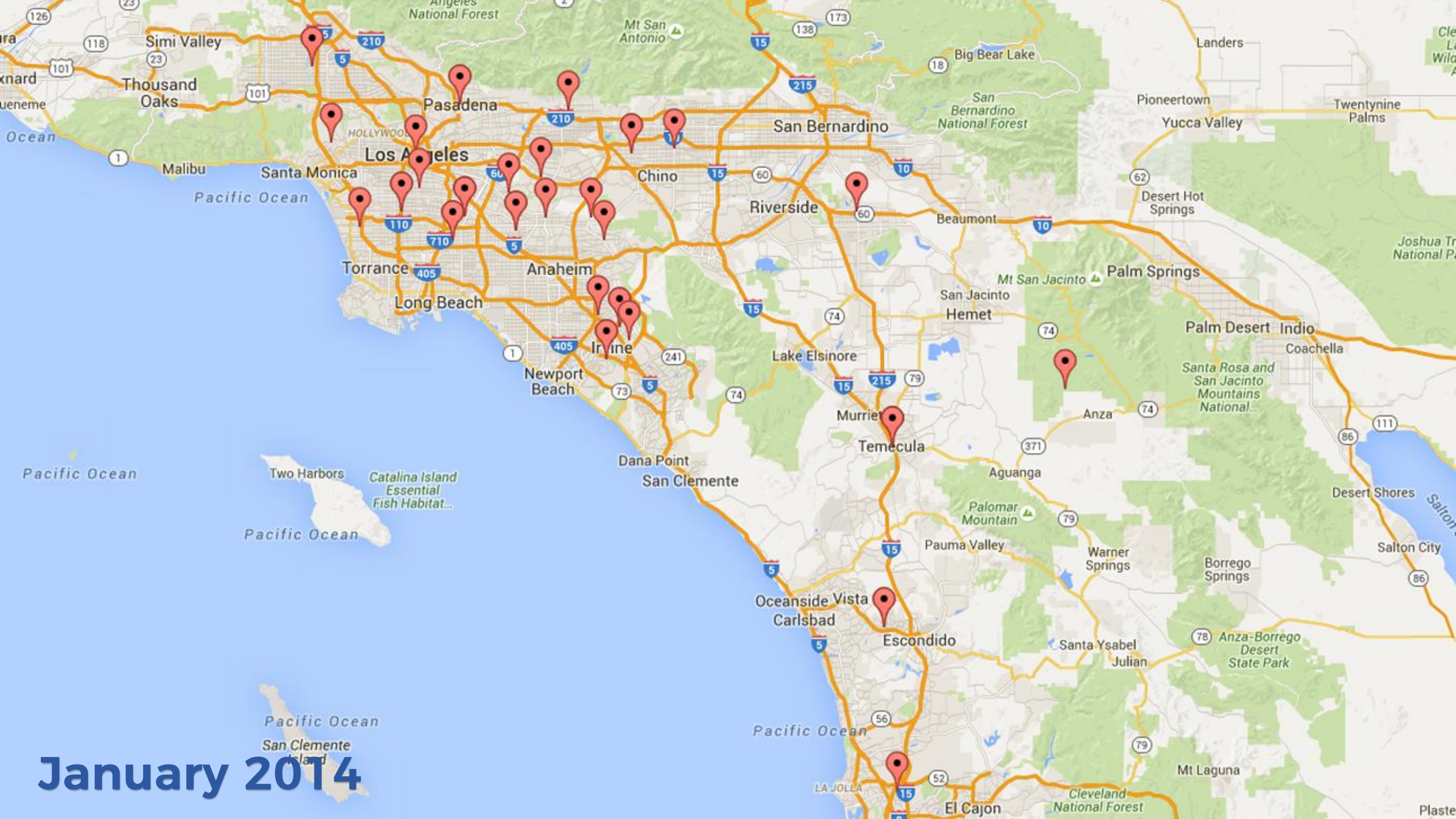
November 2013





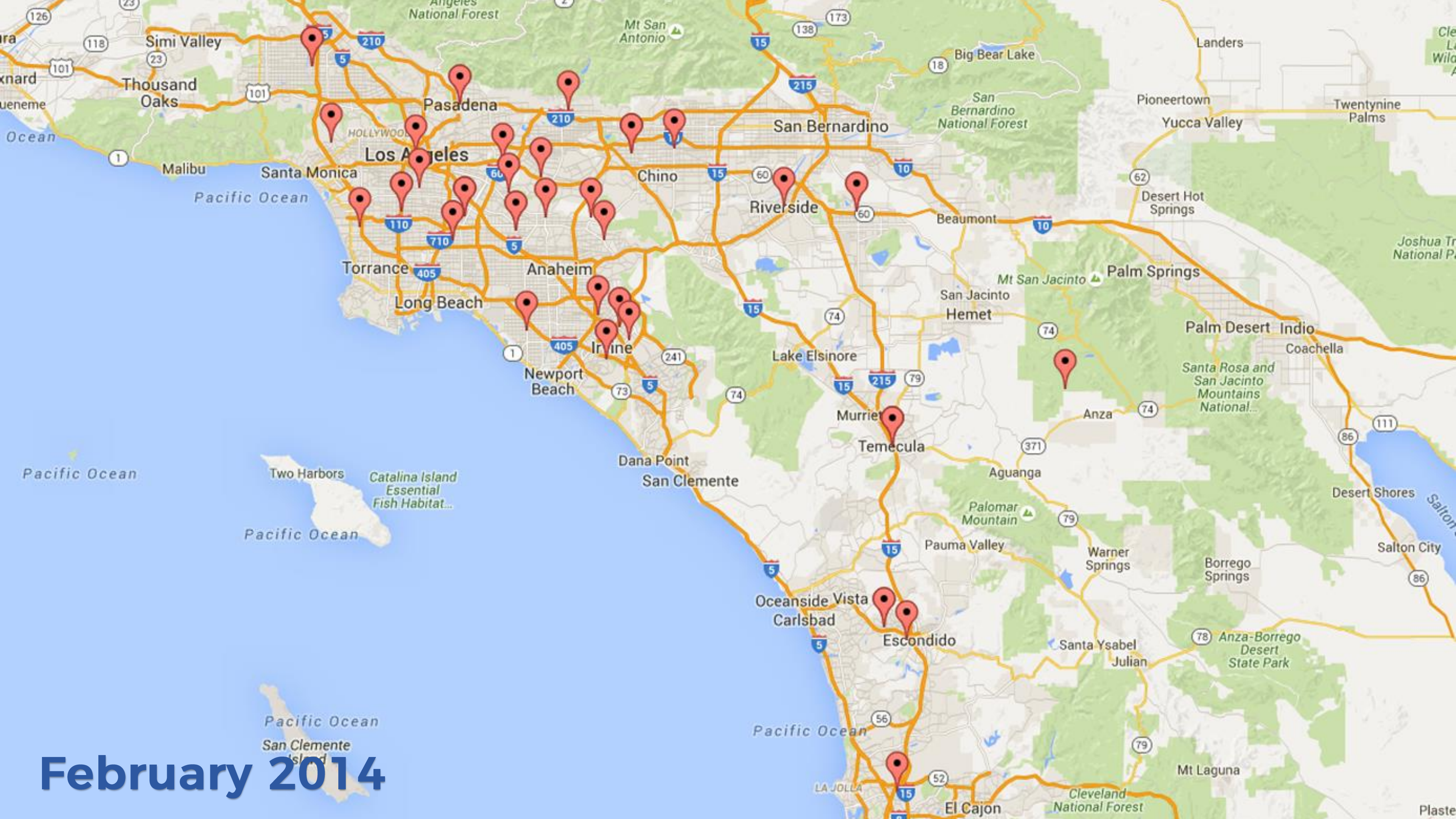
December 2013





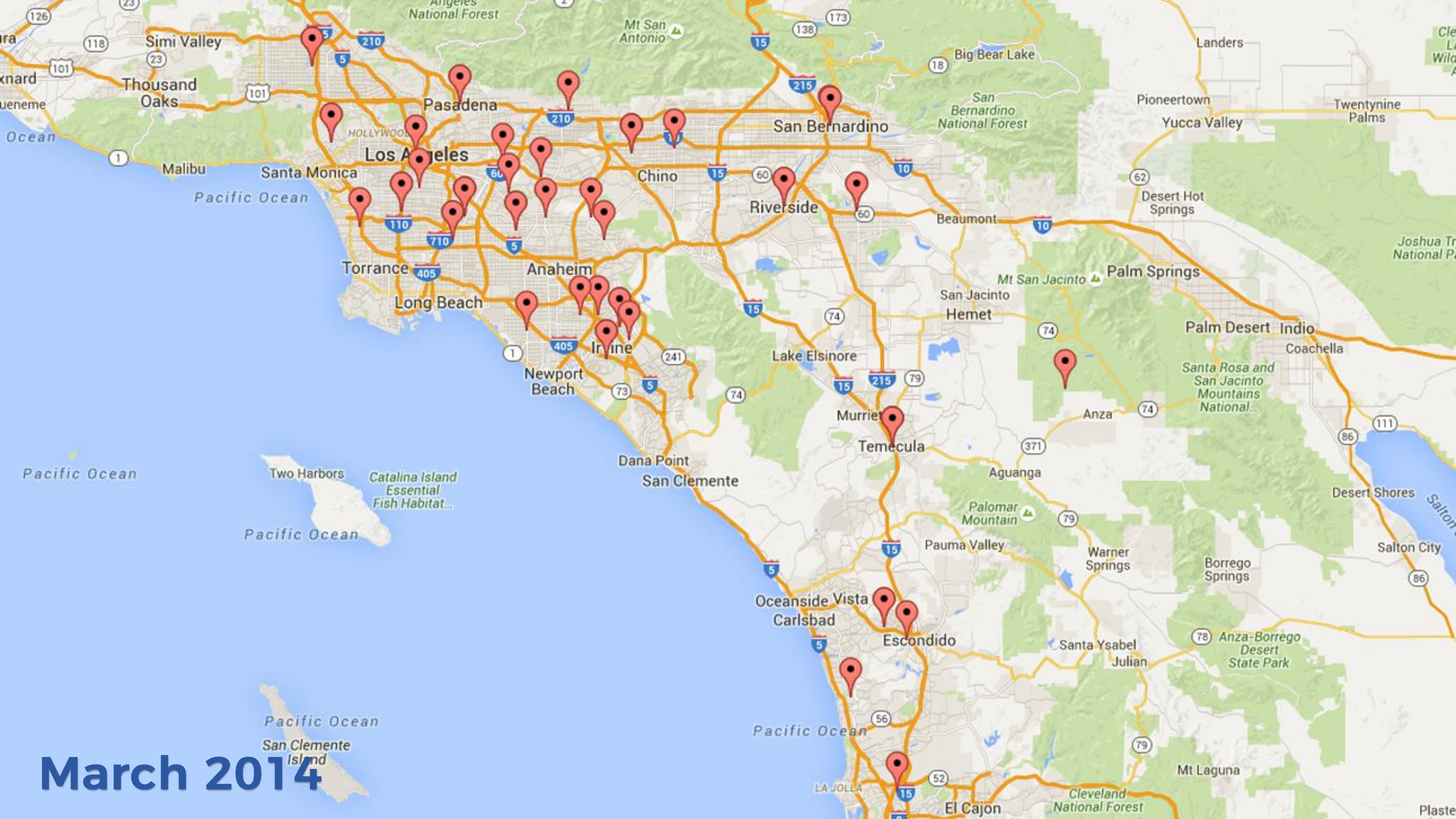
January 2014





February 2014





March 2014

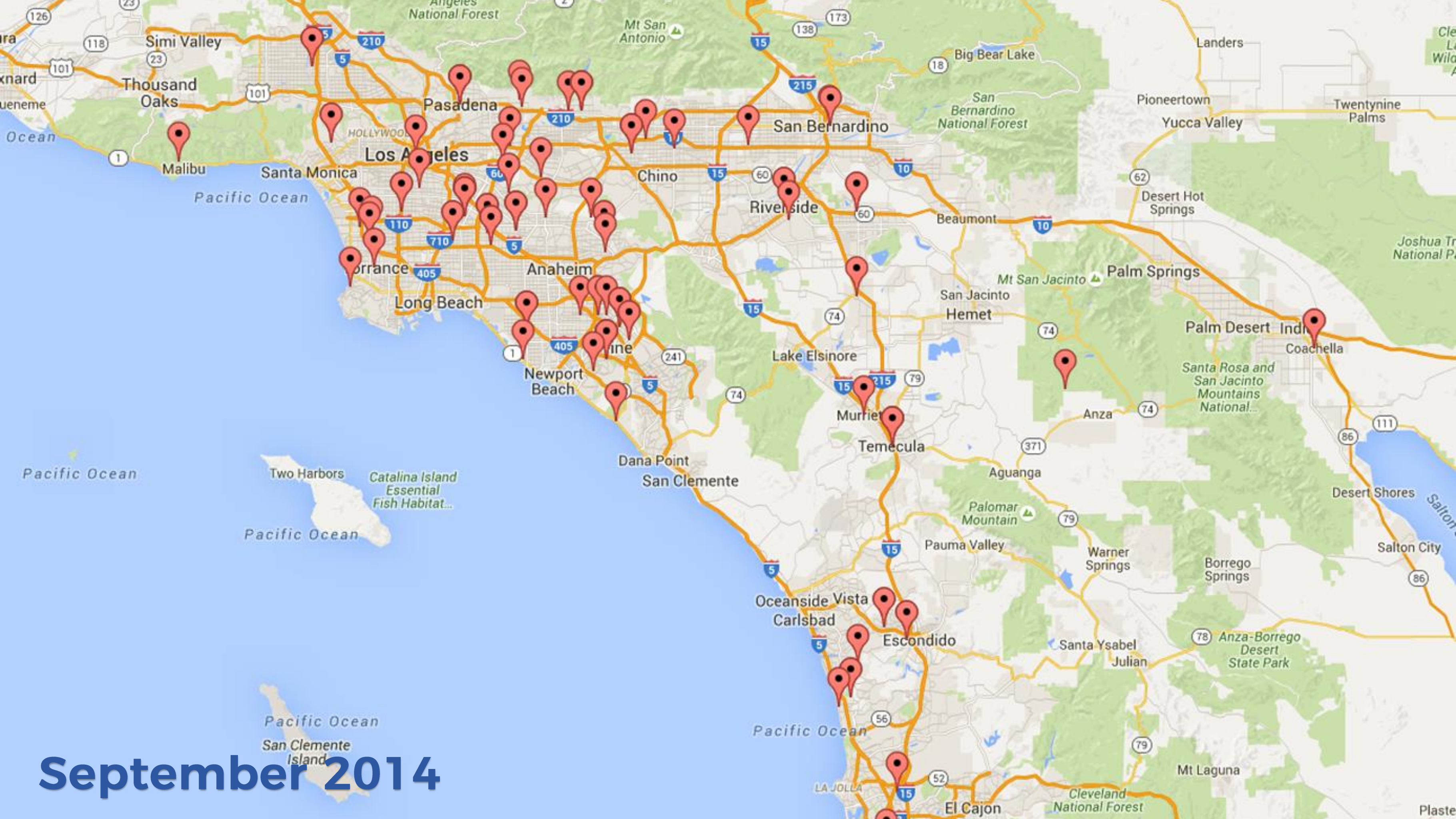






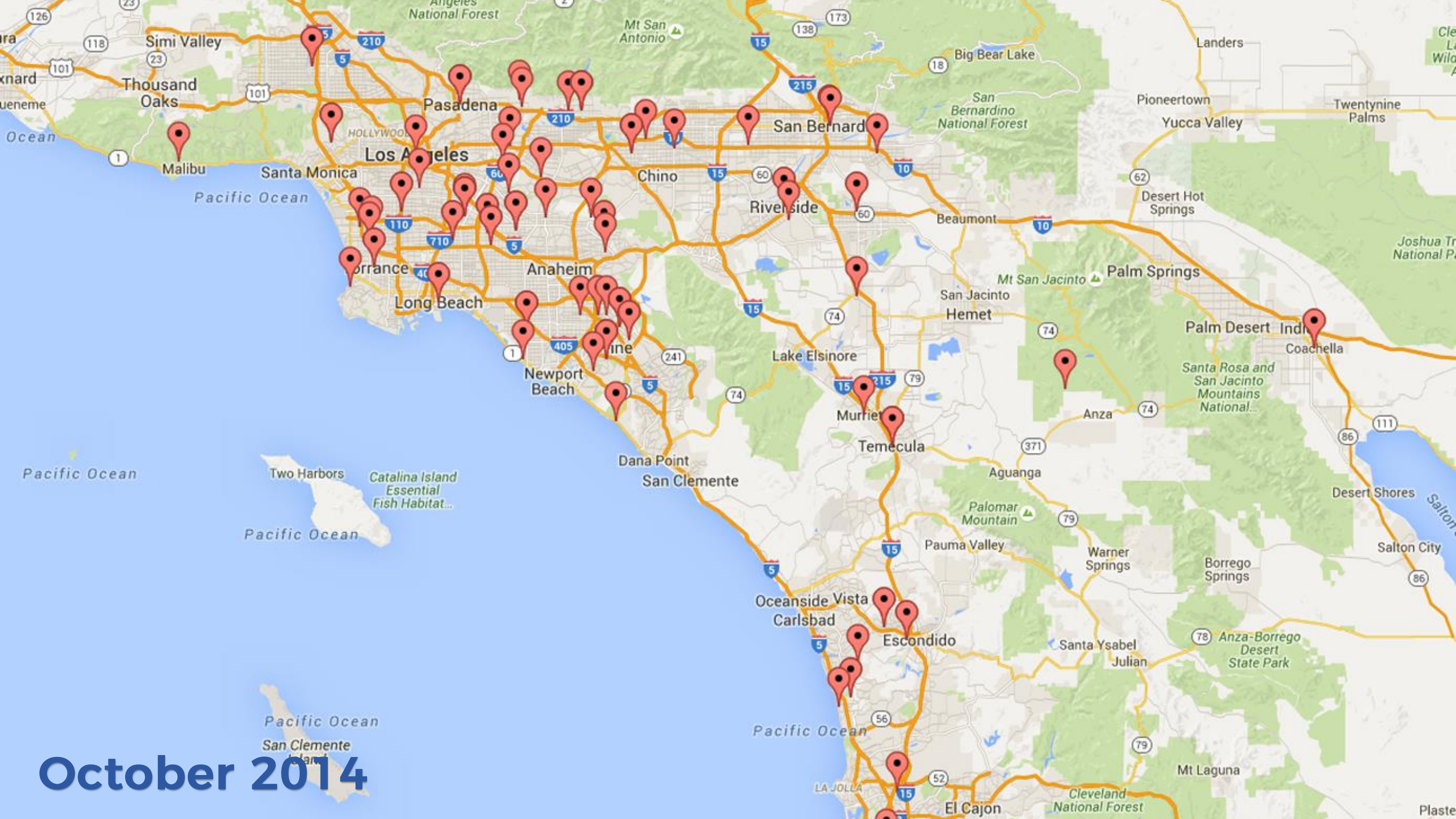






September 2014



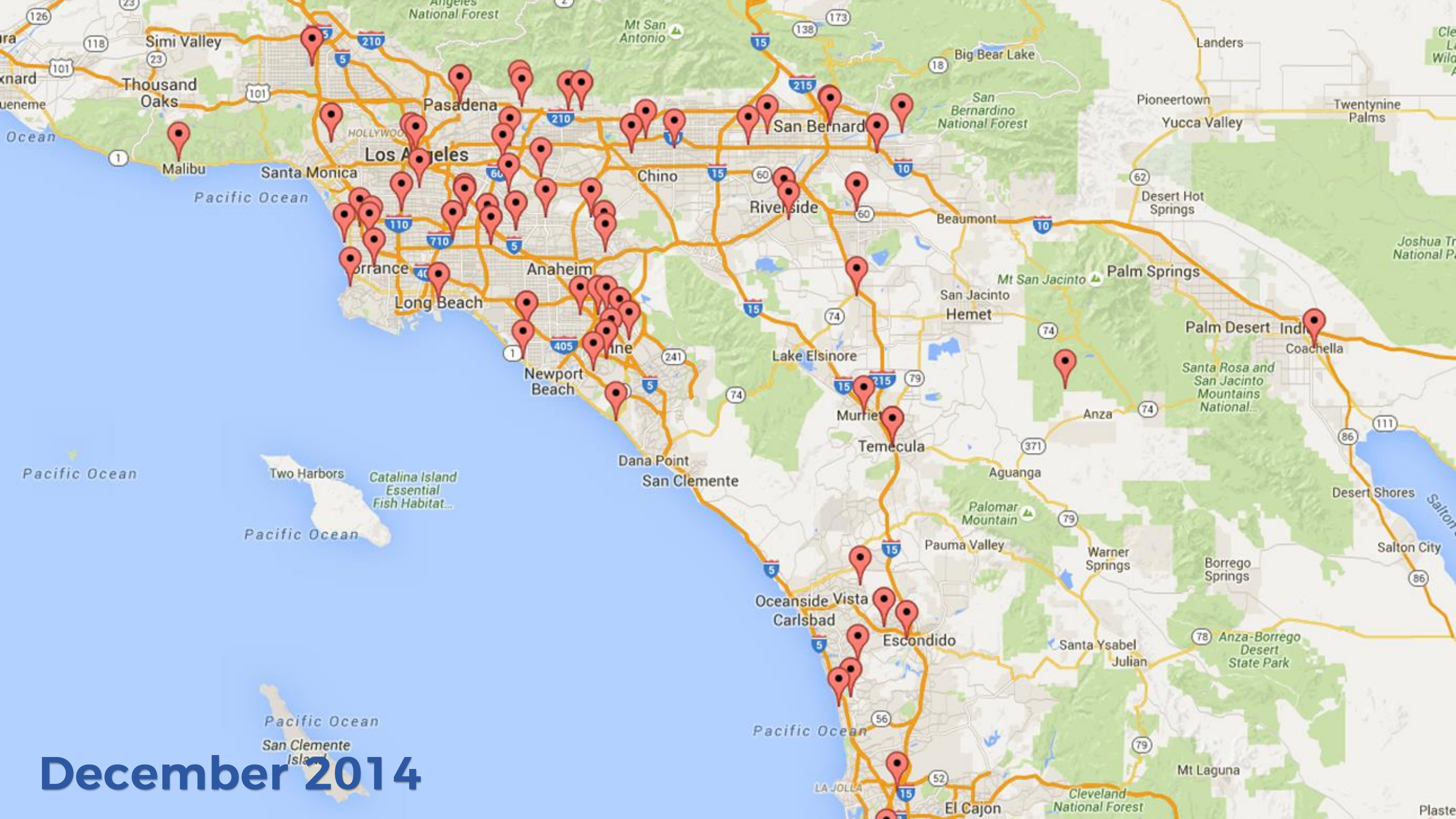


October 2014



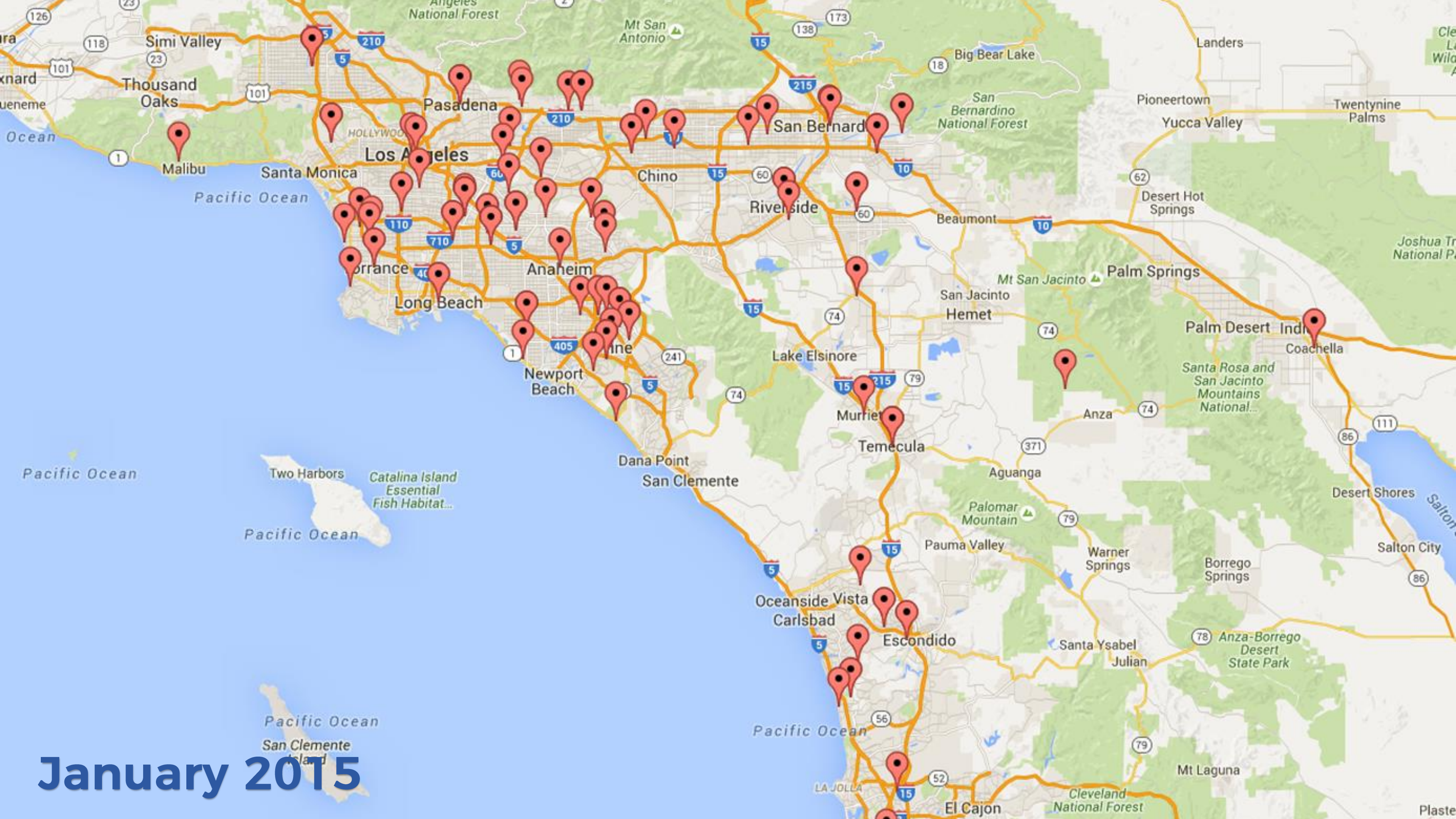






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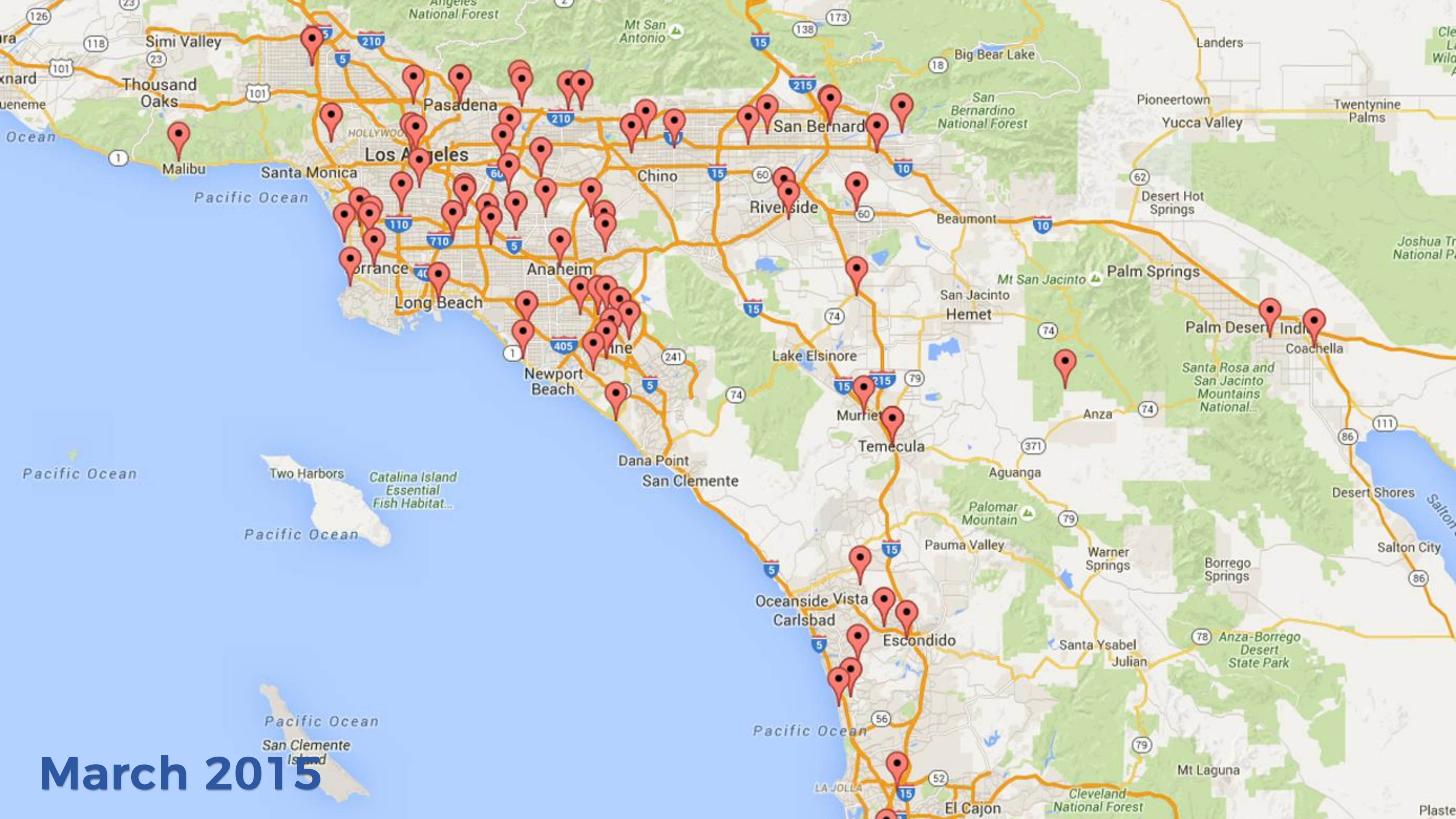


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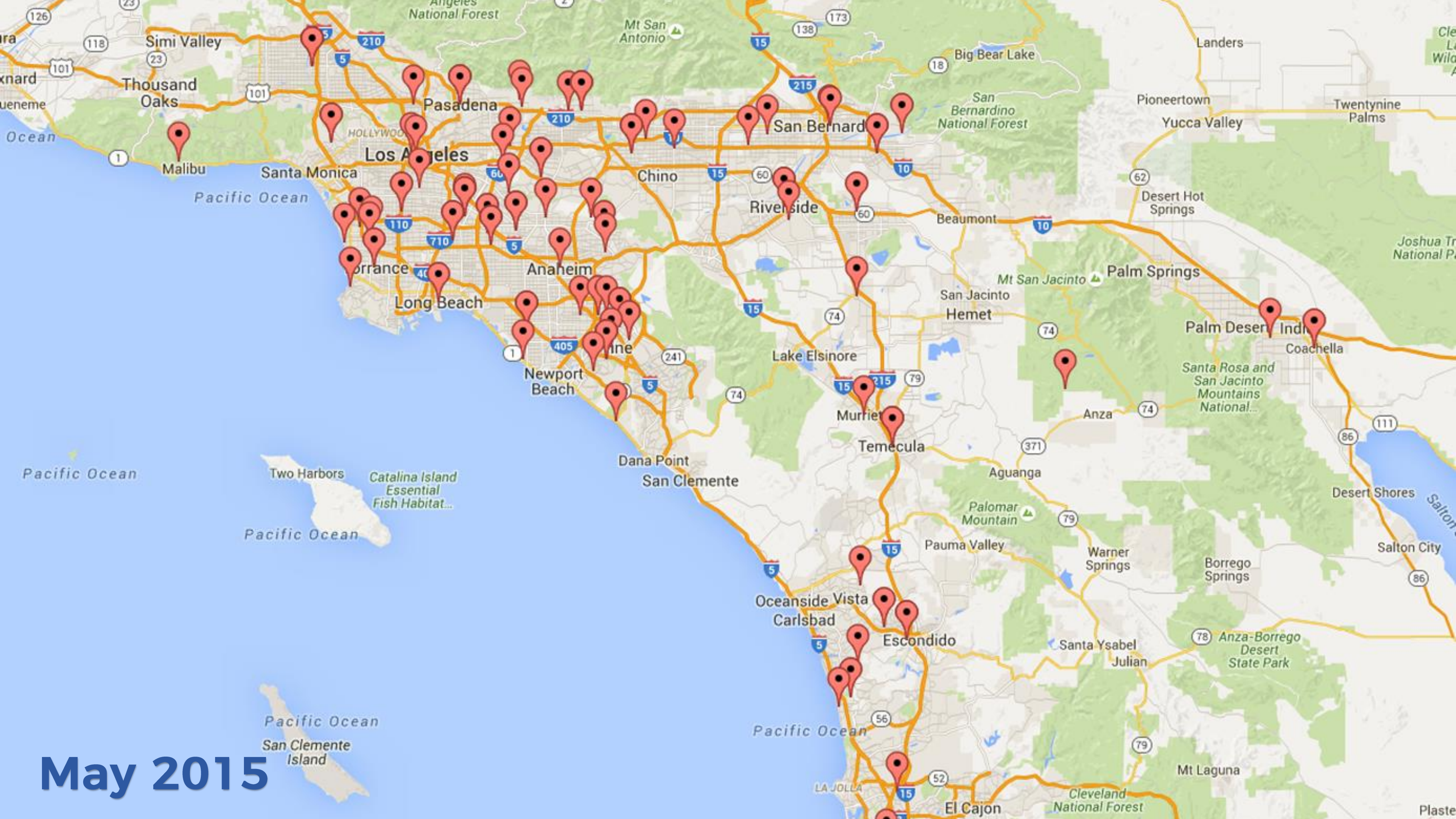






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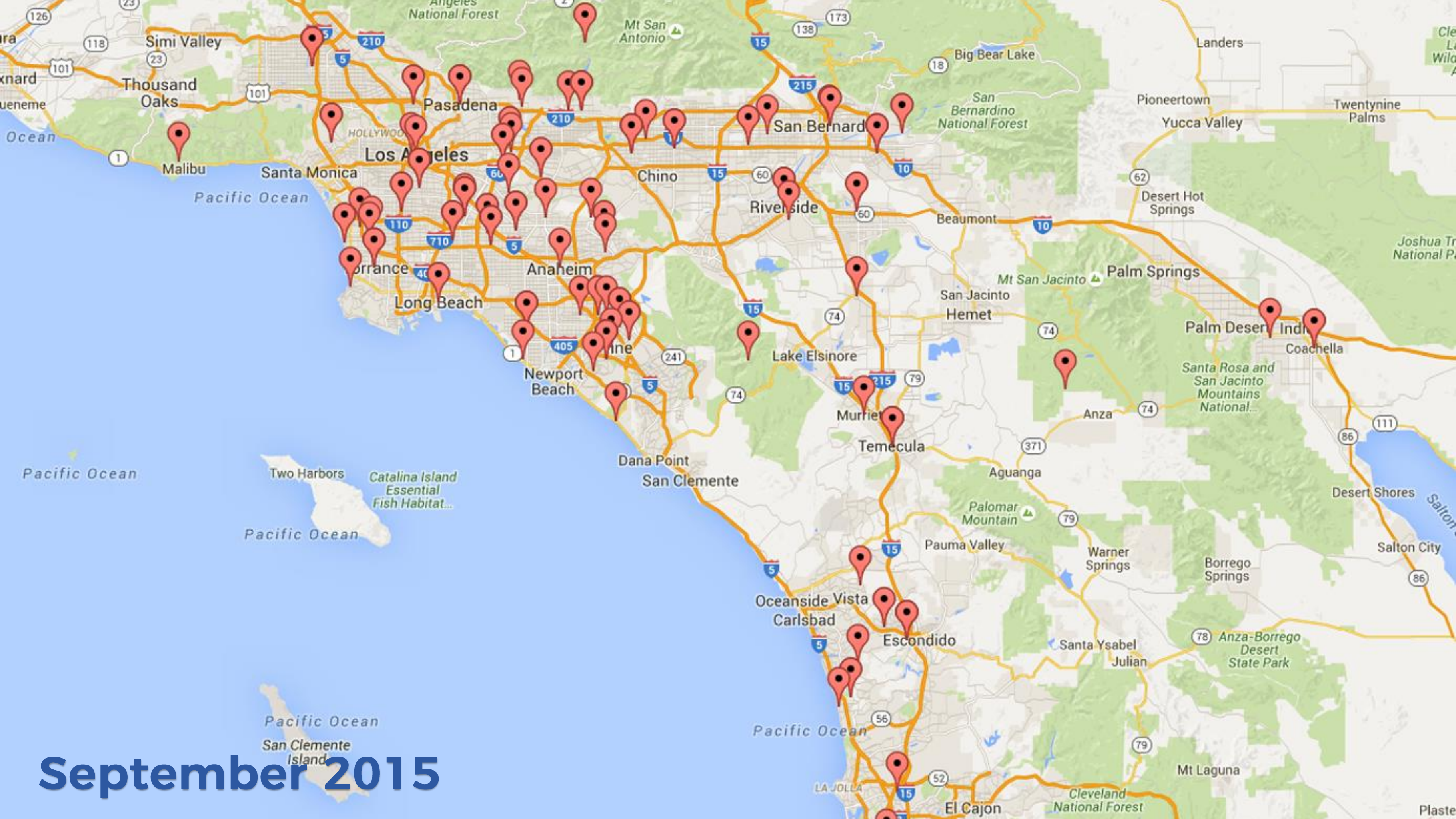


May 2015



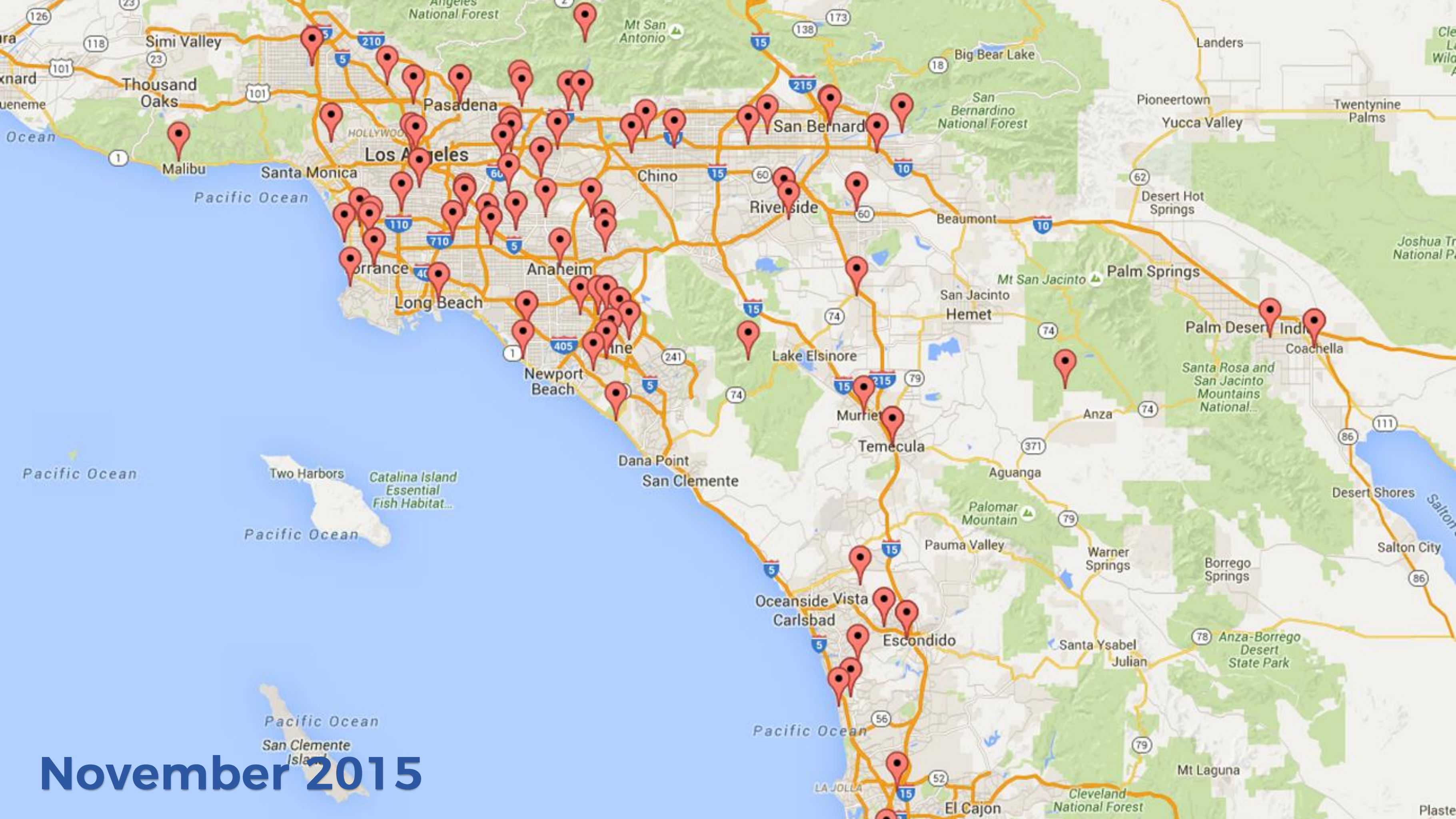






September 2015



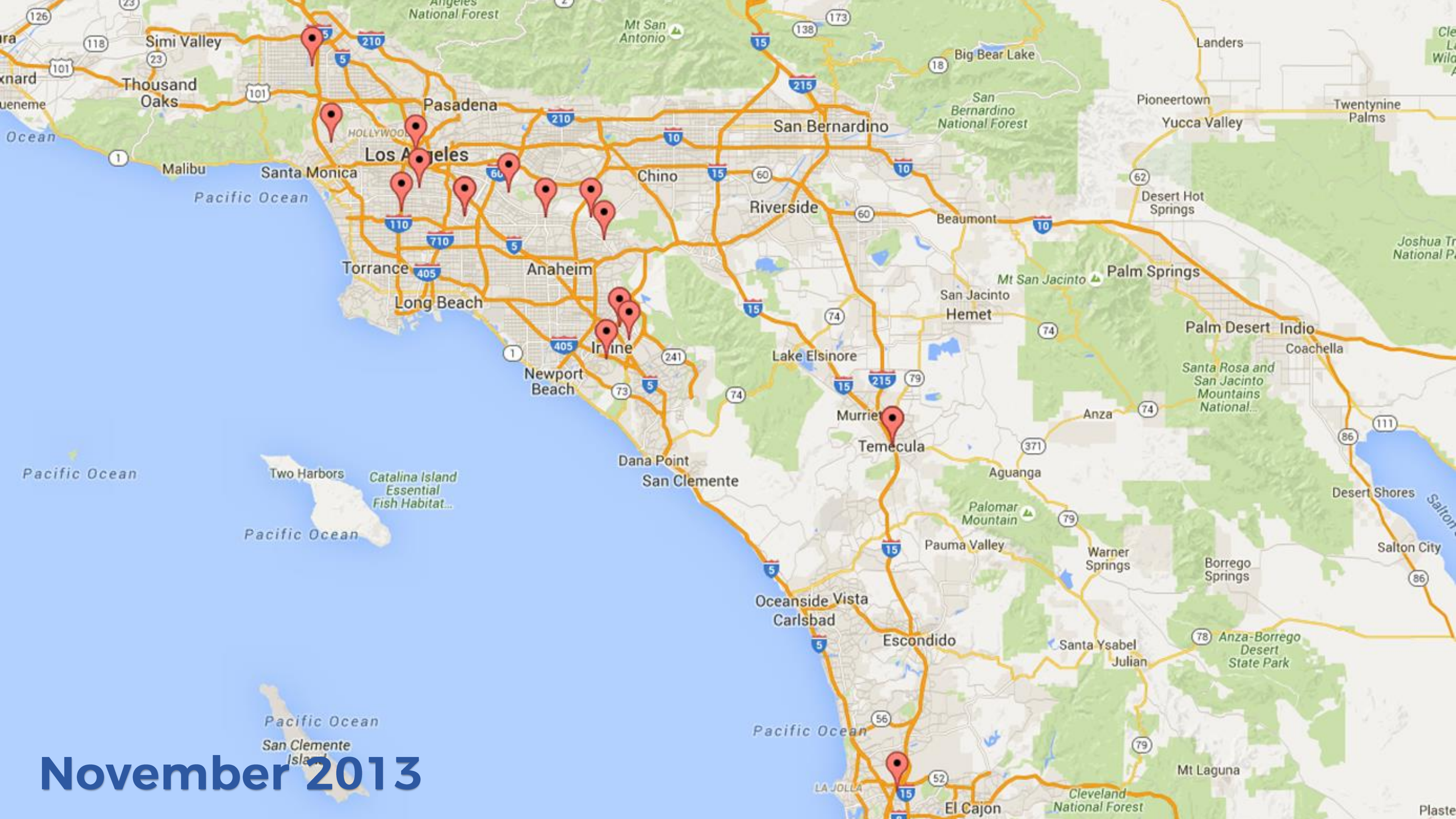


November 2015









November 2013





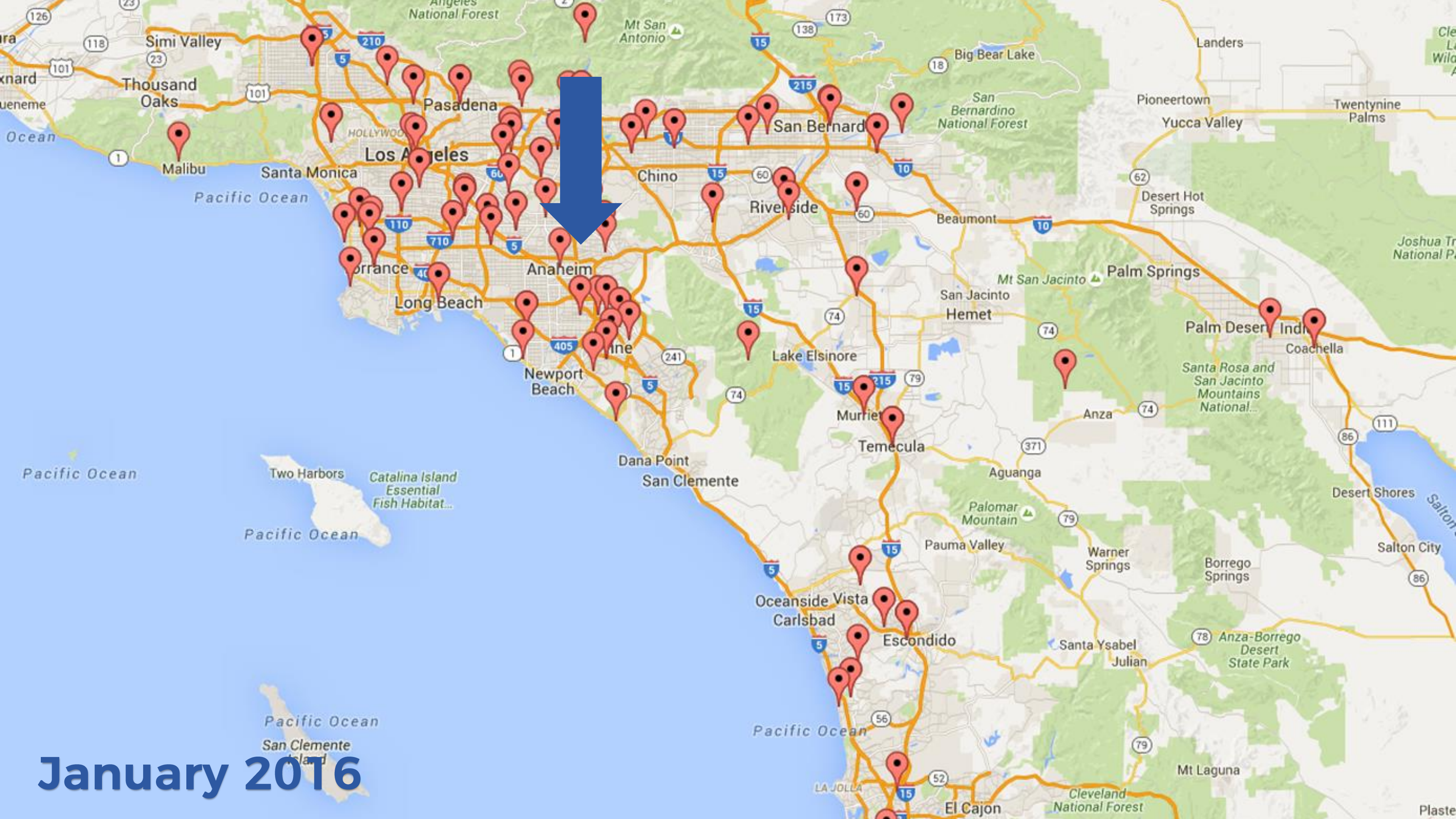
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graph TD; Spies --> Analysts; Analysts --> Model;
```

Spies

Analysts

Model





January 2016





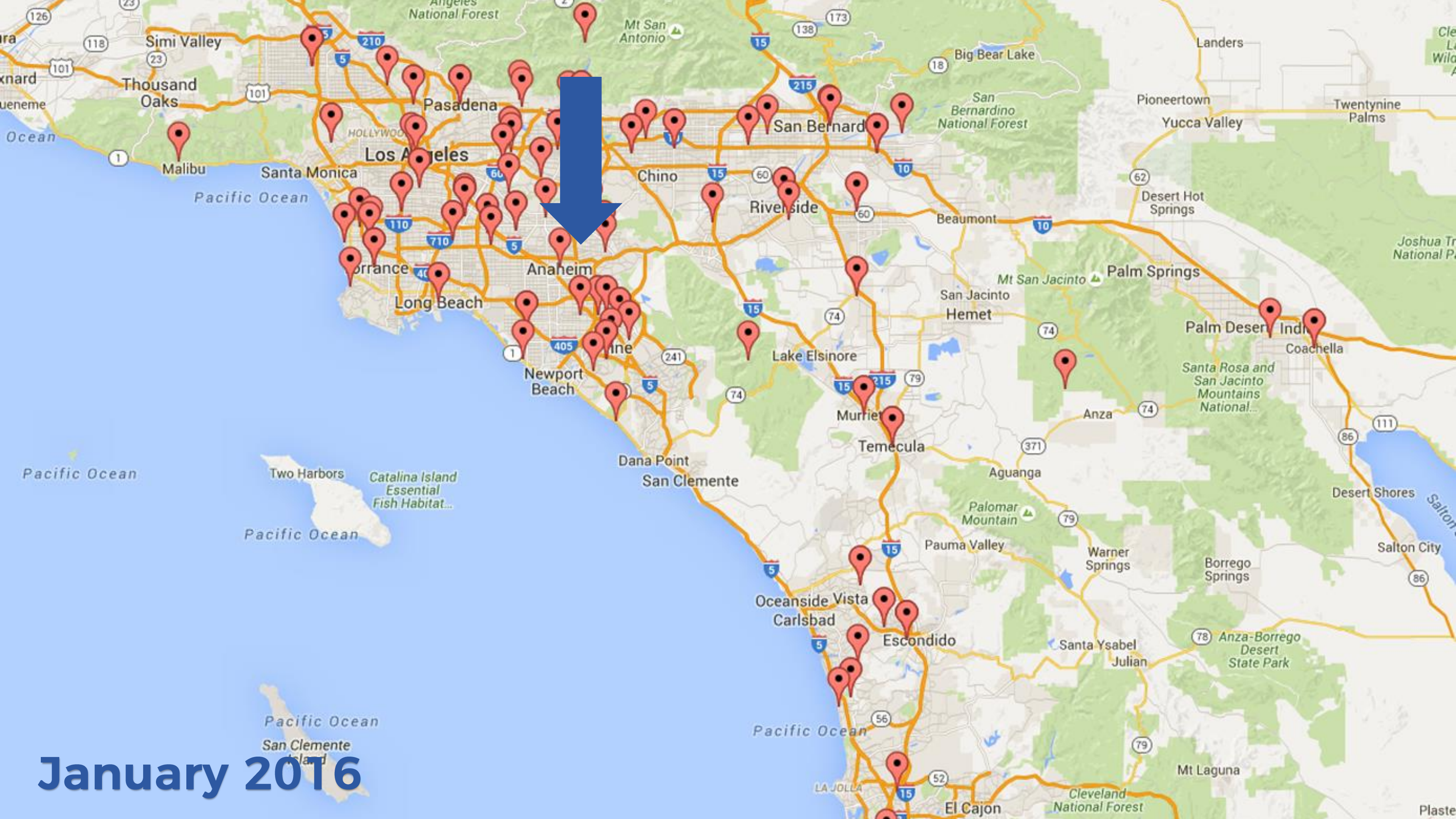
```
graph TD; Spies --> Analysts; Analysts --> Model; Model --> Spies;
```

Spies

Analysts


Model





January 2016





All models are  
wrong, but some  
are useful.

**GEORGE E. P. BOX**





# Classic Mix

**20**  
Singles

LAY'S® Classic Potato Chips. DORITOS® Nacho Cheese Flavored Tortilla Chips. DORITOS® COOL RANCH® Flavored Tortilla Chips. CHEETOS® Crunchy Cheese Flavored Snacks. SUNCHIPS® Original Multigrain Snacks. FRITOS® Original Corn Chips (All 1 OZ. Each)

20 INDIVIDUAL BAGS: 1 OZ. EACH, TOTAL NET WT. 20 OZ. (1 LB. 4 OZ.) 567 g

⚠ WARNING: PREVENT ENTANGLEMENT AND STRANGULATION. KEEP THIS BAG AWAY FROM YOUNG CHILDREN. IT IS NOT A TOY.





```
graph TD; Spies --> Analysts; Analysts --> Model; Model --> Spies;
```

Spies

Analysts

Model



# THINKING TIME

EASY TO STORE.



Classic Mix **20**  
Singles

4 LAY'S® Classic Potato Chips, 4 DORITOS® Nacho Cheese Flavored Tortilla Chips, 2 DORITOS® COOL RANCH® Flavored Tortilla Chips, 4 CHEETOS® Crunchy Cheese Flavored Snacks, 2 SUNCHIPS® Original Multigrain Snacks, 4 FRITOS® Original Corn Chips (All 1 OZ. Each)

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**Robert Kaplinsky**

@robertkaplinsky



Hey #MTBoS, can you do me a favor and complete this 3 question anonymous survey about your favorite chips? I need data for a presentation. Please RT.

[goo.gl/forms/etPtujll...](https://goo.gl/forms/etPtujll...) #iteachmath



**Favorite Chips**

Please complete this anonymous survey. I'll be using this data in a presentation.

[docs.google.com](https://docs.google.com)

8:05 PM - 4 Feb 2018

**63** Retweets **45** Likes



18



63



45







# Favorite Chips (Responses)



File Edit View Insert Format Data Tools Form Add-ons Help

Comments [Share](#)

100% 123 Arial 10

*fx*

Timestamp

|    | A                 | B              | C                      | D                    | E                 | F                    | G                 | H                  |
|----|-------------------|----------------|------------------------|----------------------|-------------------|----------------------|-------------------|--------------------|
| 1  | Timestamp         | Lays (Classic) | Doritos (Nacho Cheese) | Doritos (Cool Ranch) | Cheetos (Crunchy) | Sun Chips (Original) | Fritos (Original) | Time Zone          |
| 2  | 2/4/2018 20:06:53 | 6              | 5                      | 4                    | 2                 | 3                    | 1                 | Central Time Zone  |
| 3  | 2/4/2018 20:06:55 | 1              | 5                      | 6                    | 3                 | 2                    | 4                 | Eastern Time Zone  |
| 4  | 2/4/2018 20:06:56 | 5              | 2                      | 1                    | 3                 | 6                    | 4                 | Central Time Zone  |
| 5  | 2/4/2018 20:06:57 | 2              | 1                      | 6                    | 3                 | 5                    | 4                 | Pacific Time Zone  |
| 6  | 2/4/2018 20:07:36 | 4              | 1                      | 2                    | 3                 | 5                    | 6                 | Pacific Time Zone  |
| 7  | 2/4/2018 20:08:02 | 5              | 1                      | 6                    | 4                 | 2                    | 3                 | Pacific Time Zone  |
| 8  | 2/4/2018 20:08:05 | 6              | 2                      | 4                    | 3                 | 5                    | 1                 | Pacific Time Zone  |
| 9  | 2/4/2018 20:08:07 | 4              | 2                      | 1                    | 5                 | 3                    | 6                 | Pacific Time Zone  |
| 10 | 2/4/2018 20:08:29 | 5              | 3                      | 4                    | 1                 | 6                    | 2                 | Central Time Zone  |
| 11 | 2/4/2018 20:08:56 | 4              | 5                      | 6                    | 1                 | 2                    | 3                 | Central Time Zone  |
| 12 | 2/4/2018 20:09:54 | 5              | 6                      | 5                    | 6                 | 5                    | 4                 | Pacific Time Zone  |
| 13 | 2/4/2018 20:10:01 | 4              | 2                      | 3                    | 1                 | 5                    | 6                 | Pacific Time Zone  |
| 14 | 2/4/2018 20:10:04 | 6              | 2                      | 3                    | 1                 | 5                    | 4                 | Central Time Zone  |
| 15 | 2/4/2018 20:10:04 | 3              | 5                      | 6                    | 1                 | 4                    | 2                 | Central Time Zone  |
| 16 | 2/4/2018 20:10:05 | 4              | 2                      | 6                    | 1                 | 3                    | 5                 | Eastern Time Zone  |
| 17 | 2/4/2018 20:10:06 | 3              | 2                      | 6                    | 5                 | 1                    | 2                 | Pacific Time Zone  |
| 18 | 2/4/2018 20:10:10 | 4              | 2                      | 6                    | 3                 | 5                    | 1                 | Mountain Time Zone |
| 19 | 2/4/2018 20:10:12 | 3              | 1                      | 5                    | 6                 | 2                    | 4                 | Eastern Time Zone  |
| 20 | 2/4/2018 20:10:26 | 5              | 3                      | 6                    | 2                 | 4                    | 1                 | Pacific Time Zone  |



# THINKING TIME


- The available data includes:
  - Lays, Nacho Cheese Doritos, Cool Ranch Doritos, Cheetos, Sun Chips, and Fritos ranked from 1 to 6
  - Geographic region: West, Central, or Eastern



# ANALYSTS' JOB FOR THE TOP 1

1. **Count** all the first place votes for each chip type.
2. **Divide** the total first place votes for each chip type by the total number of votes.
3. **Multiply that fraction** by 20 to find how many bags there would be in a twenty pack, **rounding** as necessary.





Teaching students skills  
without chances to apply  
them is like teaching a child  
to walk and expecting them  
to safely exit during a fire.



# ANALYSTS' EXAMPLE

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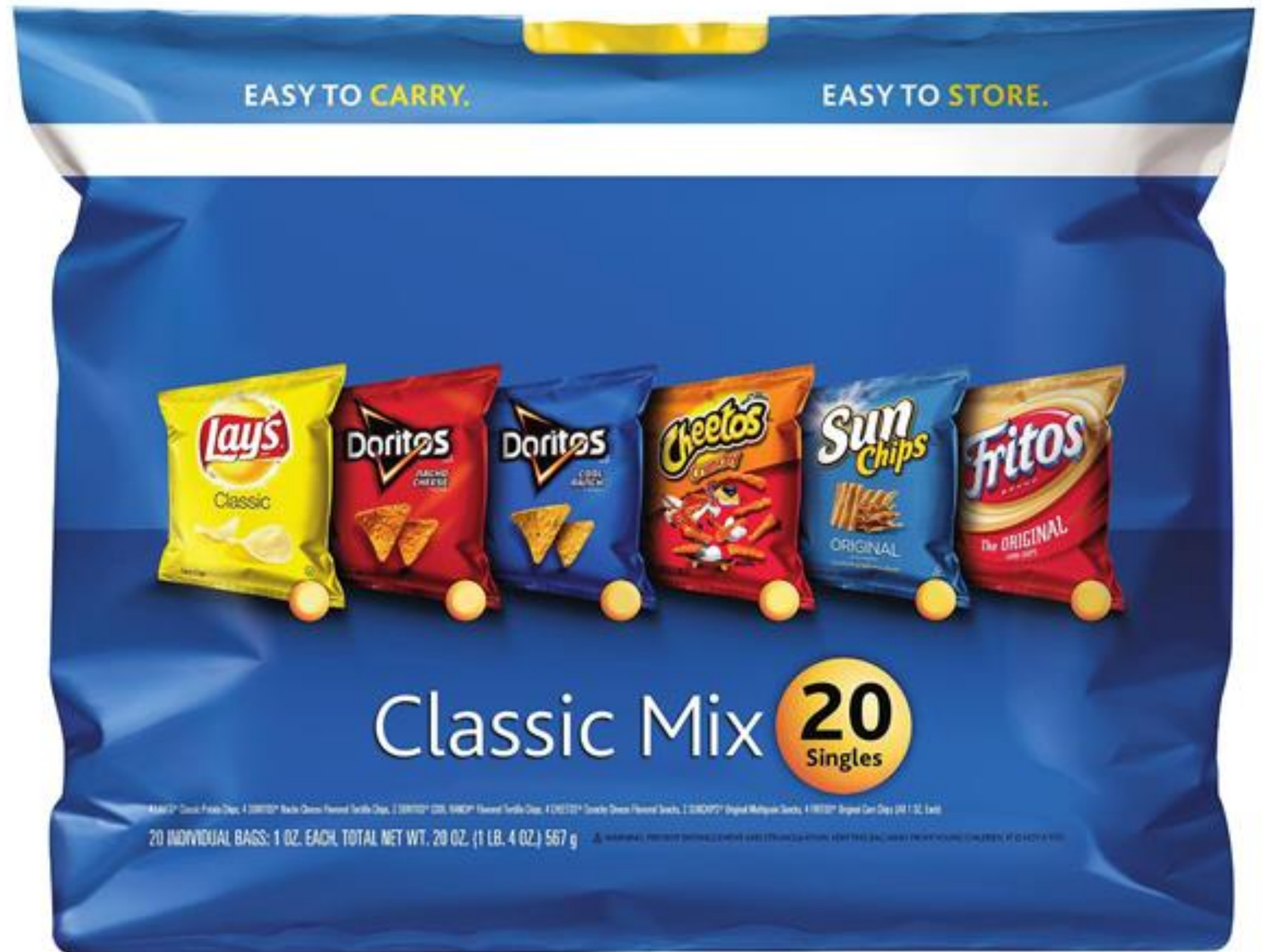


# CHIP BAG RESULTS

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- 20. Food** Frito-Lay puts a variety of flavors in each package of chips. Survey your classmates and use proportions to figure out how many of each flavor there should be, then fill in the blanks.







**Kate Hayes**

@MsHayesOG

Follow



We used a [@robertkaplinsky](#) video scenario to talk about chip bags. The lesson here was how math applies to the real world, solving word problems, and spies/analysts 🤔🕶️👓 the best part was reading student reflections of what they learned today ❤️👏

[@oakgrovees](#) [@WCPSS](#) [@OtterBias](#)



8:32 AM - 14 Feb 2019

2 Retweets 18 Likes



2




2



18







Spies get the info.  
Analysts use the info  
to create the model.



# MATH MODELING

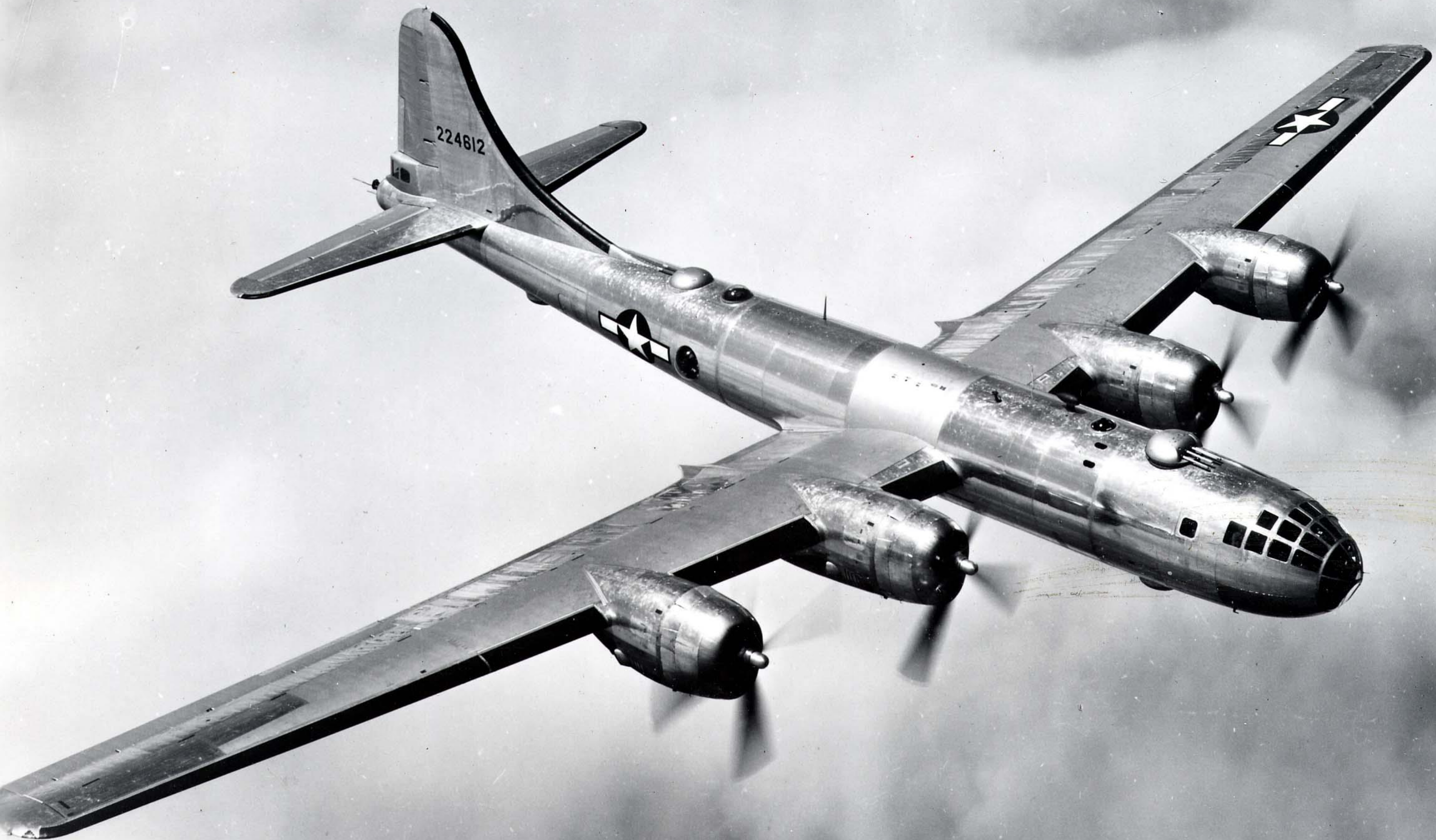
☒ HOW DO WE MAKE SENSE OF MATH MODELING?

☐ IS IT JUST ANSWERING QUESTIONS?

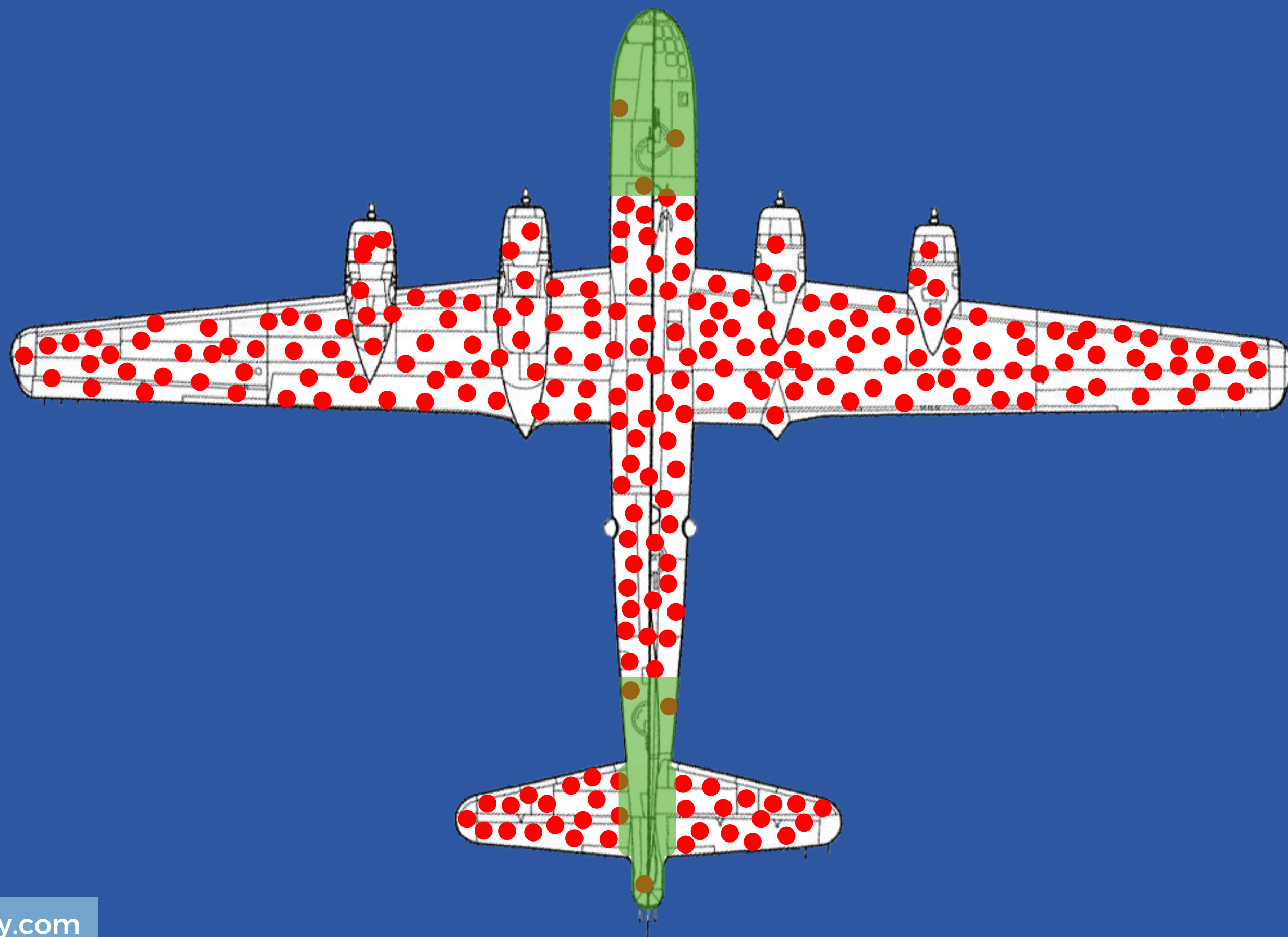
☐ HOW IS MATH MODELING USED IN REAL LIFE?

☐ HOW DO WE HELP OUR STUDENTS IMPROVE?





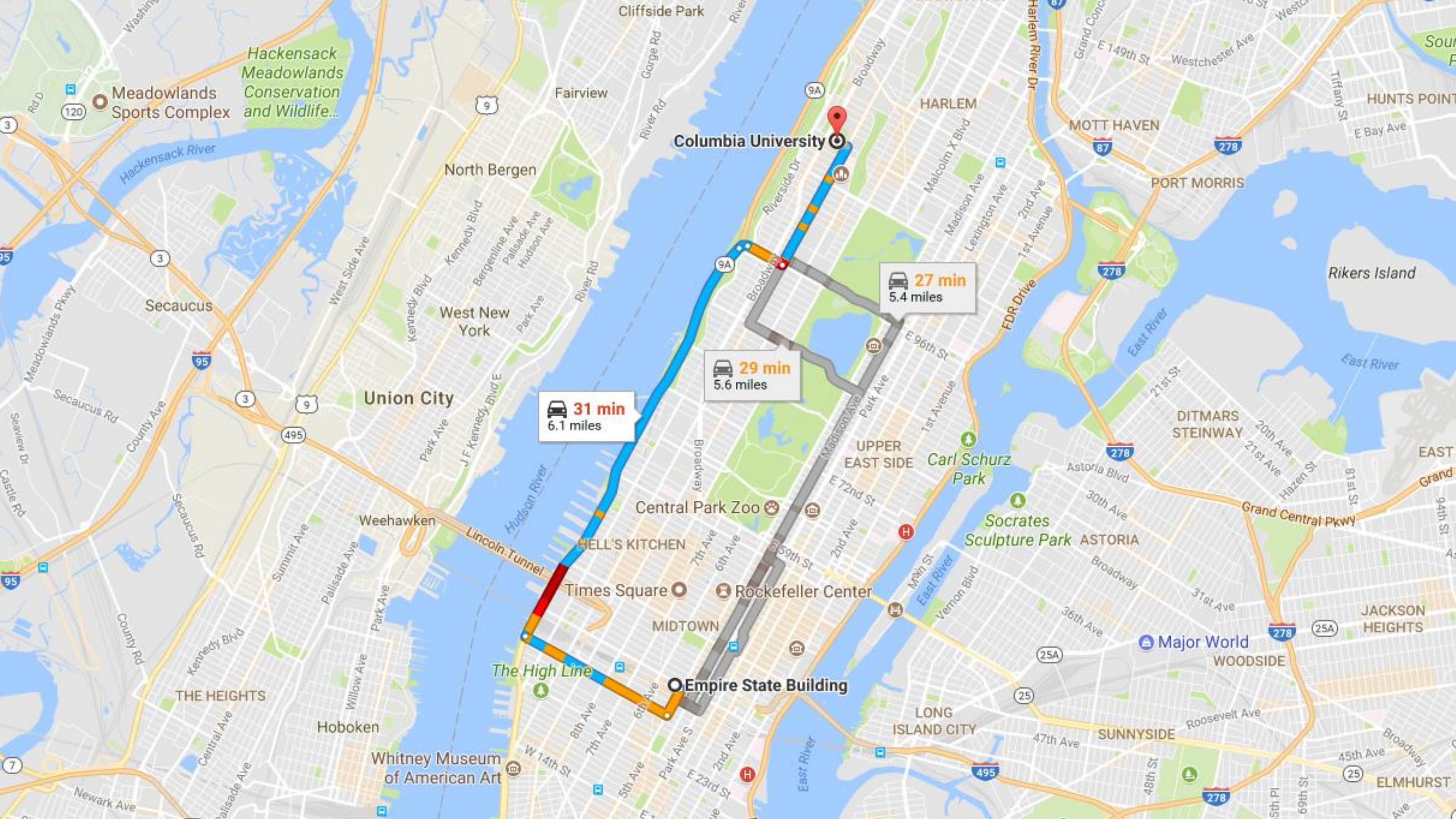






- ~~How do we protect our planes?~~
- ~~Which parts of the plane are being hit by the most bullets?~~
- Which parts of the plane are the most critical to protect?







- ~~How do we find the fastest route for each customer?~~
- How do we find the fastest route for each customer without impacting our other customers?





# Classic Mix

**20**  
Singles

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- ~~How many of each flavor should we put in a package?~~
- ~~How many of each flavor should we put in a package for each region?~~
- How can we determine if the extra cost of creating different packages will make us more money?






Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

## CCSS MATH PRACTICE 4






They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## CCSS MATH PRACTICE 4





Mathematical modeling is not just about answering a question. It's also about determining if you're asking the right question.



# MATH MODELING

☒ HOW DO WE MAKE SENSE OF MATH MODELING?

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☐ HOW IS MATH MODELING USED IN REAL LIFE?

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```
graph TD; Spies --> Analysts; Analysts --> Model; Model --> Spies;
```

Spies

Analysts

Model





They used 25 products for a pregnancy prediction' score including:

- unscented lotion
- mineral supplements
- cotton balls

Source: New York Times









```
graph TD; Spies --> Analysts; Analysts --> Model; Model --> Spies;
```

Spies

Analysts

Model



# Priority is determined by:

- passenger's fare class
- itinerary
- frequent flyer program membership
- check-in time

Source: United Airlines





Robert

Home



Robert Kaplinsky



News Feed



Messenger



Watch



Marketplace

### Explore



Pages



Events



Groups



Friend Lists



On This Day

3



Insights



Games

7



Fundraisers



Live Video



Pokes

See More...

### Create

Ad · Page · Group · Event · Fundraiser



Make Post



Photo/Video Album



Live Video



What's on your mind, Robert?



Photo/Video



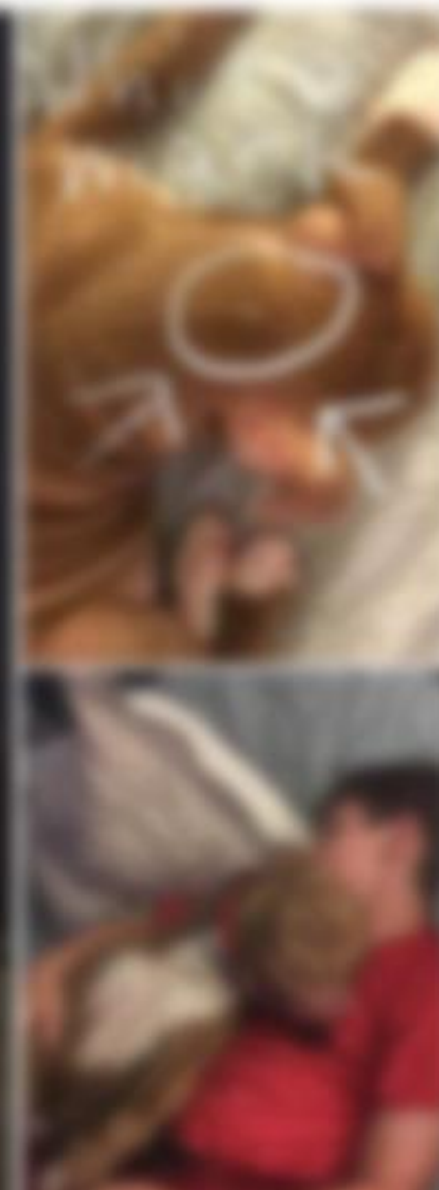
Feeling/Activity



Additively shared another Tom Janggen's post

17 views · 2h

hoping this boy gets back to his family



New Father Blogger's birthday is today

### Trending



Jared Kushner

The Disappearance of Jared and Ivanka's Personal Email Server - [truthaboutkush.com](#)

Fredericksburg, Virginia

Mother recovering from copperhead snake bite at Virginia restaurant - [chronicle.com](#)

Anthony Warner

Anthony Warner Sentenced to 21 Months in Prison - [washingtonpost.com](#)

See More

### Watchlist: Latest Episodes



Episode: The Making of a Leader

Part in The Family



Episode: The Making of a Leader

Part in The Family

See All

### Sponsored

Create Ad







```
graph TD; Spies --> Analysts; Analysts --> Model; Model --> Spies;
```

Spies

Analysts

Model





# The stories that show in your News Feed are influenced by:

- friends you interact with the most
- the number of comments and likes a post receives
- what kind of story it is (ex: photo, video, status update)

Source: Facebook



# MORE EXAMPLES

- How does US News and World Reports rank colleges?
- How does Google know which results to show?
- How do sports teams know who to draft?
- How does Amazon know what products to recommend?
- How does Zillow estimate home prices?
- How does eHarmony know which people to show you?
- How does a school decide which students should take advanced math classes?
- How do they figure out who should speak at a conference?



# MATH MODELING

☒ HOW DO WE MAKE SENSE OF MATH MODELING?

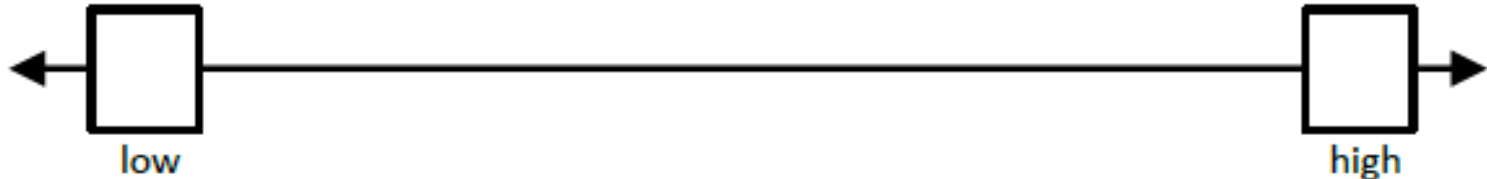
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☐ HOW DO WE HELP OUR STUDENTS IMPROVE?

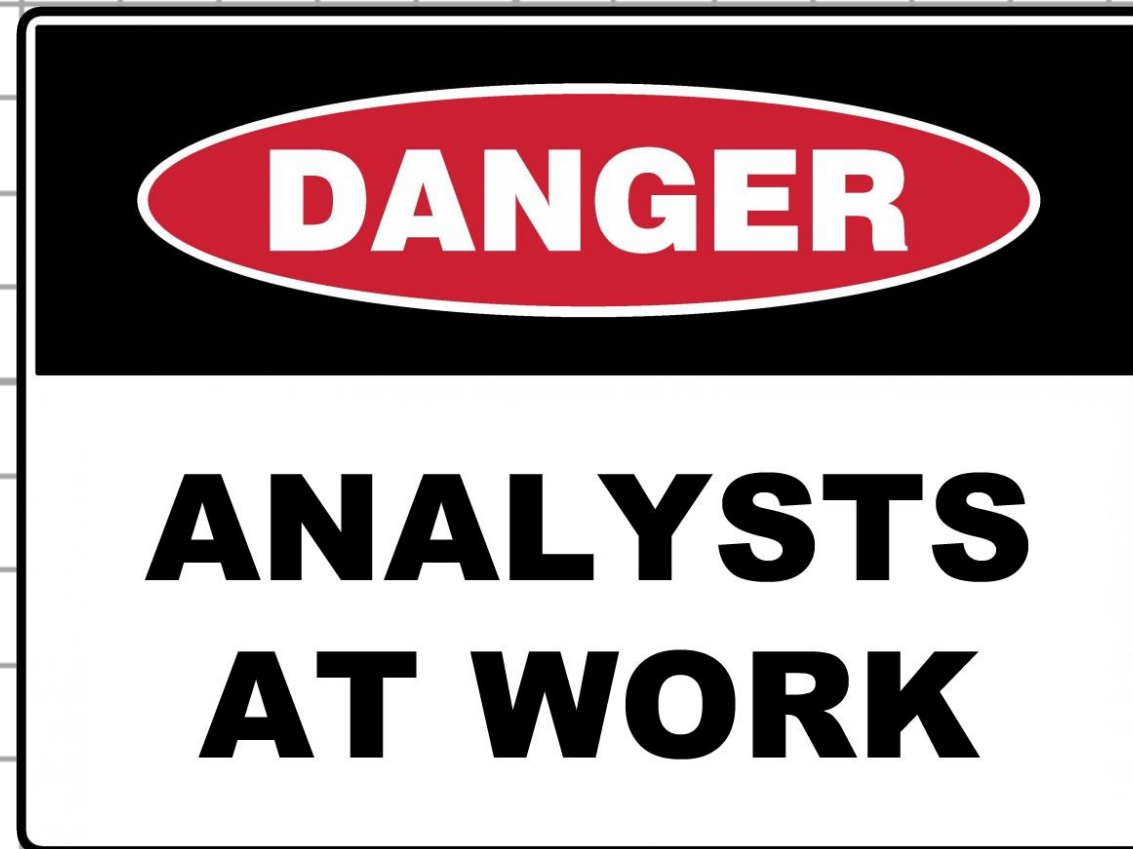


Name: \_\_\_\_\_ Period: \_\_\_\_\_ Date: \_\_\_\_\_

| What problem are you trying to figure out?                             | What estimates do you have?   |
|--|---|
|  | <div data-bbox="1725 315 3085 480"></div> <p data-bbox="2059 714 2768 752">Place your estimate on the number line.</p> |
| What info do you already know about the problem?                       | What info do you need about the problem?  |
| <div data-bbox="736 761 2558 1365"><b>TOP SECRET! SPIES ONLY</b></div> |   |
|  |   |
| What is your conclusion? How did you reach that conclusion?            |   |



Your work





# MODELING EXAMPLES

☐ ELEMENTARY SCHOOL

☐ MIDDLE SCHOOL

☐ HIGH SCHOOL









```
graph TD; Spies --> Analysts; Analysts --> Model; Model --> Spies;
```

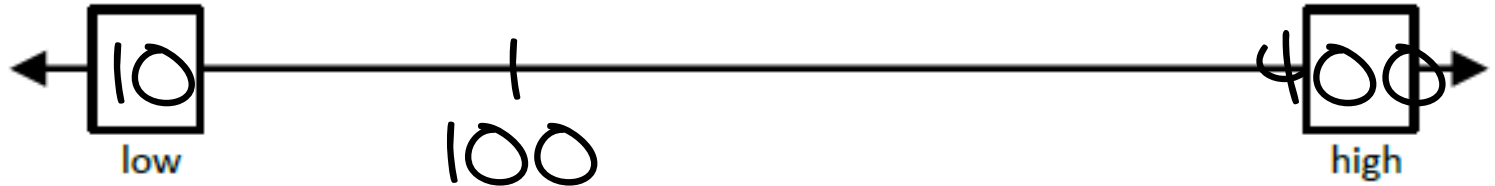
Spies

Analysts

Model



Name: \_\_\_\_\_ Period: \_\_\_\_\_ Date: \_\_\_\_\_

| What problem are you trying to figure out?   | What estimates do you have?  |
|--|--|
| How many beverage choices are there?   |  <p>Place your estimate on the number line.</p>                                     |
| What info do you already know about the problem?   | What info do you need about the problem?   |
| <ul style="list-style-type: none"><li>• There are main flavors and added flavors.</li><li>• Lemonade is yummy.</li></ul> | <ul style="list-style-type: none"><li>• How many main flavors are there?</li><li>• How many added flavors are there?</li><li>• Can we mix them all together?</li></ul> |
| What is your conclusion? How did you reach that conclusion?  |  |



# COUNT ALL



## low/no calories





# COUNT GROUPS



low/no calories



8

7

7

8

8

2

8

8

6

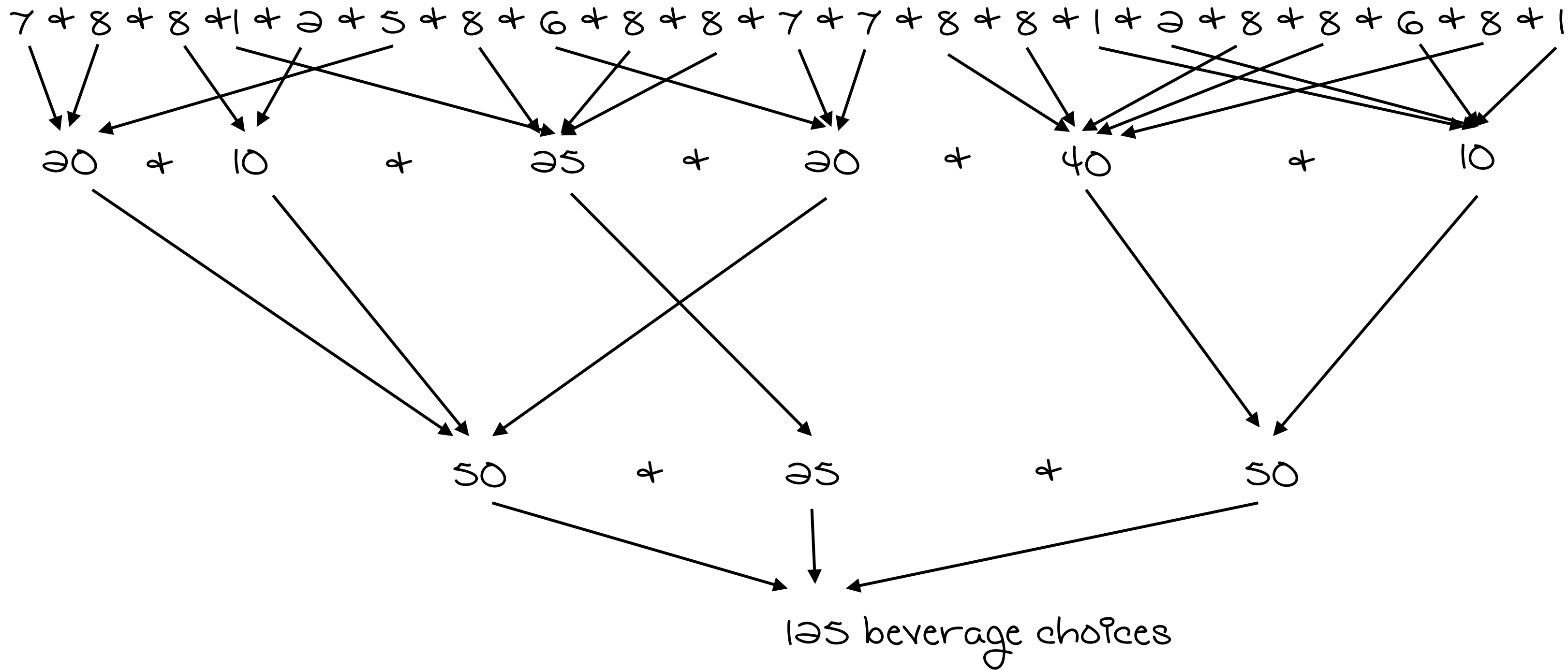
8



# INVENTED STRATEGY









The main attraction for a busload of Dover fifth-graders was supposed to be the Museum of Fine Arts, but that all changed when they stopped by Kelly's Roast Beef and got a glimpse of their soda-drinking future.

At the entrance of Kelly's sat a sleek **Coca-Cola** Freestyle fountain crafted to resemble an old-fashioned vending machine, but with a twist: a touchscreen computer embedded in the machine gives customers **the option of 125 flavors**. You can quench your thirst with a Coke or a Sprite, or try something more exotic — Sprite with Grape or a Hi-C Orange Vanilla.





**Kelly Hall**  
@hAllStars4th

Follow



@VgEagles sharing strategies for figuring out @robertkaplinsky's 3 Act math task: How many possible combinations of soda are there in a Coke Freestyle machine?  
#WEareLakota #CocaCola #3actmathtasks



12:53 PM - 5 Feb 2019

4 Retweets 30 Likes



2



4



30





# MODELING EXAMPLES

☒ ELEMENTARY SCHOOL

☐ MIDDLE SCHOOL

☐ HIGH SCHOOL







LIVE



Source: [robertkaplinsky.com/lessons](http://robertkaplinsky.com/lessons)





```
graph TD; Spies --> Analysts; Analysts --> Model; Model --> Spies;
```

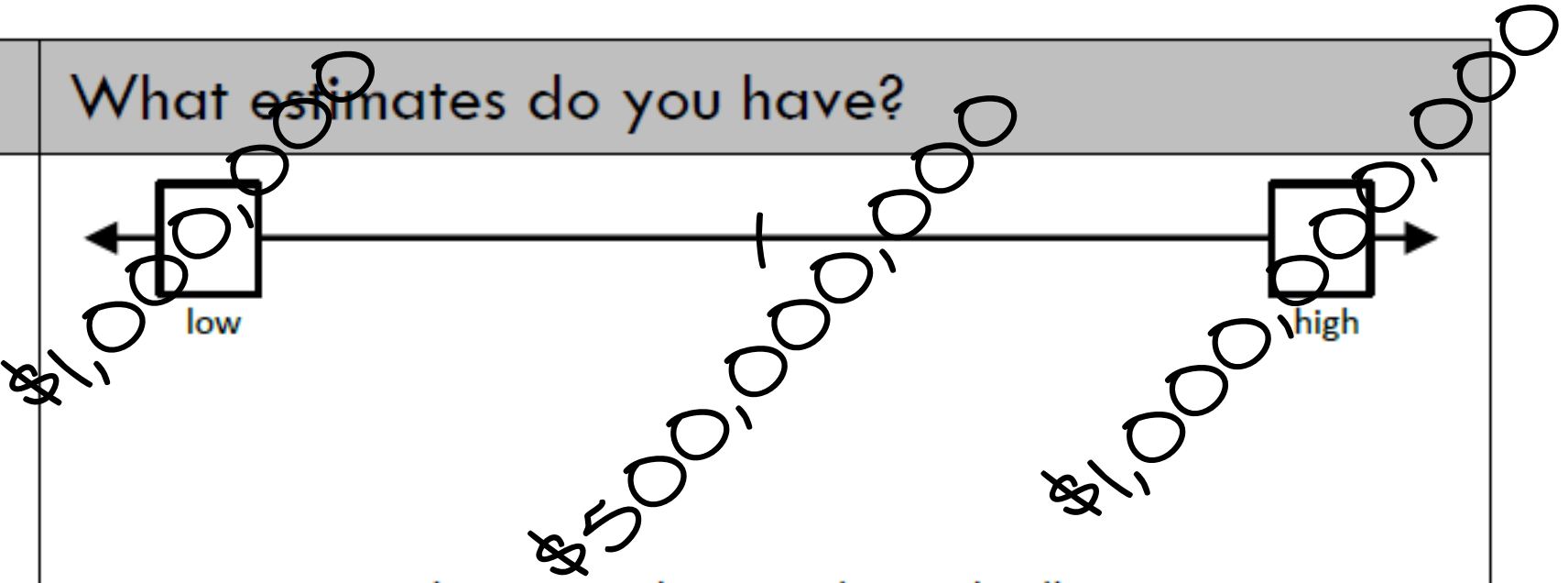
Spies

Analysts

Model



Name: \_\_\_\_\_ Period: \_\_\_\_\_ Date: \_\_\_\_\_

| What problem are you trying to figure out?  | What estimates do you have?   |
|---|---|
| <p>How much money was that?</p>   |  <p>Place your estimate on the number line.</p>  |
| What info do you already know about the problem?  | What info do you need about the problem?  |
| <ul style="list-style-type: none"><li>• There is a lot of money.</li><li>• It is in a pile.</li><li>• It is in bundles.</li></ul> | <ul style="list-style-type: none"><li>• Is it all the same denomination?</li><li>• <del>How much does one bill weigh?</del></li><li>• <del>How much does all the money weigh?</del></li></ul> |
| What is your conclusion? How did you reach that conclusion?   |   |



Name: \_\_\_\_\_ Period: \_\_\_\_\_ Date: \_\_\_\_\_

| What problem are you trying to figure out?  | What estimates do you have?  |
|---|--|
| How much money was that?  |  |
| What info do you already know about the problem?  | What info do you need about the problem?   |
| <ul style="list-style-type: none"><li>• There is a lot of money.</li><li>• It is in a pile.</li><li>• It is in bundles.</li></ul> | <ul style="list-style-type: none"><li>• Is it all the same denomination?</li><li>• How many rows and columns are there?</li><li>• How many bills are in one stack?</li></ul> |
| What is your conclusion? How did you reach that conclusion?   |  |









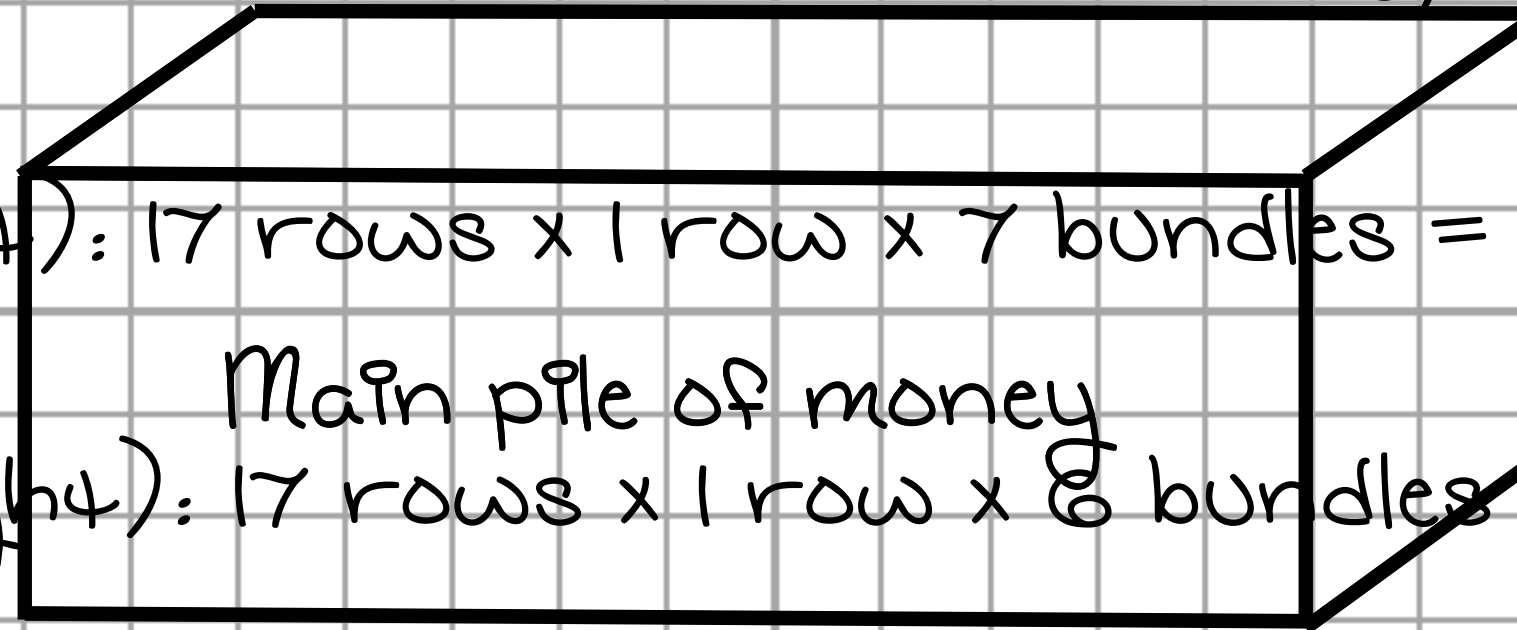


## Your work

Main pile: 34 rows x 11 rows across bundles = 3,740 bundles

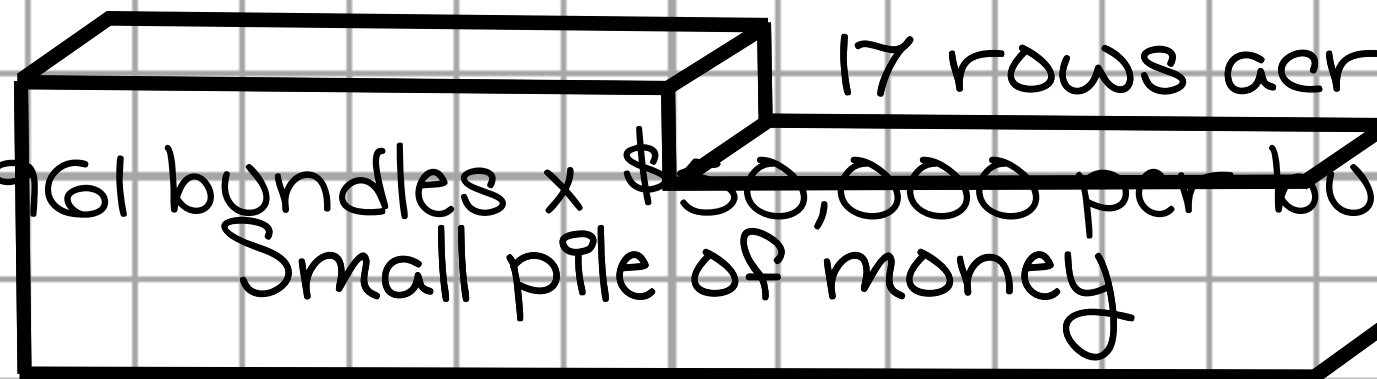
Small pile (left): 17 rows x 1 row x 7 bundles = 19 bundles  
10 bundles

Small <sup>high</sup> pile (right): 17 rows x 1 row x 8 bundles = 102 bundles  
11 rows deep



Total bundles: 3,740 + 19 + 102 = 3,961 bundles  
17 rows across

Total money: 3,961 bundles x \$50,000 per bundle = \$198,050,000



8 bundles  
high

7 bundles  
high



LIVE



Source: [robertkaplinsky.com/lessons](http://robertkaplinsky.com/lessons)





**Holly Keeton**  
@holly\_keeton

Follow



@WGBulldogs 5th graders were VERY ENGAGED yesterday with @robertkaplinsky 's math lesson "How Much Money Is That?!". Students worked with money, adding & multiplying numbers, and solving for volume to estimate the cash! 💵💵  
#AllMeansAllMCSS #mathisfun #mathisreal



2:49 PM - 6 Mar 2019

3 Retweets 15 Likes



3



15



# MODELING EXAMPLES

☒ ELEMENTARY SCHOOL

☒ MIDDLE SCHOOL

☐ HIGH SCHOOL

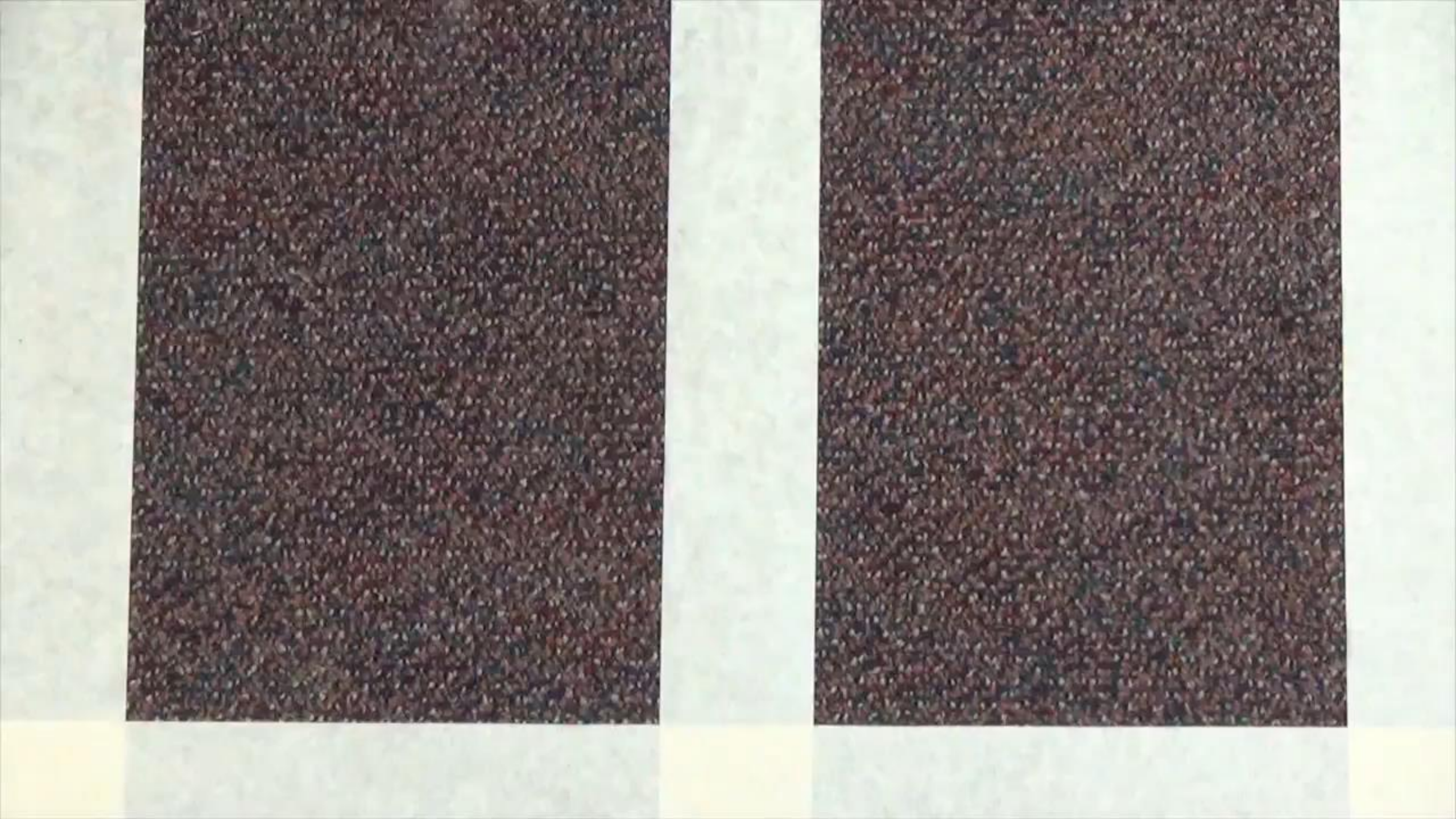




**NON-STAGGERED**

**STAGGERED**









```
graph TD; Spies --> Analysts; Analysts --> Model; Model --> Spies;
```

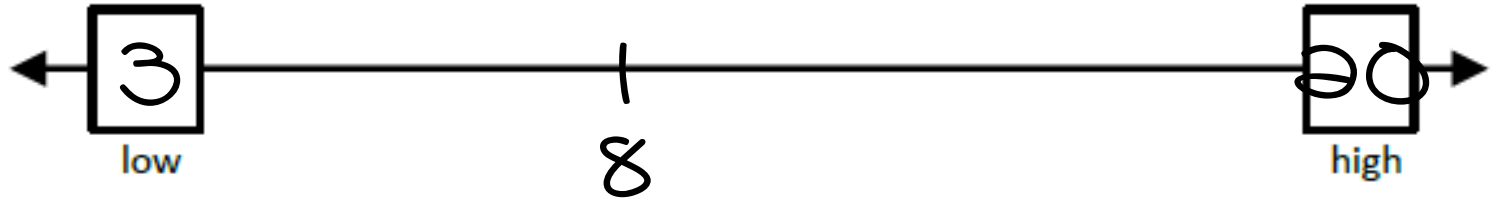
Spies

Analysts

Model



Name: \_\_\_\_\_ Period: \_\_\_\_\_ Date: \_\_\_\_\_

| What problem are you trying to figure out?  | What estimates do you have?   |
|---|---|
| How much shorter are 20 layers of staggered pipes?  |  <p>(in inches)</p> <p>Place your estimate on the number line.</p> |
| What info do you already know about the problem?  | What info do you need about the problem?  |
| <ul style="list-style-type: none"><li>• One pile of pipes is staggered.</li><li>• One pile of pipes is not staggered.</li><li>• We have to compare 20 layers of each.</li></ul> | <ul style="list-style-type: none"><li>• What are the dimensions of a pipe?</li><li>• What units are we using to measure?</li></ul>                    |
| What is your conclusion? How did you reach that conclusion?   |   |



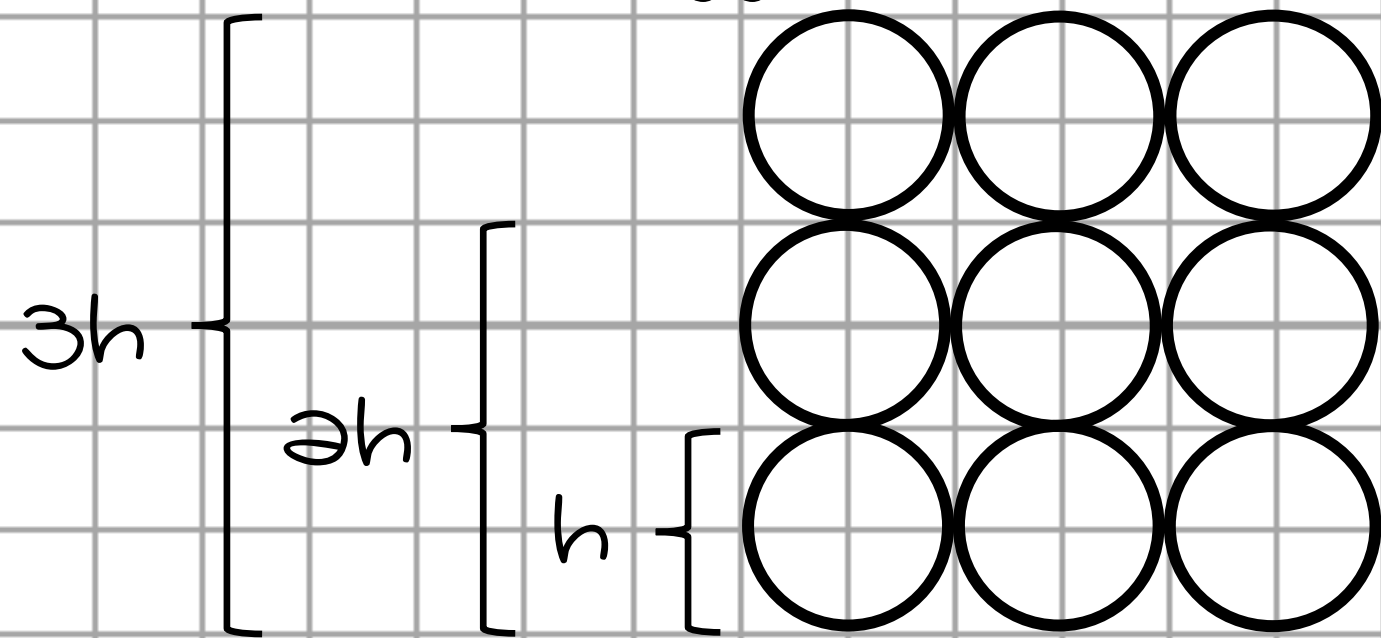








Non-staggered pipes



1 pipe =  $h$  cm

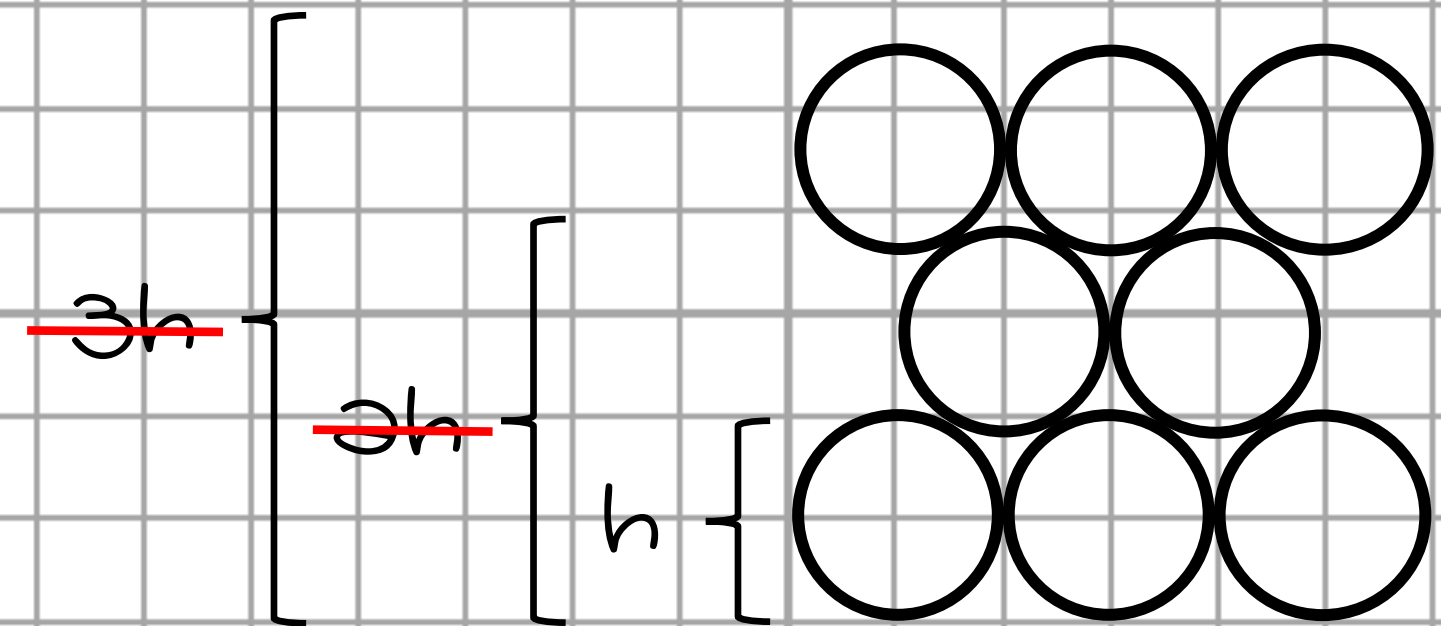
2 pipes =  $2h$  cm

3 pipes =  $3h$  cm

⋮

20 pipes =  $20h$  cm

Staggered pipes



1 pipe =  $h$  cm

2 pipes =  $33$  cm

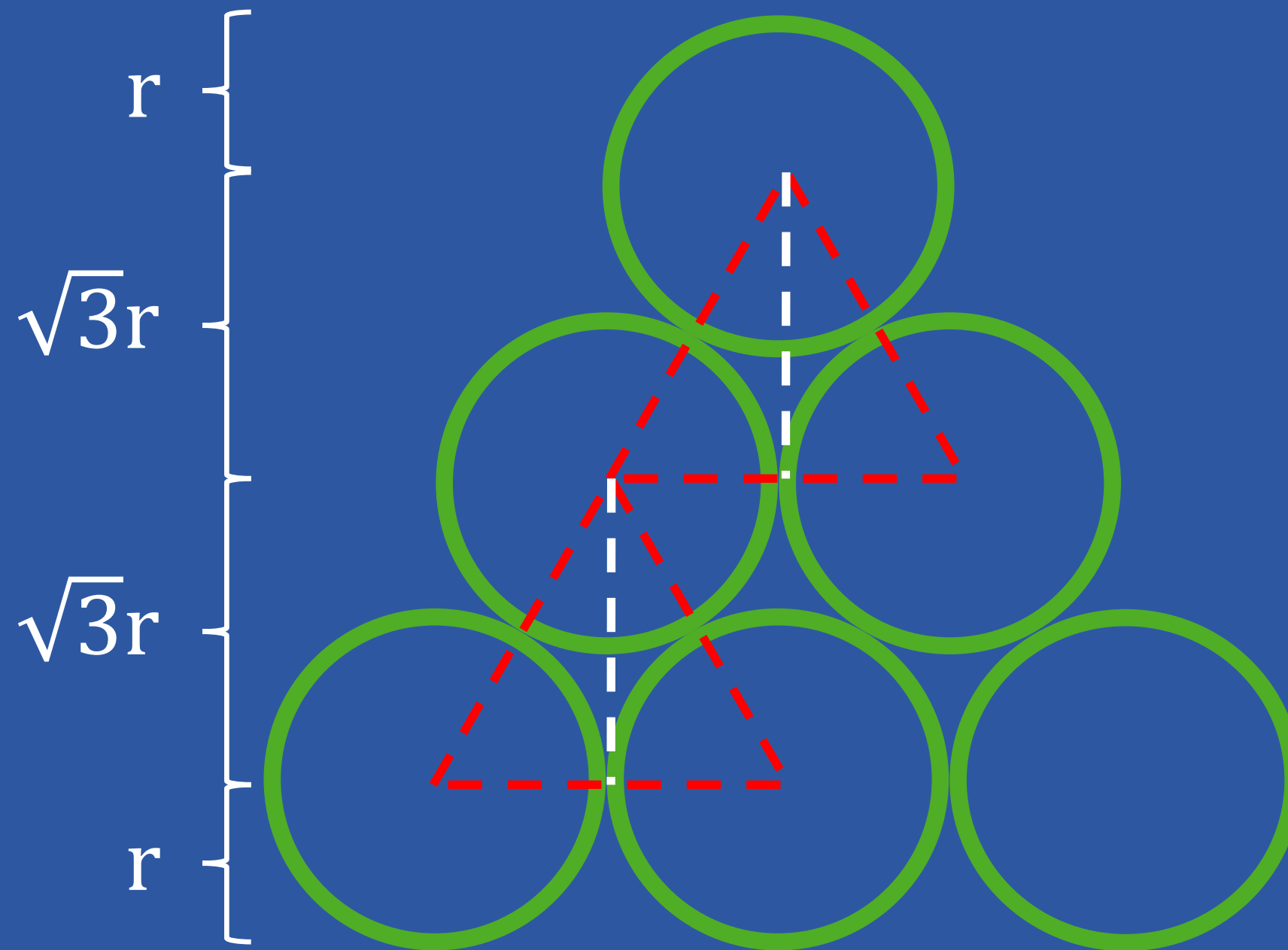
3 pipes =  $33$  cm

⋮

20 pipes =  $33$  cm



# STAGGERED PIPES







**John Mahlstedt**  
@jdmahlstedt

Following



Working on @robertkaplinsky's Staggered Pipes prob 2day and modeling it w/ cups. Great prob!  
[robertkaplinsky.com/work/staggered...](http://robertkaplinsky.com/work/staggered...)



11:19 AM - 10 Apr 2015

2 Likes





# MODELING EXAMPLES

 **ELEMENTARY SCHOOL**

 **MIDDLE SCHOOL**

 **HIGH SCHOOL**



# MATH MODELING

 **HOW DO WE MAKE SENSE OF MATH MODELING?**

 **IS IT JUST ANSWERING QUESTIONS?**

 **HOW IS MATH MODELING USED IN REAL LIFE?**

 **HOW DO WE HELP OUR STUDENTS IMPROVE?**



# DISCUSSION TIME

- Why should we reconsider using word problems?
- How is math modeling different from traditional word problems?



# GOALS

☒ CORRECT ANSWERS = UNDERSTANDING?

☒ RECONSIDER USING WORD PROBLEMS

☐ RECONSIDER USING WORKSHEETS



# GOALS

- ☐ WHAT'S WRONG WITH WORKSHEETS?
- ☐ WHAT SHOULD WE BE DOING INSTEAD?
- ☐ HOW DO WE DO IT IN OUR CLASSROOMS?
- ☐ WHERE DO WE GET MORE PROBLEMS?
- ☐ WHAT COMES NEXT?



Algebra 1

Name\_\_\_\_\_

## One-Step Equations

Date\_\_\_\_\_ Period\_\_\_\_

**Solve each equation.**

1)  $26 = 8 + v$

2)  $3 + p = 8$

3)  $15 + b = 23$

4)  $-15 + n = -9$

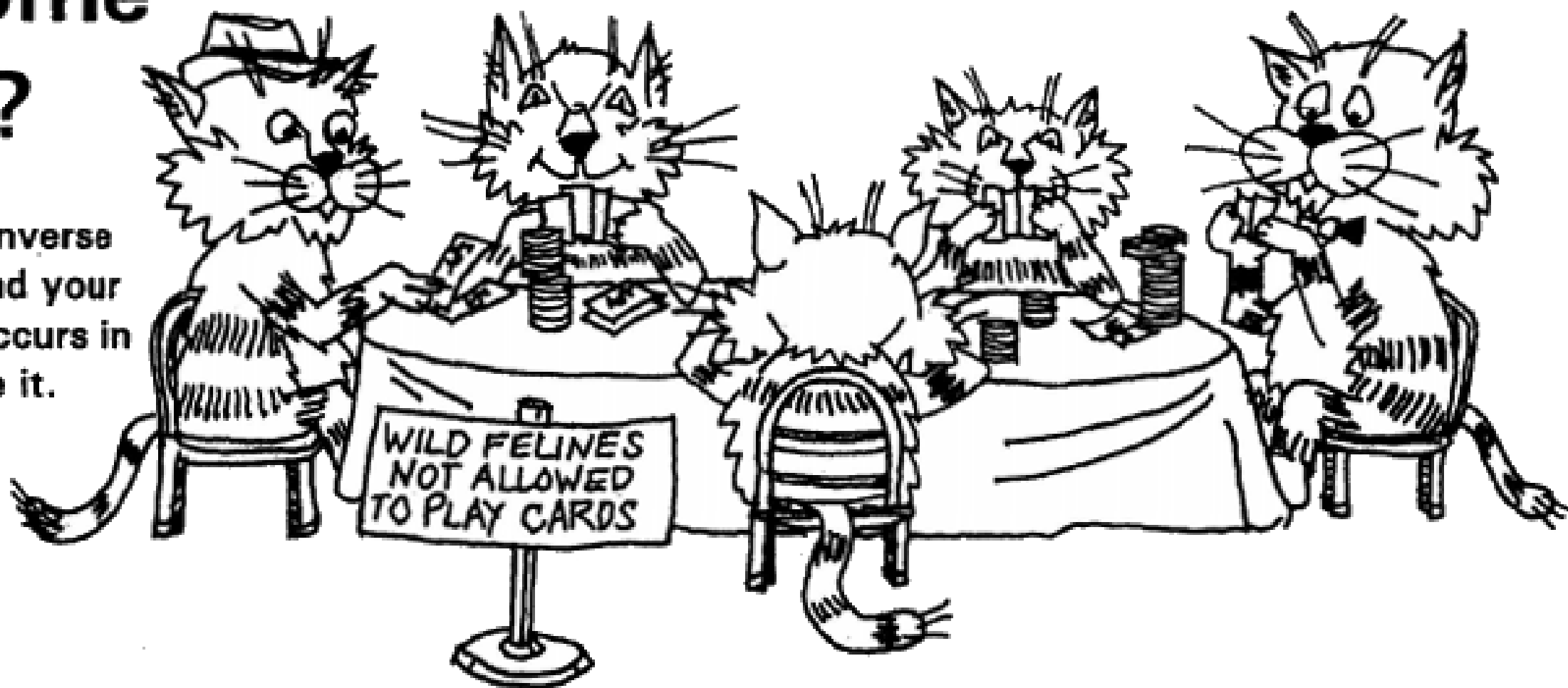
5)  $m + 4 = -12$

6)  $x - 7 = 13$



# Why shouldn't some cats play cards?

**DIRECTIONS:** Solve each equation by using the inverse operation. Use a calculator where necessary. Find your answer in the decoder. Each time your answer occurs in the decoder, write the letter of the problem above it.



1.  $3 + g = 13$  ( $g =$  \_\_\_\_\_)

2.  $34 = a \cdot 2$  ( $a =$  \_\_\_\_\_)

3.  $\frac{h}{15} = 10$  ( $h =$  \_\_\_\_\_)

4.  $15 = d - 18$  ( $d =$  \_\_\_\_\_)

5.  $132 = m \times 11$  ( $m =$  \_\_\_\_\_)

6.  $15o = 210$  ( $o =$  \_\_\_\_\_)

7.  $\frac{i}{2.3} = 6.7$  ( $i =$  \_\_\_\_\_)

8.  $2.5e = 40$  ( $e =$  \_\_\_\_\_)

9.  $180 = t - 35$  ( $t =$  \_\_\_\_\_)

10.  $90 = 3l$  ( $l =$  \_\_\_\_\_)

11.  $7.2 = 0.36n$  ( $n =$  \_\_\_\_\_)

12.  $\frac{b}{5} = 31$  ( $b =$  \_\_\_\_\_)

13.  $4c = 60$  ( $c =$  \_\_\_\_\_)



# WORKSHEET CONCERNS

- **OFTEN FEELS LIKE BUSY WORK**
- **DON'T REALLY BUILD SENSE MAKING**
- **RARELY LEAD TO GREAT CONVERSATIONS**
- **DON'T GIVE US RICH INFORMATION**



# WORKSHEET CONCERNS

- ~~OFTEN FEELS LIKE BUSY WORK~~
- ~~DON'T REALLY BUILD SENSE MAKING~~
- ~~RARELY LEAD TO GREAT CONVERSATIONS~~
- ~~DON'T GIVE US RICH INFORMATION~~



# GOALS

- ☒ WHAT'S WRONG WITH WORKSHEETS?
- ☐ WHAT SHOULD WE BE DOING INSTEAD?
- ☐ HOW DO WE DO IT IN OUR CLASSROOMS?
- ☐ WHERE DO WE GET MORE PROBLEMS?
- ☐ WHAT COMES NEXT?



# PROBLEM ONE

Solve for  $x$ .

$$21 + x = 70$$



# PROBLEM TWO

Using the digits 1 to 9 at most one time each, fill in the boxes to create two equations: one where  $x$  has a positive value and one where  $x$  has a negative value. You may reuse digits for each equation.

$$\boxed{3} \boxed{6} + x = \boxed{8} \boxed{4}$$



# PROBLEM THREE

Using the digits 1 to 9 at most one time each, fill in the boxes to create an equation where  $x$  has the greatest possible value.

$$\boxed{1}\boxed{2} + x = \boxed{9}\boxed{8}$$





Robert Kaplinsky

@robertkaplinsky

MS & HS #MTBoS Ts, please ask your Ss these 3 ?s and put the % who answered correctly here:

[docs.google.com/forms/d/e/1FAI](https://docs.google.com/forms/d/e/1FAI) .... Answers at top of form.

## PROBLEM ONE

Solve for x.

$$21 + x = 7$$

RobertKaplinsky.com

## PROBLEM TWO

Using the digits 1 to 9, at most one time each, create two equations: one where x has a positive value and one where x has a negative value.

$$\square\square + x = \square\square$$

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## PROBLEM THREE

Using the digits 1 to 9, at most one time each, create an equation where x has the greatest possible value.

$$\square\square + x = \square\square$$

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RETWEETS

36

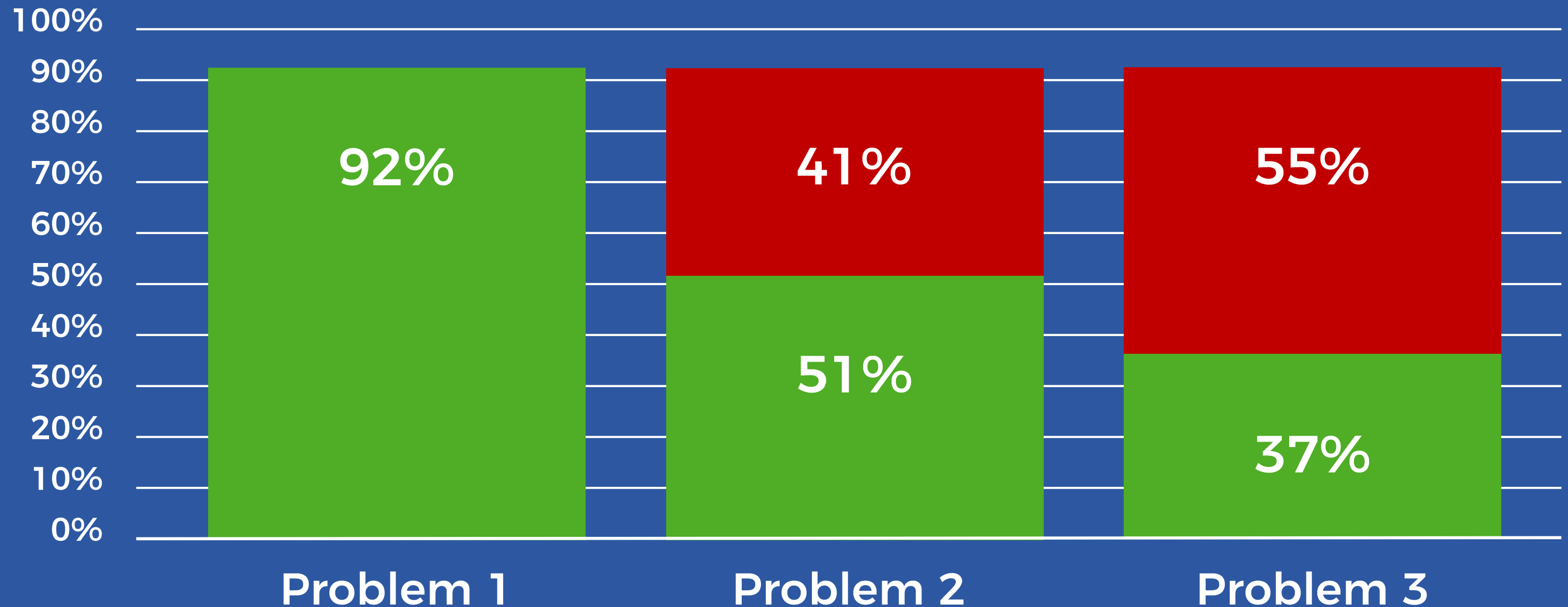
LIKES

54





# PROBLEM RESULTS





What is the perimeter  
of a rectangle that  
measures 8 units by 4  
units?







List the dimensions of a rectangle with a perimeter of 24 units.





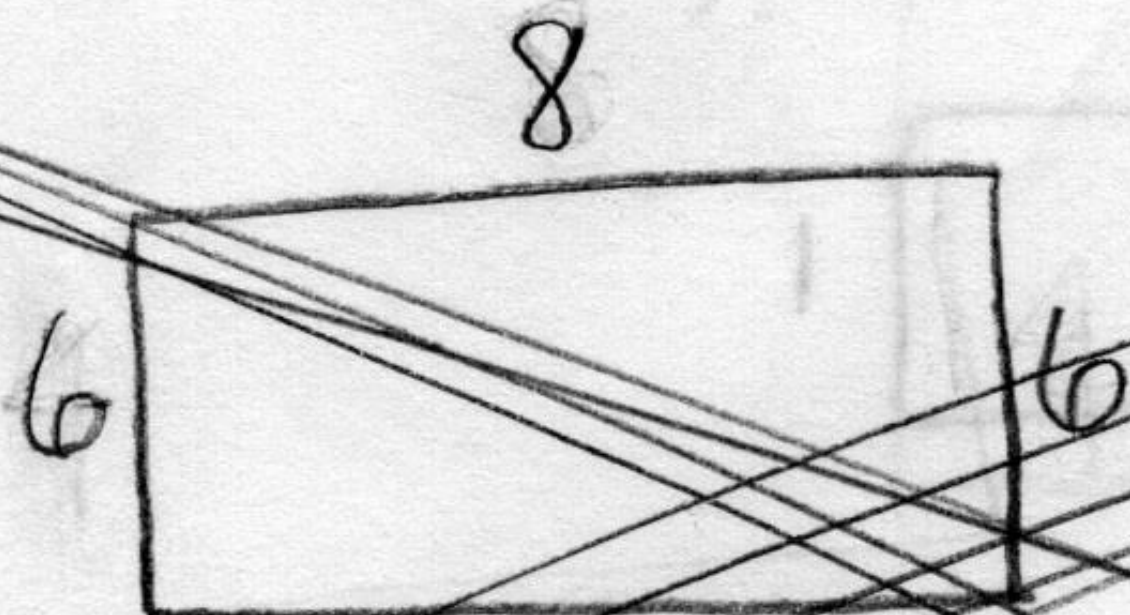


What is the greatest  
area you can make  
from a rectangle with a  
perimeter of 24 units?



First attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation



area:

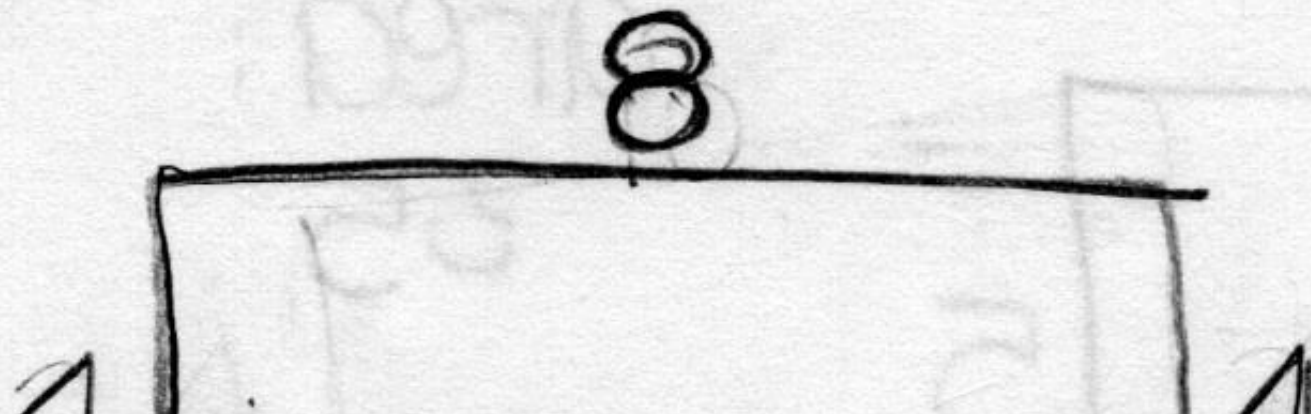
48

What did you learn from this attempt? How will your strategy change on your next attempt?

~~This attempt doesn't equal 24.~~

Second attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation



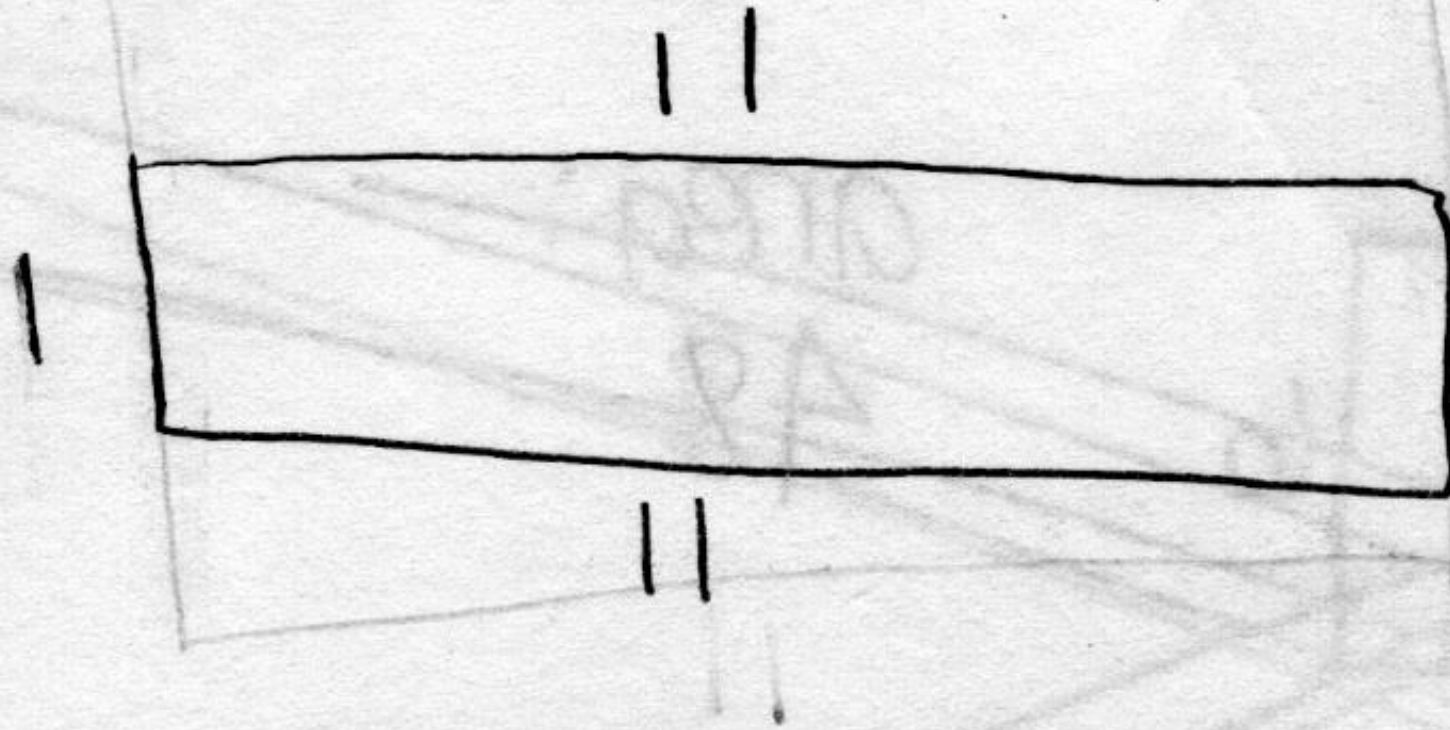
area:

32



Fourth attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation

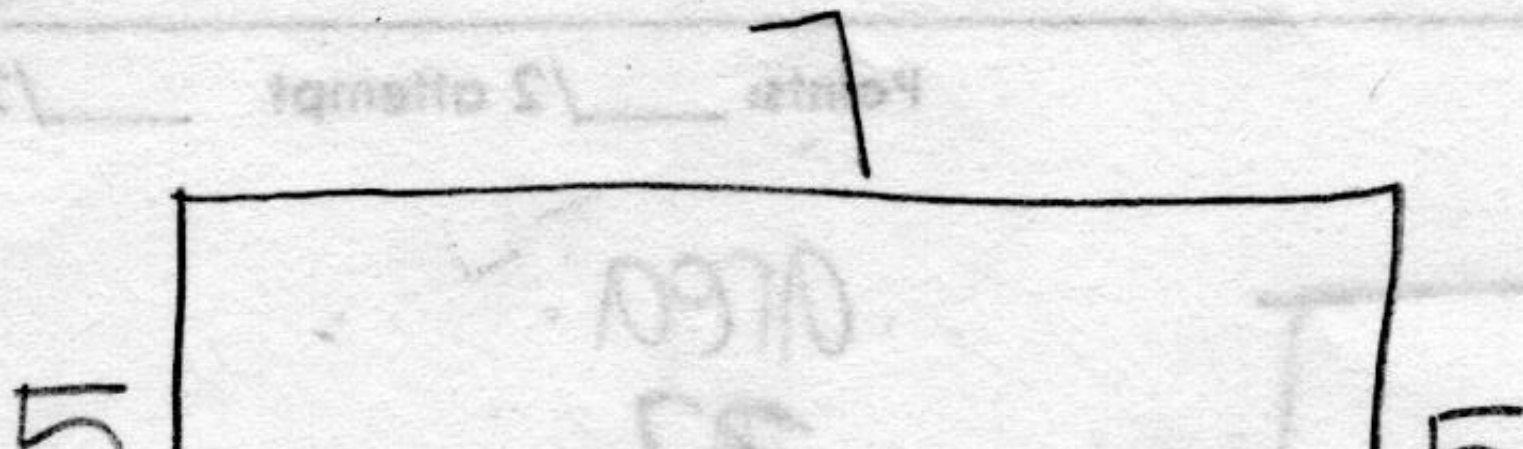


What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter is 24, but the area is 11 and attempt #2 the area is 32  
Strategy: Use #'s with more than one row.

Fifth attempt:

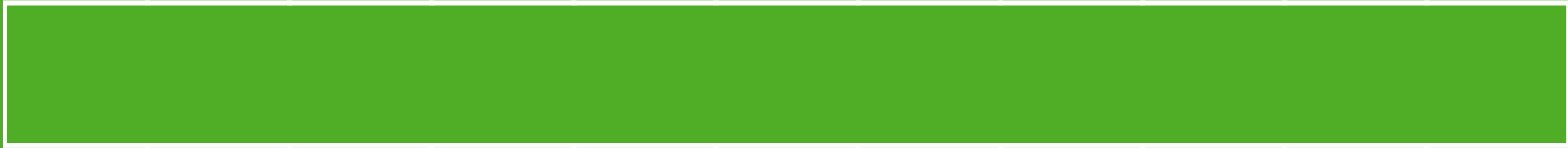
Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation



area:  
35



11 units



1 unit

10 units

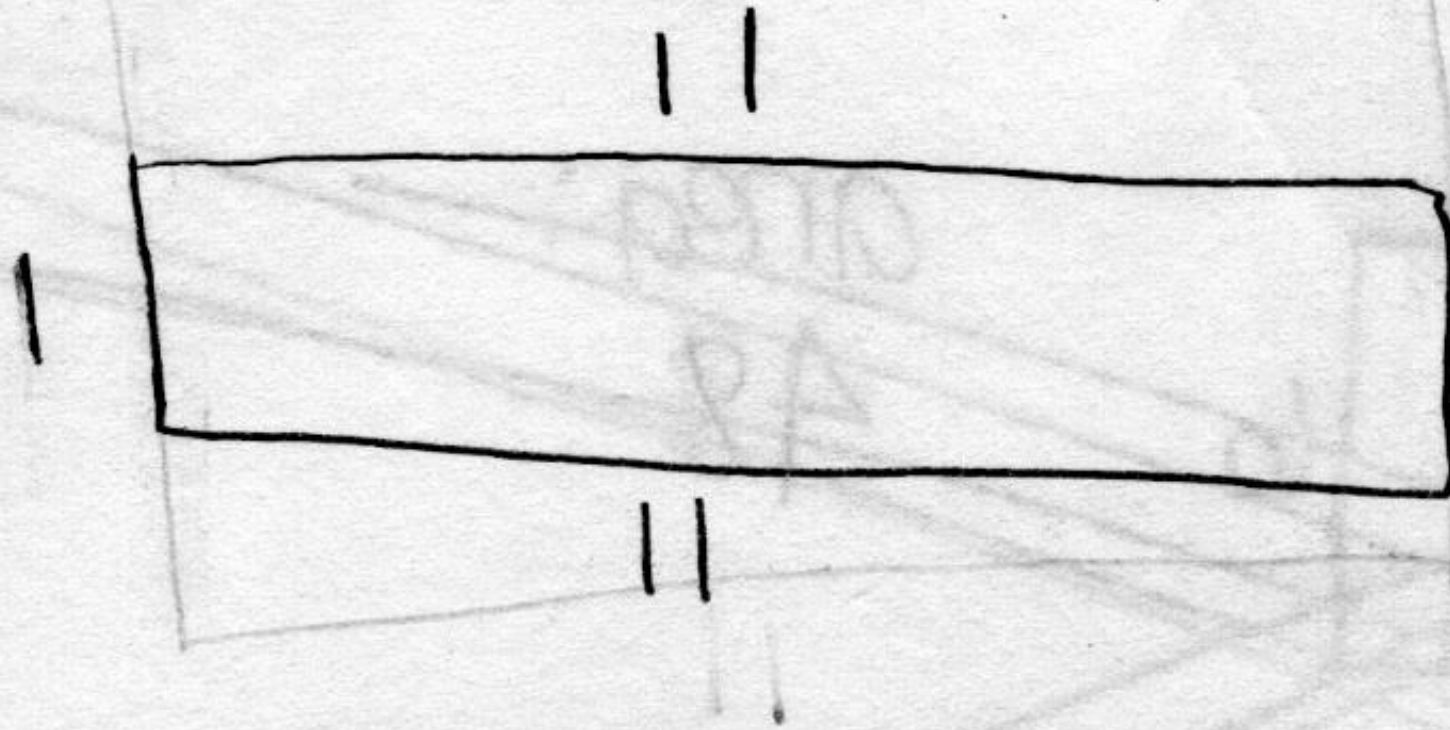


2 units



Fourth attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation

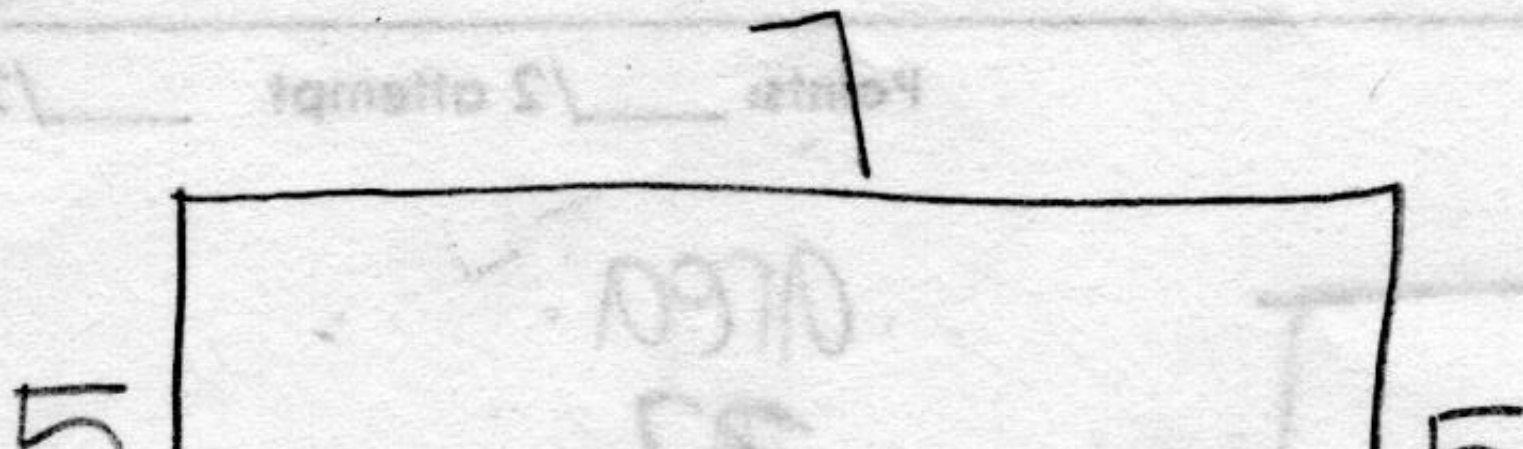


What did you learn from this attempt? How will your strategy change on your next attempt?

The perimeter is 24, but the area is 11 and attempt #2 the area is 32  
Strategy: Use #'s with more than one row.

Fifth attempt:


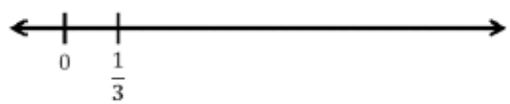

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation



area:  
35

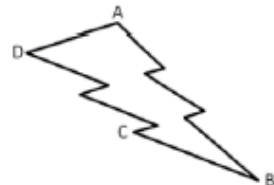
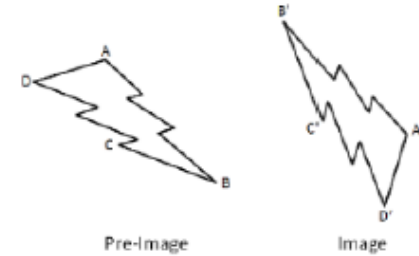
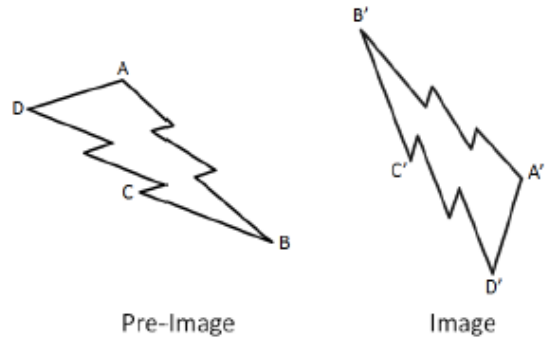


# Depth of Knowledge Matrix - Elementary & Secondary Math

| Topic            | Adding Whole Numbers   | Money   | Fractions on a Number Line   | Area and Perimeter  | Subtracting Mixed Numbers   |
|------------------|--|---|--|---|---|
| CCSS Standard(s) | <ul style="list-style-type: none"> <li>1.NBT.4</li> <li>2.NBT.5</li> </ul>   | <ul style="list-style-type: none"> <li>2.MD.8</li> </ul>                                  | <ul style="list-style-type: none"> <li>3.NF.2</li> </ul>   | <ul style="list-style-type: none"> <li>3.MD.8</li> <li>4.MD.3</li> </ul>                          | <ul style="list-style-type: none"> <li>5.NF.1</li> </ul>  |
| DOK 1 Example    | <p>Find the sum.</p> $44 + 27 =$   | <p>If you have 2 dimes and 3 pennies, how many cents do you have?</p>                     | <p>Which point is located at <math>\frac{7}{12}</math> below?</p>   | <p>Find the perimeter of a rectangle that measures 4 units by 8 units.</p>                        | <p>Find the difference.</p> $5\frac{1}{2} - 4\frac{2}{3} =$   |
| DOK 2 Example    | <p>Using the digits 1 to 9 at most one time each, fill in the boxes so that you make a true equation.</p> $\square\square + 53 = \square\square$ | <p>Make 47¢ in three different ways with either quarters, dimes, nickels, or pennies.</p> | <p>Label the point where <math>\frac{3}{4}</math> belongs on the number line below. Be as precise as possible.</p>  | <p>List the measurements of three different rectangles that each has a perimeter of 20 units.</p> | <p>Using the digits 1 to 9 at most one time each, fill in the boxes to create three different mixed numbers that will make the equation true. You may reuse the same digits for each of the three mixed numbers.</p> $5\frac{4}{5} - \square\square\frac{\square}{\square} = 3\frac{1}{20}$ |
| DOK 3 Example    | <p>Using the digits 1 to 9 at most one time each, fill in the boxes to make the largest sum.</p> $\square\square + \square\square =$             | <p>Make 47¢ using exactly 6 coins with either quarters, dimes, nickels, or pennies.</p>   | <p>Using the digits 0 to 9 at most one time each, create five fractions with a digit for each numerator and denominator and place them all on a number line.</p>                                       | <p>What is the greatest area you can make with a rectangle that has a perimeter of 24 units?</p>  | <p>Using the digits 1 to 9 at most one time each, fill in the boxes to make the smallest difference.</p>   |

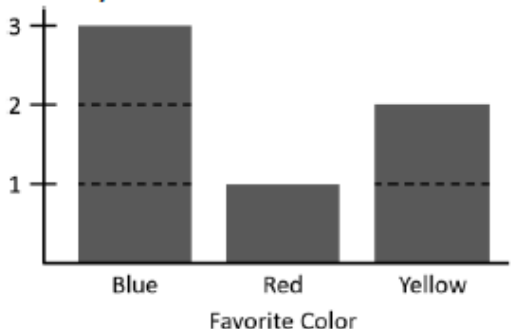
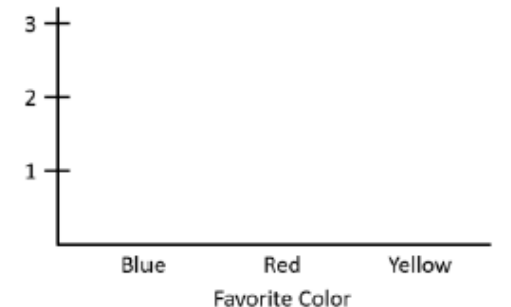
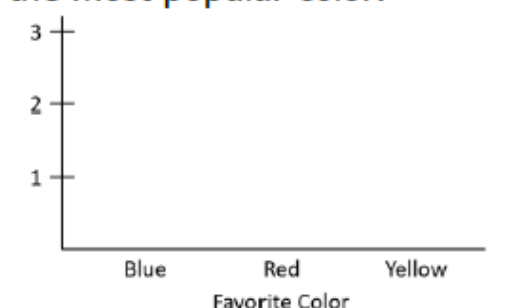


# Depth of Knowledge Matrix - Elementary & Secondary Math

| Topic            | Surface Area and Volume   | Probability  | Transformations  | Factoring Quadratics  | Quadratics in Vertex Form  |
|------------------|---|--|--|---|--|
| CCSS Standard(s) | <ul style="list-style-type: none"> <li>6.G.4</li> <li>7.G.6</li> </ul>  | <ul style="list-style-type: none"> <li>7.SP.5</li> <li>7.SP.7</li> </ul>   | <ul style="list-style-type: none"> <li>8.G.1</li> <li>G-CO.5</li> </ul>  | <ul style="list-style-type: none"> <li>A-SSE.3a</li> </ul>  | <ul style="list-style-type: none"> <li>F-IF.7a</li> </ul>  |
| DOK 1 Example    | Find the surface area of a rectangular prism that measures 3 units by 4 units by 5 units.                     | What is the probability of rolling a sum of 5 using two 6-sided dice?  | Rotate the image below 90° counterclockwise about point D and reflect it across a horizontal line.  | Find the factors:<br>$2x^2 + 7x + 3$  | Find the roots and maximum of the quadratic equation below.<br>$y = -3(x - 4)^2 - 3$   |
| DOK 2 Example    | List the measurements of three different rectangular prisms that each have a surface area of 20 square units. | What value(s) have a 1/12 probability of being rolled as the sum of two 6-sided dice?  | List three sequences of transformations that take pre-image ABCD to image A'B'C'D'.                | Find three different integers to put in the blank that will make the quadratic expression factorable.<br>$x^2 + \_\_x + 4$            | Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values.  |
| DOK 3 Example    | What is the greatest volume you can make with a rectangular prism that has a surface area of 20 square units? | Using the digits 1 to 9 at most one time each, fill in the blanks to make this sentence true.<br><br>Rolling a sum of $\_\_$ on two $\_\_$ -sided dice is the same probability as rolling a sum of $\_\_$ on two $\_\_$ -sided dice. | What is the fewest number of transformations needed to take pre-image ABCD to image A'B'C'D'?     | Fill the blank by finding the largest and smallest integers that will make the quadratic expression factorable.<br>$2x^2 + 3x + \_\_$ | Using the digits 1 to 9 at most one time each, fill in the boxes to create a quadratic equation with the largest maximum value.<br>$y = -\square(x - \square)^2 + \square$ |



# Depth of Knowledge Matrix - Elementary Math

| Topic            | Adding 1-Digit Numbers (< 5)   | Equality   | Interpreting Data   | Money   |
|------------------|--|--|---|---|
| CCSS Stand.      | • K.OA.5   | • 1.OA.7   | • 1.MD.4  | • 2.MD.8  |
| DOK 1<br>Example | Solve.<br><br>$3 + 1 =$  | Determine whether the number sentence is true or false.<br><br>$4 + 1 = 5 - 2$   | How many people were surveyed?<br>   | If you have 1 quarter, 4 dimes, 2 nickels, and 3 pennies, how many cents do you have? |
| DOK 2<br>Example | Using the digits 1 to 5 at most one time each, fill in the boxes to create two true number sentences.<br><br>$\square + \square = \square$                             | Using the digits 1 to 9 at most one time each, fill in the boxes to create two true number sentences.<br><br>$\square + \square = \square - \square$                               | Make a graph that shows a possible result of 7 students' favorite color.<br>  | Make 72¢ in two different ways with either quarters, dimes, nickels, or pennies.      |
| DOK 3<br>Example | Using the digits 1 to 5 at most one time each, fill in the boxes to create a true number sentence with the greatest possible sum.<br><br>$\square + \square = \square$ | Using the digits 1 to 9 at most one time each, fill in the boxes to create a true number sentence with the greatest possible value.<br><br>$\square + \square = \square - \square$ | Make a graph that shows a possible result of 7 students' favorite color with red being the most popular color.<br> | Make 72¢ using exactly 9 coins that are either quarters, dimes, nickels, or pennies.  |



# Depth of Knowledge Matrix - Elementary Math

| Topic            | Subtracting 3-Digit Numbers  | Operations with Time  | Comparing Fractions   | Multiplying Decimals  |
|------------------|--|---|---|---|
| CCSS Stand.      | • 3.NBT.2  | • 3.MD.1  | • 4.NF.2  | • 5.NBT.7   |
| DOK 1<br>Example | Solve.<br><br>$821 - 357 =$  | What time will it be 14 minutes after 1:27 pm?  | Place a < or > between the two fractions to make a true number sentence.<br><br>$\frac{4}{7} \quad \frac{3}{5}$   | Solve.<br><br>$3.4 \times 2.5 =$  |
| DOK 2<br>Example | Using the digits 1 to 9 at most one time each, fill in the boxes to make two different pairs of three-digit numbers that form a true number sentence.<br><br>$\square\square\square - 291 = \square\square\square$ | Using the digits 1 to 9 at most one time each, fill in the boxes to make a time that is 4:37 pm.<br><br>$\square\square$ minutes after<br>$\square:\square\square$ pm   | Using the digits 1 to 9 at most one time each, fill in the boxes to create two different fractions: one that is less than one half and one that is more than one half.<br><br>$\frac{\square}{\square} < \frac{1}{2}$ and $\frac{\square}{\square} > \frac{1}{2}$ | Using the digits 1 to 9 at most one time each, fill in the boxes to make a true number sentence.<br><br>$\square.\square \times 3.2 = \square.\square$                |
| DOK 3<br>Example | Using the digits 1 to 9 at most one time each, fill in the boxes to make a difference that is as close to 329 as possible.<br><br>$\square\square\square - \square\square\square =$                                | Using the digits 1 to 9 at most one time each, fill in the boxes to make the latest possible time.<br><br>$\square\square$ minutes after<br>$\square:\square\square$ pm | Using the digits 1 to 9 at most one time each, fill in the boxes to create a fraction that is as close to $\frac{5}{11}$ as possible.<br><br>$\frac{\square}{\square}$  | Using the digits 1 to 9 at most one time each, fill in the boxes so that the product is as close to 50 as possible.<br><br>$\square.\square \times \square.\square =$ |

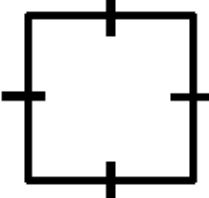


## Depth of Knowledge Matrix - Secondary Math

| Topic            | Dividing Fractions  | Solving Two-Step Equations  | Exponents  | Solving Equations with Variables on Both Sides   |
|------------------|---|---|--|--|
| CCSS Standard(s) | <ul style="list-style-type: none"> <li>6.NS.1</li> </ul>  | <ul style="list-style-type: none"> <li>7.EE.4a</li> </ul>   | <ul style="list-style-type: none"> <li>8.EE.1</li> </ul>   | <ul style="list-style-type: none"> <li>8.EE.8</li> <li>A-REI.3</li> </ul>  |
| DOK 1 Example    | <p>Evaluate.</p> $\frac{4}{9} \div \frac{2}{5}$   | <p>Solve for <math>x</math>.</p> $2x + 3 = 9$   | <p>Evaluate.</p> $3^4$   | <p>Solve for <math>x</math>.</p> $3x + 2 = -2x + 4$  |
| DOK 2 Example    | <p>Using the digits 1 to 9 at most one time each, fill in the boxes to make two different pairs of fractions that have a quotient of <math>\frac{2}{3}</math>.</p> $\frac{\square}{\square} \div \frac{\square}{\square} = \frac{2}{3}$ | <p>Using the digits 1 to 9 at most one time each, fill in the boxes to create two equations: one where <math>x</math> has a positive value and one where <math>x</math> has a negative value.</p> $\square x + \square = \square$ | <p>Using the digits 1 to 9 at most one time each, fill in the boxes to make two true number sentences.</p> $\square^{\square} = 64$  | <p>Using the digits 1 to 9 at most <u>two</u> times each, fill in the boxes to make an equation with no solutions.</p> $\square x + \square = \square x + \square$ |
| DOK 3 Example    | <p>Using the digits 1 to 9 at most one time each, fill in the boxes to make two fractions that have a quotient that is as close to <math>\frac{4}{11}</math> as possible.</p> $\frac{\square}{\square} \div \frac{\square}{\square}$    | <p>Using the digits 1 to 9 at most one time each, fill in the boxes to create an equation where <math>x</math> has the greatest possible value.</p> $\square x + \square = \square$   | <p>Using the digits 1 to 9 at most one time each, fill in the boxes to make a result that has the greatest value possible.</p> $\square^{\square} = \square\square\square$ | <p>Using the digits 1 to 9 at most one time each, fill in the boxes so that the solution is closest to zero.</p> $\square x + \square = \square x + \square$       |



## Depth of Knowledge Matrix - Secondary Math

| Topic            | Geometric Proofs   | Complex Numbers  | Trigonometric Functions  | Definite Integrals   |
|------------------|--|--|--|--|
| CCSS Standard(s) | <ul style="list-style-type: none"> <li>G-CO.11</li> </ul>  | <ul style="list-style-type: none"> <li>N-CN.2</li> </ul>   | <ul style="list-style-type: none"> <li>F-TF.3</li> </ul>   | <ul style="list-style-type: none"> <li>N/A</li> </ul>  |
| DOK 1 Example    | Add one geometric marking to demonstrate the quadrilateral is a square.  | Multiply the binomials. $(3 + 4i)(2 + 3i)$   | Evaluate. $\sin \frac{\pi}{3}$   | Solve. $\int_2^6 x^3 dx$   |
| DOK 2 Example    | Use exactly five geometric markings to show that a quadrilateral is a square.  | Using the integers -9 to 9 at most one time each, fill in the boxes twice: once to make a positive real number product and once to make a negative real number product. $(\square + \square i)(\square + \square i)$ | Using the digits 1 to 9 at most one time each, fill in the boxes to make two true number sentences. $\sin \frac{\square \pi}{\square} = 1$   | Using the digits 1 to 9 at most one time each, fill in the boxes to make a positive and a negative solution. $\int_{\square}^{\square} x^{\square} dx$             |
| DOK 3 Example    | What is the least number of geometric markings needed to demonstrate that a quadrilateral is a square?   | Using the integers -9 to 9 at most one time each, fill in the boxes to make a real number product with the greatest value. $(\square + \square i)(\square + \square i)$  | Using the digits 1 to 9 at most one time each, fill in the boxes to find the function's greatest possible value. $\sin \frac{\square \pi}{\square} = \frac{\sqrt{\square}}{\square}$ | Using the digits 1 to 9 at most one time each, fill in the boxes to make a solution that is as close to 100 as possible. $\int_{\square}^{\square} x^{\square} dx$ |

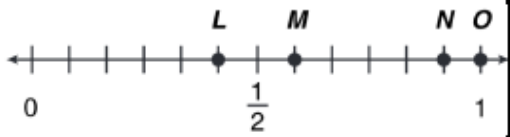



## Depth of Knowledge Matrix – Third Grade Math

| Topic            | Rounding  | Adding 3-Digit Numbers  | Subtracting 3-Digit Numbers  | Multiplying Multiples Of Ten   |
|------------------|---|---|--|--|
| CCSS Stand.      | • 3.NBT.1   | • 3.NBT.2   | • 3.NBT.2  | • 3.NBT.3  |
| DOK 1<br>Example | Round to the nearest hundred.<br><br>436  | Add.<br><br>$253 + 419 =$   | Solve.<br><br>$821 - 357 =$  | Multiply.<br><br>$4 \times 60$   |
| DOK 2<br>Example | Using the digits 0 to 9 at most one time each, place a digit in each box to make two different three-digit numbers that round (to the nearest hundred) to 500.<br><br>□□□ and □□□       | Using the digits 1 to 9 exactly one time each, place a digit in each box two times: once to make a sum that is greater than 700 and once to make a sum that is less than 700. You may reuse all the digits for each sum.<br><br>$\begin{array}{r} \square\square\square \\ + \square\square\square \\ \hline \end{array}$ | Using the digits 1 to 9 at most one time each, place a digit in each box to make two different pairs of three-digit numbers that form a true number sentence. You may reuse all the digits each difference.<br><br>□□□ - 291 = □□□ | Using the digits 0 to 9 at most one time each, place a digit in each box to make two different true number sentences: one with a product that's less than 500 and one with a product that's greater than 500. You may reuse all the digits each product.<br><br>□ × □0 = □□□ |
| DOK 3<br>Example | Using the digits 0 to 9 at most one time each, place a digit in each box to make the greatest possible three-digit number that still rounds (to the nearest hundred) to 500.<br><br>□□□ | Using the digits 1 to 9 exactly one time each, place a digit in each box to make the sum as close to 1000 as possible.<br><br>$\begin{array}{r} \square\square\square \\ \square\square\square \\ + \square\square\square \\ \hline \end{array}$  | Using the digits 1 to 9 at most one time each, place a digit in the boxes to make a difference that is as close to 329 as possible.<br><br>□□□ - □□□ =   | Using the digits 0 to 9 at most one time each, place a digit in each box to make a product that's as close to 500 as possible.<br><br>□ × □0 = □□□   |



# Depth of Knowledge Matrix – Fourth Grade Math

| Topic            | Fractions on a Number Line   | Comparing Fractions   | Adding Mixed Numbers   | Comparing Decimals   |
|------------------|--|---|--|--|
| CCSS Stand.      | • 4.NF.2   | • 4.NF.2  | • 4.NF.3a  | • 4.NF.7   |
| DOK 1<br>Example | Which point is located at $\frac{7}{12}$ below?<br>  | Compare the fractions using a <, >, or = sign.<br>$\frac{3}{8}$ $\frac{4}{7}$   | Find the sum.<br>$3\frac{5}{8} + 2\frac{7}{8} =$   | Compare the decimals using a <, >, or = sign.<br>6.714   8.023   |
| DOK 2<br>Example | Label the point where $\frac{3}{5}$ belongs on the number line below. Be as precise as possible.<br>   | Using the digits 1 to 9 at most one time each, place a digit in each box to create a true statement.<br>$\frac{\square}{\square} < \frac{\square}{\square} < \frac{\square}{\square}$ | Using the digits 1 to 9 at most one time each, place a digit in each box to make a true equation.<br>$\square\frac{\square}{8} + \square\frac{\square}{8} = \square\frac{\square}{8}$                                | Using the digits 0 to 9 at most one time each, place a digit in each box to create two different decimals: one that is greater than 5 and one that is less than 5.<br>$\square.\square\square\square$<br>$\square.\square\square\square$ |
| DOK 3<br>Example | Using the digits 0 to 9 at most one time each, place a digit in each box to create five fractions and place them all on a number line with the correct order and spacing.<br>$\frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}$ | Using the digits 1 to 9 at most one time each, place a digit in each box to create a fraction as close to one as possible.<br>$\frac{\square\square}{\square\square}$                 | Using the digits 1 to 9 at most one time each, place a digit in each box to make a true equation with the smallest possible sum.<br>$\square\frac{\square}{8} + \square\frac{\square}{8} = \square\frac{\square}{8}$ | Using the digits 0 to 9 at most one time each, place a digit in each box to create two decimals that are close to 5 as possible but also equally far away from 5.<br>$\square.\square\square\square$<br>$\square.\square\square\square$  |



## Depth of Knowledge Matrix – Fifth Grade Math

| Topic            | Evaluating Expressions   | Rounding Decimals  | Multi-Digit Multiplication   | Multiplying Decimals  |
|------------------|--|--|--|---|
| CCSS Stand.      | • 5.OA.1   | • 5.NBT.4  | • 5.NBT.5  | • 5.NBT.7   |
| DOK 1<br>Example | Evaluate the expression.<br><br>$56 \div (8 - 1)$  | Round the decimal to the nearest tenth.<br><br>7.163   | Find the product.<br><br>$37 \times 45$  | Solve.<br><br>$3.4 \times 2.5 =$  |
| DOK 2<br>Example | Using the digits 0 through 9, at most one time each, place a digit in each box to create two true statements: one where the value on each side of the equal sign is greater than 30 and one where it's less than 30. You may reuse all the digits for each equation.<br><br>$\square\square \div (\square - \square) = \square + \square \times \square$ | Using the digits 0 to 9 at most one time each, place a digit in each box to create two different decimals that are equivalent when rounded to the nearest tenth.<br><br>$\square.\square\square\square$<br>$\square.\square\square\square$                                   | Using the digits 0 to 9 at most one time each, place a digit in each box to create a true equation.<br><br>$\square\square \times \square\square = \square\square\square$                                    | Using the digits 1 to 9 at most one time each, fill in the boxes to make a true number sentence.<br><br>$\square.\square \times 3.2 = \square.\square$                |
| DOK 3<br>Example | Using the digits 0 through 9, at most one time each, place a digit in each box to create the greatest possible value.<br><br>$\square\square \div (\square - \square) = \square + \square \times \square$  | Using the digits 0 to 9 at most one time each, place a digit in each box to create two different decimals that are equivalent when rounded to the nearest tenth and have the least possible value.<br><br>$\square.\square\square\square$<br>$\square.\square\square\square$ | Using the digits 0 to 9 at most one time each, place a digit in each box to create a true equation with the greatest possible product.<br><br>$\square\square \times \square\square = \square\square\square$ | Using the digits 1 to 9 at most one time each, fill in the boxes so that the product is as close to 50 as possible.<br><br>$\square.\square \times \square.\square =$ |



# Depth of Knowledge Matrix – Sixth Grade Math

| Topic            | Percent of a Quantity  | Ratios and Unit Rates   | Dividing Fractions  | Multiplying Decimals  |
|------------------|--|---|---|---|
| CCSS Stand.      | • 6.RP.3c  | • 6.RP.1 & 6.RP.2   | • 6.NS.1  | • 6.NS.3  |
| DOK 1<br>Example | Evaluate.<br><br>24 is 30% of what number?   | Fill in the blank to make an equivalent ratio.<br><br>__ : 7 = 8 : 14   | Find the quotient.<br><br>$\frac{4}{9} \div \frac{2}{5}$  | Find the product.<br><br>$3.74 \cdot 4.29$  |
| DOK 2<br>Example | Using the digits 0 to 9 at most one time each, fill in the boxes to make two true statements without rounding. You may reuse all the digits for your second statement.<br><br>$\square\square$ is $\square\square$ % of $\square\square$ | Using the digits 0 to 9 at most one time each, fill in the boxes to make an equivalent ratio.<br><br>$\square:\square=\square\square:\square$   | Using the digits 1 to 9 at most one time each, fill in the boxes to make two different pairs of fractions that have a quotient of $\frac{2}{3}$ . You may reuse all the digits for each equation.<br><br>$\frac{\square}{\square} \div \frac{\square}{\square} = \frac{2}{3}$ | Using the digits 1 to 9 at most one time each, fill in the boxes to make a whole number product.<br><br>$\square.\square \cdot \square.\square\square$                            |
| DOK 3<br>Example | Using the digits 0 to 9 at most one time each, fill in the boxes to make a true statement with the greatest possible whole without rounding.<br><br>$\square\square$ is $\square\square$ % of $\square\square$                           | Using the digits 0 to 9 at most one time each, fill in the boxes to make an equivalent ratio with a unit rate that has greatest possible value.<br><br>$\square:\square=\square\square:\square$ | Using the digits 1 to 9 at most one time each, fill in the boxes to make two fractions that have a quotient that is as close to $\frac{4}{11}$ as possible.<br><br>$\frac{\square}{\square} \div \frac{\square}{\square}$   | Using the digits 1 to 9 at most one time each, fill in the boxes to make a product with the greatest possible value.<br><br>$\square.\square\square \cdot \square.\square\square$ |

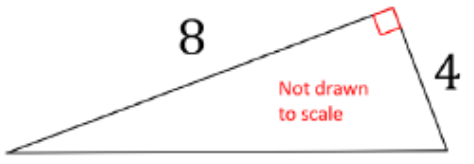
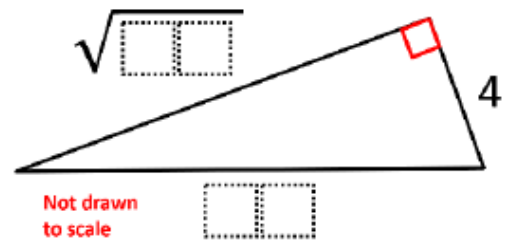
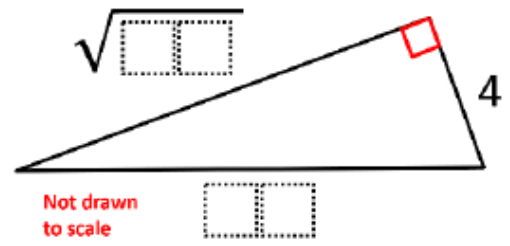


# Depth of Knowledge Matrix – Seventh Grade Math

| Topic            | Markup & Discount  | Unit Rates with Fractions   | + and – Rational Numbers  | x and ÷ Rational Numbers  |
|------------------|--|---|---|---|
| CCSS Stand.      | • 7.RP.3   | • 7.RP.1  | • 7.NS.1  | • 7.NS.2  |
| DOK 1<br>Example | Find the final price of a \$75 item after a 45% discount.  | Find the unit rate.<br>$\frac{2/9}{3/8} = \frac{\quad}{1}$  | Find the sum.<br>$-12 + -7$   | Find the quotient.<br>$\frac{-3}{4} \div \frac{7}{5}$   |
| DOK 2<br>Example | Using the digits 0 to 9 at most one time each, fill in the boxes to create two true statements without rounding. You may reuse all the digits for each statement.<br><br>\$ <input type="text"/> <input type="text"/> item at a <input type="text"/> <input type="text"/> %<br>discount costs \$ <input type="text"/> <input type="text"/> | Using the digits 0 to 9 at most one time each, fill in the boxes to create two unit rates. You may reuse all the digits each equation.<br><br>$\frac{\begin{array}{c} \square \\ \square \end{array}}{\begin{array}{c} \square \\ \square \end{array}} = \frac{\square}{1}$ | Using the integers -9 to 9 at most one time each, fill in the boxes to create two equations. You may reuse all the integers for each equation.<br><br>$-\square + \square = \square - (-\square)$ | Using the integers -9 to 9 at most one time each, fill in the boxes to create two equations. You may reuse all the integers for each equation.<br><br>$-\frac{\square}{\square} \div \frac{\square}{\square} = \frac{\square}{\square}$ |
| DOK 3<br>Example | Using the digits 0 to 9 at most one time each, fill in the boxes to create the least expensive item after discount.<br><br>\$ <input type="text"/> <input type="text"/> item at a <input type="text"/> <input type="text"/> %<br>discount costs \$ <input type="text"/> <input type="text"/>   | Using the digits 0 to 9 at most one time each, fill in the boxes to create a unit rate with the greatest possible value.<br><br>$\frac{\begin{array}{c} \square \\ \square \end{array}}{\begin{array}{c} \square \\ \square \end{array}} = \frac{\square}{1}$               | Using the integers -9 to 9 at most one time each, fill in the boxes to create an equation where each side has the greatest possible value.<br><br>$-\square + \square = \square - (-\square)$     | Using the integers -9 to 9 at most one time each, fill in the boxes to create a quotient with the greatest possible value.<br><br>$-\frac{\square}{\square} \div \frac{\square}{\square} = \frac{\square}{\square}$                     |



# Depth of Knowledge Matrix – Eighth Grade Math

| Topic            | Approximating Irrationals  | Properties of Exponents  | Scientific Notation   | Pythagorean Theorem   |
|------------------|--|--|---|---|
| CCSS Stand.      | • 8.NS.2   | • 8.EE.1   | • 8.EE.4  | • 8.G.8   |
| DOK 1<br>Example | The irrational number $\sqrt{70}$ is between which two integers?   | Simplify.<br>$4^3 \cdot -6^2$  | Simplify.<br>$2 \cdot 10^{-4} \cdot 5 \cdot 10^7$   | Find the length of the missing side.<br>   |
| DOK 2<br>Example | Using the digits 0 to 9 at most one time each, fill in the boxes twice to make two different true statements. You may reuse all the digits for each statement.<br><br>$\sqrt{\square\square}$ is greater than $\square$<br>and less than $\square$ | Using the integers -9 to 9 at most one time each, fill in the boxes twice to make a positive product and a negative product. You may reuse all the integers each product.<br><br>$\square^{\square} \cdot \square^{\square}$ | Using the digits 1 to 9 at most one time each, fill in the boxes twice to make a product that equals 800,000,000. You may reuse all the digits for each product.<br><br>$\square \cdot 10^{\square} \cdot \square \cdot 10^{\square}$ | Using the digits 0 to 9 at most one time each, fill in the boxes to find two pairs of possible lengths for the missing sides.<br><br>   |
| DOK 3<br>Example | Using the digits 0 to 9 at most one time each, fill in the boxes twice to make the greatest possible irrational number.<br><br>$\sqrt{\square\square}$ is greater than $\square$<br>and less than $\square$  | Using the integers -9 to 9 at most one time each, fill in the boxes to make a product that is as close to zero as possible without being exactly zero.<br><br>$\square^{\square} \cdot \square^{\square}$                    | Using the digits 1 to 9 at most one time each, fill in the boxes to make the greatest product.<br><br>$\square \cdot 10^{\square} \cdot \square \cdot 10^{\square}$   | Using the digits 0 to 9 at most one time each, fill in the boxes to find the lengths of the missing sides such that the missing leg's length is as long as possible.<br><br> |

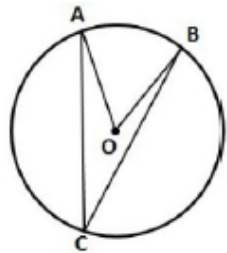
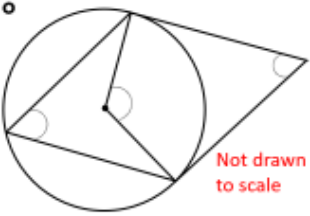
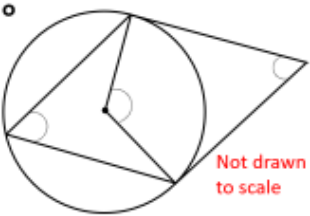


## Depth of Knowledge Matrix – Algebra 1 (Integrated 1)

| Topic         | Solving Equations with Variables on Both Sides   | Factoring Quadratics  | Quadratics in Vertex Form  | Adding polynomials   |
|---------------|--|---|--|--|
| CCSS Stand.   | • A-REI.3  | • A-SSE.3a  | • F-IF.7a  | • A-APR.1  |
| DOK 1 Example | Solve for $x$ .<br>$3x + 2 = -2x + 4$  | Find the factors:<br>$2x^2 + 7x + 3$  | Find the roots and maximum of the quadratic equation below.<br>$y = -3(x - 4)^2 - 3$   | Add the polynomials.<br>$(4x^2 - 3x + 1) + (-6x^2 + 5x)$   |
| DOK 2 Example | Using the digits 1 to 9 at most <u>two</u> times each, fill in the boxes to make an equation with no solutions.<br>$\square x + \square = \square x + \square$ | Find three different integers to put in the blank that will make the quadratic expression factorable.<br>$x^2 + \_\_x + 4$            | Create three equations for quadratics in vertex form that have roots at 3 and 5 but have different maximum and/or minimum values.  | Using the integers -9 to 9 at most one time each, place an integer in each box to make two expressions: one that has three or more terms and one that has fewer than three terms. You may reuse all the integers for each expression.<br>$(\square x^2 - \square x + \square) + (\square x^2 + \square x)$ |
| DOK 3 Example | Using the digits 1 to 9 at most one time each, fill in the boxes so that the solution is closest to zero.<br>$\square x + \square = \square x + \square$       | Fill the blank by finding the largest and smallest integers that will make the quadratic expression factorable.<br>$2x^2 + 3x + \_\_$ | Using the digits 1 to 9 at most one time each, fill in the boxes to create a quadratic equation with the largest maximum value.<br>$y = -\square(x - \square)^2 + \square$ | Using the integers -9 to 9 at most one time each, place an integer in each box to create a polynomial with the least amount of terms.<br>$(\square x^2 - \square x + \square) + (\square x^2 + \square x)$   |



# Depth of Knowledge Matrix – Geometry (Integrated 2)

| Topic            | Equation of a Circle   | Central, Inscribed, & Circumscribed Angles   | Perpendicular Lines   | Area on a Coordinate Plane  |
|------------------|--|--|---|---|
| CCSS Stand.      | • G-MG.1   | • G-C.2  | • G-GPE.5   | • G-GPE.7   |
| DOK 1<br>Example | Write the equation of a circle with a radius of 7 units.   | If the measure of angle AOB is $40^\circ$ , what is the measure of angle ACB?<br>   | Determine whether the lines are perpendicular.<br>$3x + 4y = 7$<br>$y = \frac{2}{3}x + 5$   | Find the area of the triangle with vertices at $(-4, -1)$ , $(-2, 5)$ , and $(3, -3)$   |
| DOK 2<br>Example | Using the digits 1 to 9 at most two times each, place a digit in each box to make two circles: one with an area of less than 100 units <sup>2</sup> and one with more than 100 units <sup>2</sup> .<br><br>$\square x^2 + \square y^2 = \square$ | Using the digits 0 to 9 at most one time each, place a digit in each box two times: once where the central angle is greater than $130^\circ$ and once where it is less than $130^\circ$ . You may reuse all the digits each time.<br><br>central angle measure = $\square\square\square^\circ$<br>inscribed angle measure = $\square\square\square^\circ$<br>circumscribed angle measure = $\square\square\square^\circ$<br> | Using the digits 0 to 9 at most one time each, fill in the boxes to create two perpendicular lines.<br><br>$y = \frac{\square}{\square}x + \square$<br>$\square x + \square y = \square$  | Using the integers -9 to 9 at most one time each, fill in the boxes to create coordinates that represent the vertices of two triangles: one with an area of less than 55 units <sup>2</sup> and one with an area of more than 55 units <sup>2</sup> .<br>You may reuse all the integers each time.<br>$A: (\square, \square)$<br>$B: (\square, \square)$<br>$C: (\square, \square)$ |
| DOK 3<br>Example | Using the digits 1 to 9 at most two times each, place a digit in each box to make a circle with the least possible area.<br><br>$\square x^2 + \square y^2 = \square$  | Using the digits 0 to 9 at most one time each, place a digit in each box so that the central angle has the greatest possible value.<br><br>central angle measure = $\square\square\square^\circ$<br>inscribed angle measure = $\square\square\square^\circ$<br>circumscribed angle measure = $\square\square\square^\circ$<br>  | Using the digits 0 to 9 at most one time each, fill in the boxes to create two perpendicular lines whose solution is as close to the origin as possible.<br><br>$y = \frac{\square}{\square}x + \square$<br>$\square x + \square y = \square$ | Using the integers -9 to 9 at most one time each, fill in the boxes to create coordinates that represent the vertices of the triangle with the smallest possible area.<br><br>$A: (\square, \square)$<br>$B: (\square, \square)$<br>$C: (\square, \square)$   |



## Depth of Knowledge Matrix – Algebra 2 (Integrated 3)

| Topic            | Rational Function Features   | Square Root Function Features   | Exponential Function Features  | Logarithmic Function Features   |
|------------------|--|---|--|---|
| CCSS Stand.      | • F-IF.7d  | • F-IF.7b   | • F-IF.7e  | • F-IF.7e   |
| DOK 1<br>Example | Identify the function's vertical asymptote and its solution.<br>$y = \frac{5}{x+8} + -3$   | Find the domain and x-intercept of the square root function.<br>$y = -5\sqrt{x+7} + 3$  | Find the y-intercept of the exponential function.<br>$y = -2 \cdot 3^{(x+1)} + 4$  | Find the y-intercept of the logarithmic function.<br>$y = 3 \log_6(x - (-4)) + 4$   |
| DOK 2<br>Example | Using the integers -9 to 9, at most one time each, fill in the boxes to create a rational function, its vertical asymptote, and its solution.<br>$y = \frac{\square}{x + \square} + \square$<br>solution: $x = \square$<br>vertical asymptote: $x = \square$                   | Using the integers -9 to 9, at most one time each, fill in the boxes to create a square root function, its domain, and the x-intercept.<br>$y = \square\sqrt{x + \square} + \square$<br>domain: $x \geq \square$<br>x-intercept: $(\square, \square)$                   | Use the integers -9 to 9, at most two times each, fill in the boxes to create an exponential growth function with its y-intercept.<br>$y = \square \cdot \square^{(x + \square)} + \square$<br>y-intercept: $(0, \square)$                   | Using the integers -9 to 9, at most one time each, fill in the boxes and create a logarithmic function with its corresponding y-intercept.<br>$y = \square \log_{\square}(x - \square) + \square$<br>y-intercept: $(0, \square)$    |
| DOK 3<br>Example | Using the integers -9 to 9, at most one time each, fill in the boxes to create a rational function, its vertical asymptote, and the greatest possible solution.<br>$y = \frac{\square}{x + \square} + \square$<br>solution: $x = \square$<br>vertical asymptote: $x = \square$ | Using the integers -9 to 9, at most one time each, fill in the boxes to create a square root function, its domain, and the greatest possible x-intercept.<br>$y = \square\sqrt{x + \square} + \square$<br>domain: $x \geq \square$<br>x-intercept: $(\square, \square)$ | Use the integers -9 to 9, at most two times each, fill in the boxes to create an exponential growth function with the greatest possible y-intercept.<br>$y = \square \cdot \square^{(x + \square)} + \square$<br>y-intercept: $(0, \square)$ | Using the integers -9 to 9, at most one time each, fill in the boxes to create a logarithmic function with the greatest possible y-intercept.<br>$y = \square \log_{\square}(x - \square) + \square$<br>y-intercept: $(0, \square)$ |





**Chrissy Day**

@ChrissyDay1974



I LOVE Open Middle [@robertkaplinsky](#) second graders were working on \_\_\_\_ - \_\_\_\_ Make the smallest difference possible using the digits 1-9 once only. The conversation and perseverance was something I had never seen from these kids!

5:20 PM · Mar 9, 2019 · [Twitter for iPhone](#)

**6** Retweets   **62** Likes





**DeLaina Ellis** @dellis5th · Jan 11

It was an @openmiddle showdown in 5th grade! They could NOT stop! One student even asked me for his paper during recess so he could try to get even closer! #wearegrandview #iteachmath #mtbos #productivestruggle



1



2



8

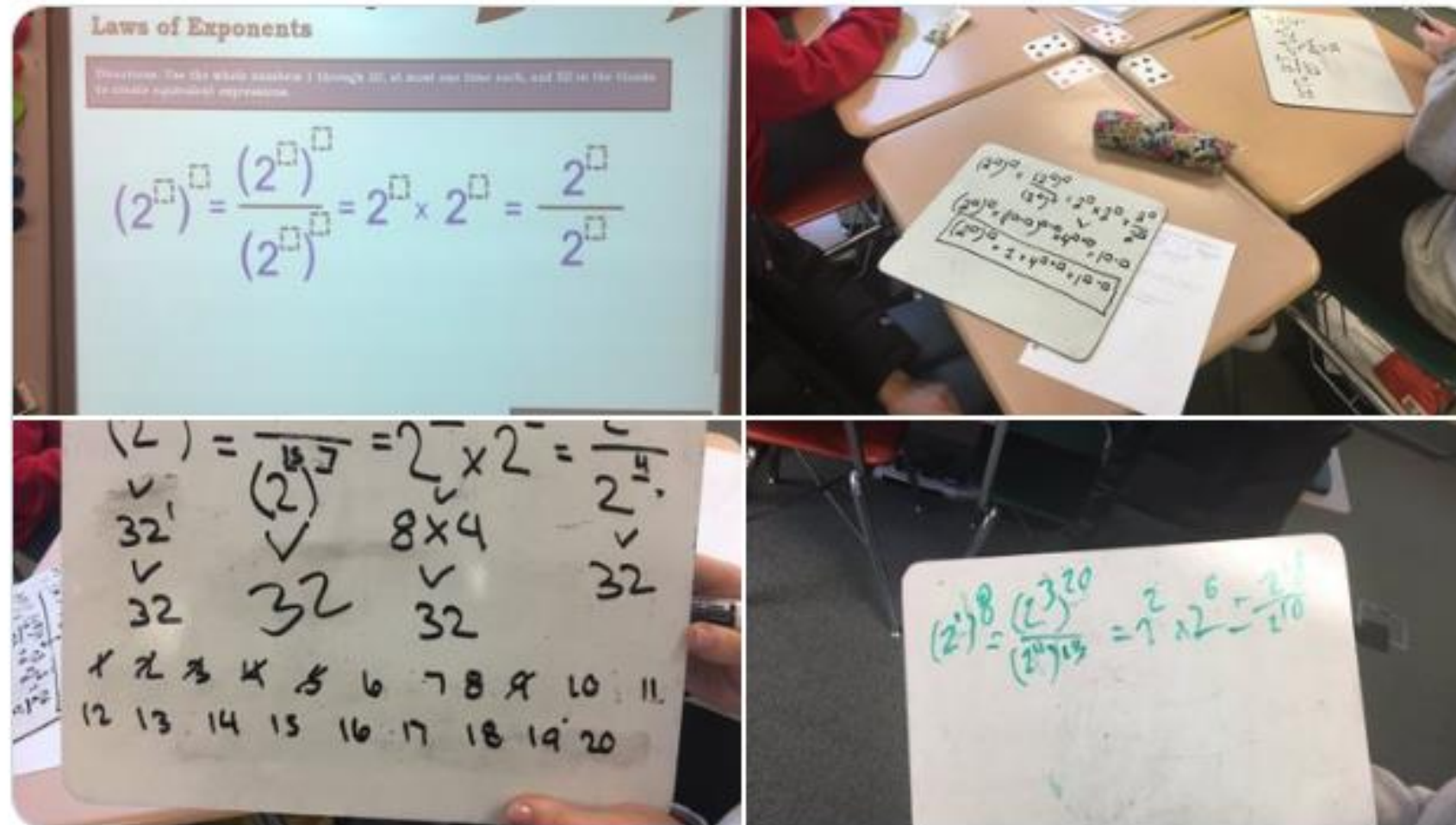






Wendy Kozina  
@wkozina

Kids begging for more time and yelling, "No" when I asked if they wanted a hint! Amazing activity  
[@robertkaplinsky](#) [@openmiddle](#)



1:30 PM · Mar 8, 2019 · [Twitter for iPhone](#)

14 Retweets 98 Likes





**Marguerite Spriggs** @mspriggs30 · Nov 16, 2018

My **first time trying** an @openmiddle problem with my students today. Wasn't sure how it would go or if they'd solve it. After a few minutes going at it (and coming up with more than one solution) they asked "can we do another one?" "That was fun - we should do it more!"

## Radical Challenge

### RADICAL EQUATIONS

Directions: Using the digits 0-9 at most one time each, make both of these equations true.

$$\sqrt{\boxed{\phantom{00}}} = \boxed{\phantom{0}} \sqrt{\boxed{\phantom{0}}}$$
$$\sqrt{\boxed{\phantom{00}}} - \boxed{\phantom{0}}$$



1



1



7







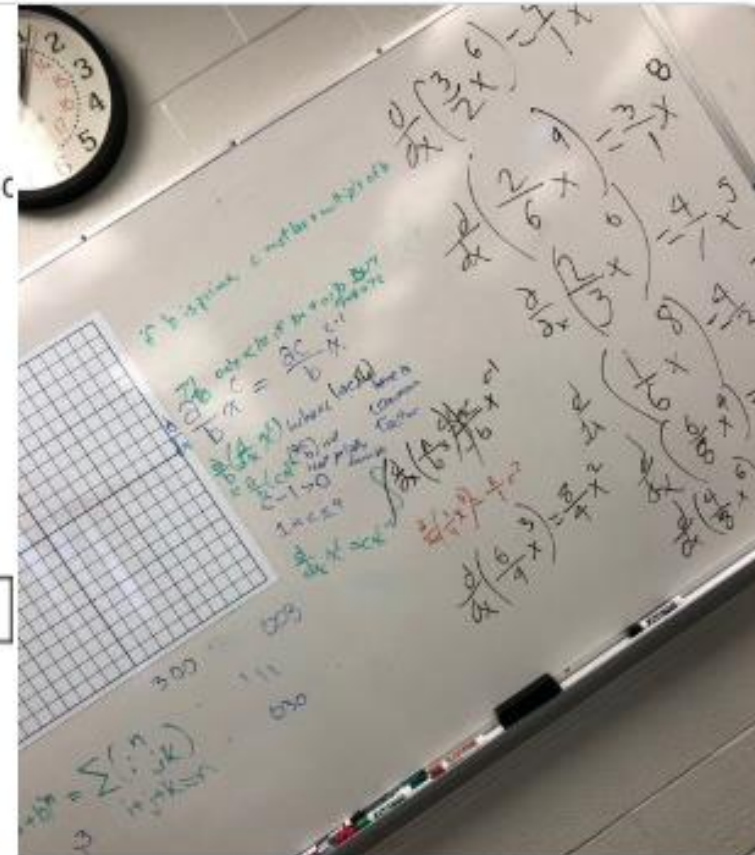
**Katherine MacKenzie**  
@kmackenzie7

Tried an [@openmiddle](#) problem (for the 1st time) with my calculus crew. Left it on the board went to grab a photocopy before class start. Came back and Ss were crowded around the board sharing ideas. It's was magical. I \*must\* bring these to all my classes [#MTBoS](#) [#iteachmath](#)

### DERIVATIVE POWER RULE

Directions: Use the digits 1 to 9, at most one time each, to create a true derivative statement.

$$\frac{d}{dx} \left( \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} x^{\boxed{\phantom{00}}} \right) = \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} x^{\boxed{\phantom{00}}}$$



2:17 PM · Apr 18, 2019 · [Twitter for iPhone](#)

20 Retweets 156 Likes



# OPEN MIDDLE PROBLEM BENEFITS

- KIDS LOVE DOING THEM
- BUILD CONCEPTUAL UNDERSTANDING
- OFTEN LEAD TO GREAT CONVERSATIONS
- REVEAL HIDDEN MISCONCEPTIONS



# GOALS

☒ WHAT'S WRONG WITH WORKSHEETS?

☒ WHAT SHOULD WE BE DOING INSTEAD?

☐ HOW DO WE DO IT IN OUR CLASSROOMS?

☐ WHERE DO WE GET MORE PROBLEMS?

☐ WHAT COMES NEXT?



# HOW DO WE DO IT?

- Open Middle Worksheet



Name: \_\_\_\_\_ Period: \_\_\_\_\_ Date: \_\_\_\_\_

First attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?

Second attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation



First attempt:

Points: \_\_\_\_/2 attempt \_\_\_\_/2 explanation

What did you learn from this attempt? How will your strategy change on your next attempt?



**Directions:** Using the digits 1 to 9 exactly one time each, place a digit in each box to make the sum as close to 1000 as possible.

+  +

**Directions:** Using the digits 1 to 9 exactly one time each, place a digit in each box to make the sum as close to 1000 as possible.

459 + 368 + 217

What did you learn from this attempt? How will your strategy change on your next attempt?

10 44

Students, draw anywhere on this slide!

😊 Robert Thomas

**Directions:** Using the digits 1 to 9 exactly one time each, place a digit in each box to make the sum as close to 1000 as possible.

176 + 358 + 294

What did you learn from this attempt? How will your strategy change on your next attempt?

i tried

## Student Strategy Tracker (Grades 6 to 12) ☆ 📁 ☁

File Edit View Insert Format Tools Add-ons Help Last edit was yesterday at 11:...

### Strategy

Guess and check  
(Students are picking all  
digits randomly)

| Student Name(s) and Notes |
|---------------------------|
|---------------------------|

## Order

Guess and check followed  
by strategic digit swapping

Begin with estimated values that roughly sum to 1000 and then try to make the numbers accordingly.

Begin with estimated values that roughly sum to 1000 and then try to make the numbers accordingly followed by strategic digit swapping

Students don't understand which digits can be swapped to without changing values.

Students don't understand which digits can be swapped to change values.



# HOW DO WE DO IT?

- Open Middle Worksheet
- Classwork
- Homework
- Assessments



# GOALS

☒ WHAT'S WRONG WITH WORKSHEETS?

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Home > [Grade 2](#) > Close to 1000

## CLOSE TO 1000

Directions: Using the digits 1 to 9 exactly one time each, place a digit in each box to make the sum as close to 1000 as possible.

$$\boxed{\phantom{000}} + \boxed{\phantom{000}} + \boxed{\phantom{000}}$$

Hint

How do you know you can't get any closer to 1000? What should be true about the hundreds places of your three numbers? How do the tens places affect your answer?

Answer

Lots of answers will get you 999. One would be  $247 + 563 + 189$ .  
Is exactly 1000 even possible? Here's a [Geogebra tool](#) you can use to check your answer.

Source: [John Ulbright](#) and [Robert Kaplinsky](#)



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# GOALS

☒ WHAT'S WRONG WITH WORKSHEETS?

☒ WHAT SHOULD WE BE DOING INSTEAD?

☒ HOW DO WE DO IT IN OUR CLASSROOMS?

☒ WHERE DO WE GET MORE PROBLEMS?

☐ WHAT COMES NEXT?



# WHAT COMES NEXT?

| Action  | Do Now | Start Planning | Don't Do |
|---|--------|----------------|----------|
| Try Open Middle problems out with your students             | ✓      |                |          |
| Find more problems I can use on the Open Middle website.    | ✓      |                |          |
| Incorporate Open Middle problems on assessments.            |        | ✓              |          |
| Replace all traditional problems with Open Middle problems. |        |                | ✓        |
| Share these resources with colleagues to make them aware.   | ✓      |                |          |



# GOALS

- ✓ **WHAT'S WRONG WITH WORKSHEETS?**
- ✓ **WHAT SHOULD WE BE DOING INSTEAD?**
- ✓ **HOW DO WE DO IT IN OUR CLASSROOMS?**
- ✓ **WHERE DO WE GET MORE PROBLEMS?**
- ✓ **WHAT COMES NEXT?**



# OPEN MIDDLE PROBLEMS

✓ ☐ WHY DO WE NEED THEM?

✓ ☐ WHY ARE THEY DIFFERENT?

✓ ☐ HOW DO YOU IMPLEMENT THEM?

✓ ☐ HOW DO YOU CREATE YOUR OWN?





Open Middle @openmiddle · Jan 11

Open Middle @openmiddle · Jan 11

Hey @openmiddle fans, we want to hear from you. Why you use our problems



**Open Middle**

@openmiddle

Hey @openmiddle fans, we want to hear from you. Why do you use our problems with your students? Share your success stories or lessons learned.

RETWEETS

7

LIKES

6



2:10 PM - 11 Jan 2017



8



7



6



1



2





# DISCUSSION TIME

- Why should we reconsider using word problems?
- Why do Open Middle problems help build conceptual understanding, lead to great conversations, and help uncover hidden misconceptions?



# GOALS

☒ **CORRECT ANSWERS = UNDERSTANDING?**

☒ **RECONSIDER USING WORD PROBLEMS**

☒ **RECONSIDER USING WORKSHEETS**





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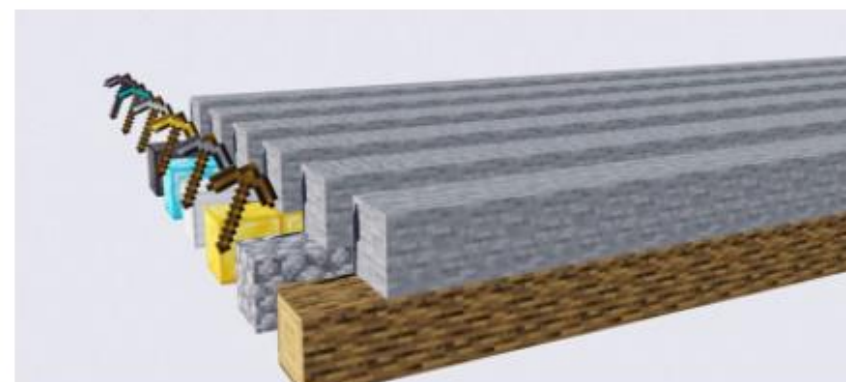
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# Scary & Dangerous











# WHY WE SHOULD RECONSIDER USING WORKSHEETS AND WORD PROBLEMS (AND WHAT WE SHOULD BE DOING INSTEAD)

**ROBERT KAPLINSKY**

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robertkaplinsky.com

@robertkaplinsky

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